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# MULTIOBJECTIVE OPTIMIZATION FOR DYNAMIC BALANCING OF FOUR-BAR MECHANISM

**Marina Bošković[1], Slaviša Šalinić[1], Radovan Bulatović[1], Goran Miodragović[2]**

[1] The Faculty of Mechanical and Civil Engineering in Kraljevo, University of Kragujevac
Dositejeva 19, 36000 Kraljevo, Serbia
e-mail: boskovic.m@mfkv.kg.ac.rs, salinic.s@mfkv.kg.ac.rs, bulatovic.r@mfkv.kg.ac.rs
[2] School of Mechanical Engineering of Applied Study Trstenik,
Radoja Krstića 19, 37240 Trstenik, Serbia
e-mail: mijac037@yahoo.com

**Abstract**

This paper presents an optimization technique for dynamic balancing of four-bar mechanism in order to minimize shaking force and shaking moment. The balancing problem is solved as multi-objective optimization problem and thus avoids the use of weighting factors. Kinematic and dynamic parameters of four-bar mechanism are taken as design variables. The eight objective functions, that contain joint reaction forces, input torque, shaking force and shaking moment, are simultaneously minimized. A new algorithm, named sub-population firefly algorithm is used for solving the optimization problem under the defined constraints. The standard FA algorithm was improved in two ways. The first improvement is related to avoidance of local minimum, and the second improvement provides satisfaction of constraints in each iteration step. By applying the proposed algorithm, a certain decrease at the shaking force and shaking moment is achieved. The effectiveness of the improved algorithm is discussed.

**Key words**: four-bar mechanism, dynamic balancing, multiobjective optimization, firefly algorithm

## 1. Introduction

Balancing of the mechanism and optimization of shaking force and shaking moment represent an important task for researchers and machine designers. By minimizing both shaking force and shaking moment improved dynamic is achieved, whereas noise and vibration are reduced.

Static balancing means balancing of shaking force [1]. For achieving dynamic balancing of mechanism, it is need to carry out static balancing [12]. Dynamic balancing means simultaneous balancing both shaking force and shaking moment [9]. The problem of balancing can be solved by using classical methods or optimization techniques.

By using counterweight method [3] or mass redistribution method [2] balancing is achieved. The method of linearly independent vectors that requires a redistribution of masses of links in such a way that the total mass center becomes stationary is proposed in [1]. Minimization of shaking force and shaking moment by finding optimum mass distribution of mechanism links using the equimomental system of point masses is considered in [4,5]. Force and moment balance of four-bar linkage by using two evolutionary algorithms named non-dominated sorting genetic algorithm and multi-objective particle swarm optimization are presented in [6]. A practical method, based on genetic algorithm, for reducing the shaking force and the shaking moment in four-bar mechanism is described in [7].

A review of various balancing methods based on the generation of different movements of counterweights is presented in [8].

In this paper, dynamic balancing of planar mechanism is considered. In contrary to [6,7], where weighting factors are used to define single objective function, the optimization problem in this paper is formulated based on multi-objective approach. Sub-population firefly algorithm is used for solving the optimization problem. The effectiveness of the proposed algorithm is shown by applying it to four-bar mechanism.

## 2. Kinematic and dynamic analysis of four-bar mechanism

Four-bar mechanism with link parameters is shown in Figure 1. Masses of the links are $m_2$, $m_3$ and $m_4$, while lengths of the links are $L_2$, $L_3$ and $L_4$. Four-bar mechanism is placed in vertical plane $Axy$. The angles $\theta_2$, $\theta_3$ and $\theta_4$ define the angular positions of mechanism links to $x$ direction, respectively.

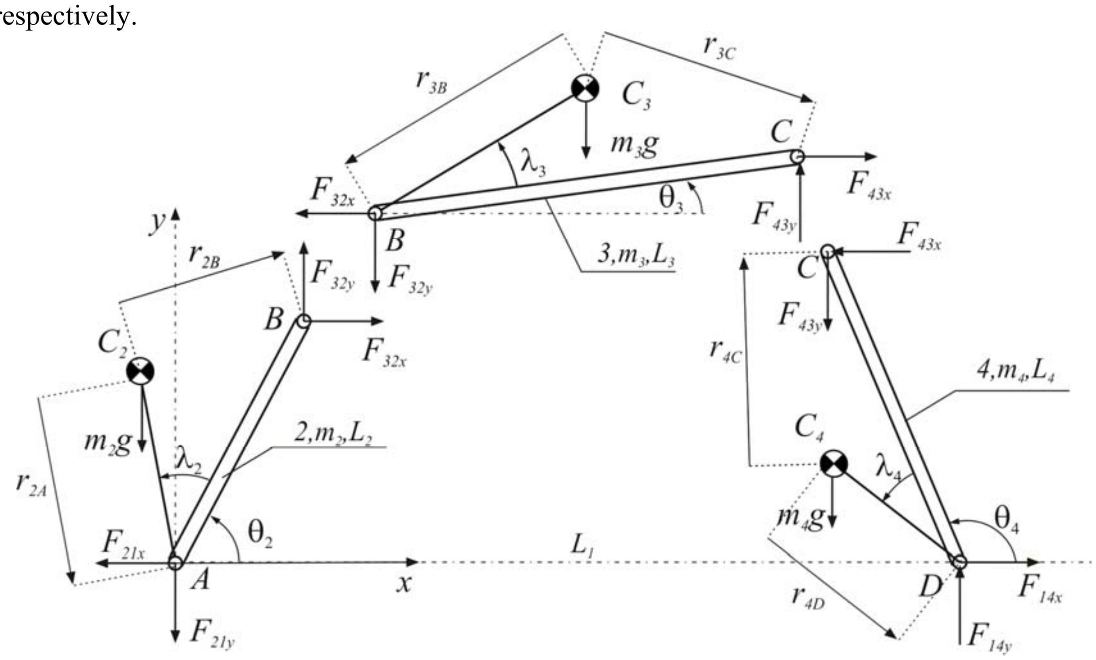


Fig. 1. Four-bar mechanism

Kinematic analysis means determination of displacements, velocities and accelerations of moving links. The expressions listed below are taken from [7] and the same are presented for better understanding of this paper. The positions of mass centers of the moving links relative to the inertial frame $Axy$ can be written as:

$$\mathbf{r_{C2}} = [x_{C2}, y_{C2}]^T = [r_{2A}\cos(\theta_2+\lambda_2), r_{2A}\sin(\theta_2+\lambda_2)]^T \tag{1}$$

$$\mathbf{r_{C3}} = [x_{C3}, y_{C3}]^T = [L_2\cos\theta_2 + r_{3B}\cos(\theta_3+\lambda_3), L_2\sin\theta_2 + r_{3B}\sin(\theta_3+\lambda_3)]^T \tag{2}$$

$$\mathbf{r_{C4}} = [x_{C4}, y_{C4}]^T = [L_1 + r_{4D}\cos(\theta_4+\lambda_4), r_{4D}\sin(\theta_4+\lambda_4)]^T \tag{3}$$

The velocities and accelerations of links mass centers are obtained taking the time-derivatives of Eqs. (1)-(3). Dynamic analysis means determination of joint reaction forces and driving torque as a function of input link angle $\theta_2$. Dynamic analysis is achieved by using differential equations of motion in the plane *Axy* as follows:

$$\mathbf{B} = \mathbf{AX} \quad \rightarrow \quad \mathbf{X} = \mathbf{A}^{-1}\mathbf{B} \tag{4}$$

where:

$\mathbf{B} = \left[ m_2\ddot{x}_{C2}, m_2\ddot{y}_{C2} + m_2 g, J_{C2}\ddot{\theta}_2, m_3\ddot{x}_{C3}, m_3\ddot{y}_{C3} + m_3 g, J_{C3}\ddot{\theta}_3, m_4\ddot{x}_{C4}, m_4\ddot{y}_{C4} + m_4 g, J_{C4}\ddot{\theta}_4 \right]^T$ ,

$\mathbf{X} = \left[ F_{21x}, F_{21y}, F_{32x}, F_{32y}, F_{43x}, F_{43y}, F_{14x}, F_{14y}, M_{21} \right]^T$ is the vector of the-joint reaction forces and the driving torque of the four-bar mechanism, and $\mathbf{A}$ is a $9\times 9$ matrix defined as:

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ r_{2Ay} & -r_{2Ax} & -r_{2By} & r_{2Bx} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & r_{3By} & -r_{3Bx} & -r_{3Cy} & r_{3Cx} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & r_{4Cy} & -r_{4Cx} & -r_{4Dy} & r_{4Dx} & 0 \end{bmatrix}.$$

Position vectors from the mass center of link *i* (*i*=2, 3, 4) to the joints *A, B, C, D* are determined from Figure 1 as:

$$\begin{bmatrix} \mathbf{r_x} & \mathbf{r_y} \end{bmatrix} = \mathbf{C} \cdot \begin{bmatrix} \mathbf{a} & \mathbf{b} \end{bmatrix} \tag{5}$$

where:

$\mathbf{r_x} = \begin{bmatrix} r_{2Ax} & r_{2Bx} & r_{3Bx} & r_{3Cx} & r_{4Cx} & r_{4Dx} \end{bmatrix}^T$; $\mathbf{r_y} = \begin{bmatrix} r_{2Ay} & r_{2By} & r_{3By} & r_{3Cy} & r_{4Cy} & r_{4Dy} \end{bmatrix}^T$

$\mathbf{C} = diag\left( \overline{AC}_2, \overline{BC}_2, \overline{BC}_3, \overline{CC}_3, \overline{CC}_4, \overline{DC}_4 \right)$;

$\mathbf{a} = \left[ -\cos(\theta_2 + \lambda_2), \cos(\theta_2 - \beta_2), -\cos(\theta_3 + \lambda_3), \cos(\theta_3 - \beta_3), \cos(\theta_4 - \beta_4), -\cos(\theta_4 + \lambda_4) \right]^T$

$\mathbf{b} = \left[ -\sin(\theta_2 + \lambda_2), \sin(\theta_2 - \beta_2), -\sin(\theta_3 + \lambda_3), \sin(\theta_3 - \beta_3), \sin(\theta_4 - \beta_4), -\sin(\theta_4 + \lambda_4) \right]^T$

The angles $\theta_3$ and $\theta_4$ define the angular positions of coupler and follower links relative to *x* direction and can be written as function of the input link position angle $\theta_2$ (see [7, 11, 12]):

$$\theta_3 = 2\arctan\left[ -\frac{B}{2A} \pm \frac{1}{2A}\left( B^2 - 4AC \right)^{1/2} \right] \tag{6}$$

$$\theta_4 = \arccos\left[ \frac{1}{L_4}\left( L_2\cos\theta_2 + L_3\cos\theta_3 - L_1\cos\theta_1 \right) \right] \tag{7}$$

where *A*, *B* and *C* are defined as:

$$A = 2L_3L_1\cos\theta_1 - 2L_2L_3\cos\theta_2 + L_1^2 + L_2^2 + L_3^2 - L_4^2 - 2L_2L_1\cos(\theta_2 - \theta_1)$$
$$B = 4L_3\left( L_2\sin\theta_2 - L_1\sin\theta_1 \right)$$

$$C = 2L_2L_3\cos\theta_2 - 2L_3L_1\cos\theta_1 + L_1^2 + L_2^2 + L_3^2 - L_4^2 - 2L_2L_1\cos(\theta_2 - \theta_1)$$

In addition, by $\beta_2$, $\beta_3$ and $\beta_4$ are denoted the angles $C_2BA$, $C_3CB$, and $C_4CD$, respectively, and these angles are determined by the following expressions [7]:

$$\beta_2 = \arccos\left(\frac{r_{22}^2 + L_2^2 - r_{21}^2}{2r_{22}L_2}\right),$$

$$\beta_3 = \arccos\left(\frac{r_{33}^2 + L_3^2 - r_{32}^2}{2r_{33}L_3}\right),$$

$$\beta_4 = \arccos\left(\frac{r_{43}^2 + L_4^2 - r_{44}^2}{2r_{43}L_4}\right).$$

The shaking force is vector sum of all the inertia forces, and the shaking moment is considered as vector sum the inertia moment and the moment of the inertia forces [10]. These forces and moment can be written as:

$$\sum F_{shx} = F_{41x} + F_{21x} \tag{8}$$

$$\sum F_{shy} = F_{41y} + F_{21y} \tag{9}$$

$$\mathbf{M_{sh}} = L_1\sin\theta_1\mathbf{j}\times\mathbf{F_{41x}}\mathbf{i} + L_1\cos\theta_1\mathbf{i}\times\mathbf{F_{41y}}\mathbf{j} \tag{10}$$

## 3. Optimization process

### *3.1 Firefly Algorithm*

Firefly algorithm was firstly introduced by X.S. Zang [13]. To create firefly algorithm, some of the flashing characteristics of firefly has to be idealized. Three idealized rules are used, [14]:

1. All fireflies are unisex so that one firefly will be attracted to other fireflies regardless of their sex,
2. Attractiveness is proportional to their brightness, thus for any two flashing fireflies, the less bright one will move towards the brighter one. Both attractiveness and brightness decrease as distance between fireflies increases. The brightest firefly moves randomly.
3. The brightness of a firefly is affected or determined by the space of objective function. For a maximization problem, the brightness can be proportional to the value of objective function.

In the FA algorithm, especially important are the variation of light intensity and formulation of the attractiveness. It is assumed that the attractiveness of a firefly is determined by its brightness which is in turn associated with objective function value [13, 15, 16].

### *3.2 Subpopulation Firefly Algorithm (SP-FA)*

Standard FA algorithm was improved in two ways. The first improvement is related to avoidance of local minimum, and the second improvement provides satisfaction of constraints in each iteration step.

Experimentation with different approaches of FA algorithm modification, in order to provide better searching of solution space and to obtain that all constraints are fulfilled in each iteration step, resulted in following modifications:

1. dividing of single fireflies population into two,
2. introduction of crossover operator,
3. continuously searching for new design variables until constraint functions are fulfilled.

Dividing of the population is carried out in each iteration step randomly. The generator of the number of fireflies is introduced in one of newly formed population. Values of design variables for each firefly are randomly placed in one of two new populations, Figure 1.

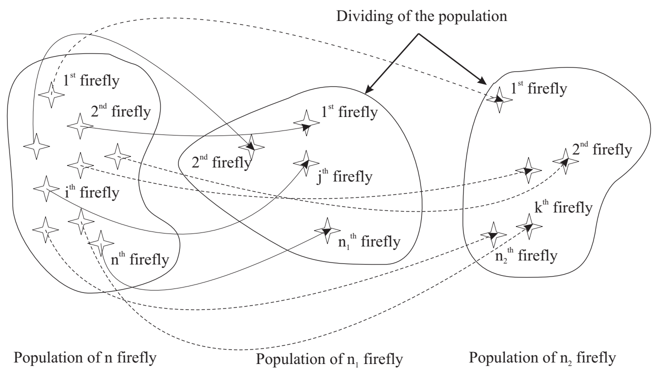


Fig. 2. Dividing single firefly population into two new populations

Where $n$ is total number of fireflies, $n_1$ is randomly generated integer of fireflies in population 1 and $n_2 = n - n_1$ is number of fireflies in population 2.

For each new population, values of objective function are determined according to standard FA algorithm. After calculation of objective function for each firefly in both populations, fireflies with best values of design variables in current iteration step, in each population are identified. Design variables are generated and value of objective function is calculated for each firefly in each iteration step. Fireflies are then ranked by value of objective function, and in each population the firefly with the best value of objective function is identified. Identification of best firefly implies determination of the best value of objective function $f_1$, and position of firefly with that value of objective function $l \rightarrow population_1(l)$, for population 1, and best value of objective function $f_2$ at position $k \rightarrow population_2(k)$ for population 2.

Next modification introduced into standard firefly algorithm had the goal to check if the space of possible solutions is well searched for each population. Basically, the idea was to switch fireflies with best values of objective function, actually, firefly's crossover is implemented. Firefly at position $l$, from population 1, replaces firefly at position $k$ in population 2 and vice versa, firefly at position $k$ from population 2 comes at position $l$ in population 1 (Algorithm 2 - lines 36,37,38). This crossover of fireflies allows population with worse solutions to get into space of better solutions, while the population with better solutions checks if the global minimum is found, or there is even better solution to search for.

After crossover of fireflies, positions of each firefly are updated in both populations, which means that firefly's positions are corrected according to "new" best firefly. Then all the fireflies are joined in one population which is again divided into two populations randomly. In that way, two new populations are created and procedure of space searching is repeated.

The third modification of standard FA provides that all solutions for each firefly in each step of iteration satisfy given constraints. The cycle of continuous search for project variables is applied in cases when all constraints are not satisfied. If the values of design variables deviate from given constraints, the firefly keeps moving in space of possible solutions until all constraints are fulfilled. In this cycle, firefly is moving according to Lèvy flight principle.

Modifications of standard FA algorithm explained above, gave the Subpopulation FA algorithm–SP-FA, which is presented by pseudocod given in Algorithm 1.

**Algorithm 1.** Subpopulation FA algorithm (SP-FA)

**begin**
Objective functions $f(\mathbf{X})$, $\mathbf{X} = (x_1, x_2, ..., x_d)^T$
Define total number of fireflies in population - n
Define the number of design variables - *d*
Define of light absorption coefficient- $\gamma$
Define the number of passes *NR*
Generating the initial firefly's population $x_i$ $(i = 1, ..., n)$
%% **1. modification –dividing the population into two**
partition_factor=rand
$n_1$=ceil(partition_factor *n) %% number of fireflies in population 1 random integer < n
$n_2 = n - n_1$
creating random array with an index of initial population: *sk*
%% Moving fireflies from original population to population 1
***for*** $i = 1 : n_1$
***population1***$(i) = x(sk(i))$
***end for*** *i*
%% Fireflies movement from original population to population 2
***for*** $i = n_1 + 1 : n$
***population2***$(i - n_1) = x(sk(i))$
***end for*** *i*
**%% End of 1st modification**
**while** $(t < NR)$
%% **3. modification checking constraint functions**
%% For population 1
***for*** $i = 1 : n_1$
***while*** not satisfied all constraints
calculate new value of project variables by Lèvy flight in ***population1***
***end while***
***end for*** *i*
%% For population 2
***for*** $i = 1 : n_2$
***while*** not satisfied all constraints
calculate new value of project variables by Lèvy flight in ***population2***
***end while***
***end for*** *i*
***%% 2. Modification – crossover of fireflies***
Light intensity determination for both populations
*Ranking of fireflies and searching for current best solution in population 1:* $f_1$ *at position l* $(1 \le l \le n_2)$.
*Ranking of fireflies and searching for current best solution in population 2:* $f_2$ *at position k* $(1 \le k \le n_1)$.
%% Crossover of fireflies with best solutions:
***temp***=***population1(l)***
***population1***(***l***) = ***population2***(***k***)
***population2***(***k***)=***temp***

```
            %% Updating the positions of fireflies in both populations
          for i = 1 : n_1 %% all n_1 fireflies from population 1
              for j = 1 : n_1 %% all n_1 fireflies from population 1
                  if (I_j^1 > I_i^1)
                      Moving firefly i towards firefly j in d-dimensional space
                      Varying attractiveness with distance r by exp[-γr]
                      Calculating the new solution and light intensity update
                  end if
              end for j
            end for i
          for i = 1 : n_2 %% all n_2 fireflies from population 2
              for j = 1 : n_2 %% all n_2 fireflies from population 2
                  if (I_j^2 > I_i^2)
                      Moving firefly i towards firefly j in d-dimensional space
                      Varying attractiveness with distance r by exp[-γr]
                      Calculating the new solution and light intensity update
                  end if
              end for j
          end for i
                    %% extension of 1st modification
            Regrouping fireflies in single population
            partition_factor =rand
    n_1 =ceil(partition_factor *n) %% number of fireflies in population 1 random integer < n
    n_2 = n - n_1
            creating random array with an index of initial population: sk
            %% Moving fireflies from original population to population 1
            for i = 1 : n_1
                    population1(i)= x(sk(i))
            end for i
            %% Moving fireflies from original population to population 2
            for i = n_1 + 1 : n
                    population2(i - n_1)= x(sk(i))
            end for i
            %% End of 1st modification
end while
Post processing and representation of results
```

### *3.3 Objective functions*

There are two different approaches in multiobjective optimization. The first one, which uses specially structured algorithms which goal is obtaining the best set of non-dominated solutions in terms of *Pareto* fronts. The second one is usually defined as multiobjective optimization, which consists of one optimization goal when different objective functions are linearly combined in unique objective function using weighting factors. The authors of this paper follow the approach which is different from two above mentioned, where optimization of several objective functions are performed simultaneously. Multi-objective optimization is applied for minimization of ground joint reaction forces of four-bar mechanism. Based on above expressions the problem is defined as follows:

$$F(\boldsymbol{X}) = minimum\left(F_1\left(\boldsymbol{X}\right), F_2\left(\boldsymbol{X}\right), ....., F_8\left(\boldsymbol{X}\right)\right), \tag{11}$$

subject to: $g_j(\boldsymbol{X}) \leq 0,\ j = 1,...,m$ where $F(\boldsymbol{X})$ is objective function, $g_j(\boldsymbol{X}) \leq 0$ are constraint functions and *m* is number of constraints. $\boldsymbol{X} = \{x_1,...,x_D\}$ represents design variables vector, and D is number of design variables. In this example, eight objective functions are simultaneously minimized:

$$f_j = \frac{1}{\delta}\sqrt{\sum_{i=0}^{\delta} F_j^2\left(t_i\right)},\ \left(j = 1,2,...,8\right), \quad \text{where } \delta = 200, \text{ and } t = 0:0.01:0.2;$$

$$f_1 = F_{21x}, f_2 = F_{21y}, f_3 = F_{41x}, f_4 = F_{41y}, f_5 = F_{shx}, f_6 = F_{shy}, f_7 = M_{sh}, f_8 = M_{21}$$

For each design variable upper and lower limit is defined. Sixteen design variables are considered, thus $\boldsymbol{X}$ is defined as:

$$\boldsymbol{X} = \{L_1, L_2, L_3, L_4, \lambda_2, \lambda_3, \lambda_4, m_2, m_3, m_4, J_{C2}, J_{C3}, J_{C4}, r_{2A}, r_{3B}, r_{4D}\}$$

and boundaries of design variables are shown in Table 1.

| Design variables | Original values | Optimized values | Boundaries |
|---|---|---|---|
| $L_1$ (mm) | 600 | **669.60** | 400-700 |
| $L_2$ (mm) | 100 | **54.76** | 50-130 |
| $r_{2A}$ (mm) | 50 | **74.30** | 20-100 |
| $m_2$ (kg) | 0.360 | **1** | 0-3 |
| $J_{C2}$ ($kgm^2$) | $4.13*10^{-4}$ | **$53.01*10^{-4}$** | $2*10^{-4}$-$60*10^{-4}$ |
| $\lambda_2$ (rad) | 0 | **3.09** | 0-6 |
| $L_3$ (mm) | 400 | **600** | 200-600 |
| $r_{3B}$ (mm) | 200 | **10.26** | 10-400 |
| $m_3$ (kg) | 1.296 | **1.21** | 0.8-1.6 |
| $J_{C3}$ ($kgm^2$) | $1.87*10^{-2}$ | **$2.42*10^{-2}$** | $2*10^{-2}$ - $7*10^{-2}$ |
| $\lambda_3$ (rad) | 0 | **2.89** | 0-3 |
| $L_4$ (mm) | 320 | **471.72** | 200-500 |
| $r_{4D}$ (mm) | 160 | **250** | 50-250 |
| $m_4$ (kg) | 1.046 | **0.50** | 0.5-1.5 |
| $J_{C4}$ ($kgm^2$) | $9.85*10^{-3}$ | **$9.94*10^{-3}$** | $8*10^{-3}$-$18*10^{-3}$ |
| $\lambda_4$ (rad) | 0 | **0.036** | 0.00001-2 |

Table 1. Original and optimized values of design variables of four-bar mechanism

## 4. Results

In this paper, the problem of dynamic balancing of the four-bar mechanism was considered as multiobjective optimization problem. Thus avoids the use of weighting factors. Sub-population firefly algorithm was used for solving the optimization problem. Driving link 2 rotates with a constant speed of 300 rpm. Design variables of the proposed mechanism were also defined. The original values (taken from [7]) and optimized values of design variables as well as boundaries of variables are given in Table 1. Joint reaction forces of original and optimized mechanism are given in Fig.3. After the optimization process, the optimized values of these forces are certain smaller than the original. Using above mentioned optimization algorithm the reduction of 99.256%, 71.883%, 92.907% and 85.120% is achieved in values of $F_{21x}, F_{21y}, F_{41x}$ and $F_{41y}$, respectively. Similarly, as shown in Fig. 4, the application of sub-population firefly algorithm results in reduction of 96.777% and 75.848% in values of shaking force for x and y directions, and 83.393% and 97.536% in values of shaking moment and driving torque, respectively.

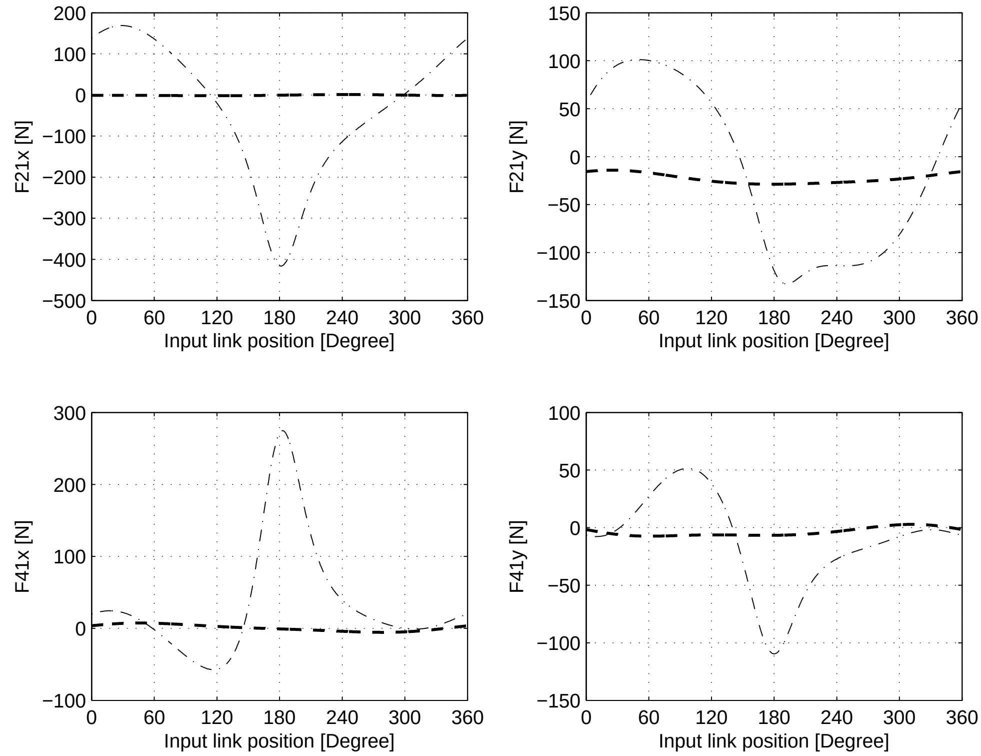


Fig. 3. Original and optimized values of ground joint reaction forces

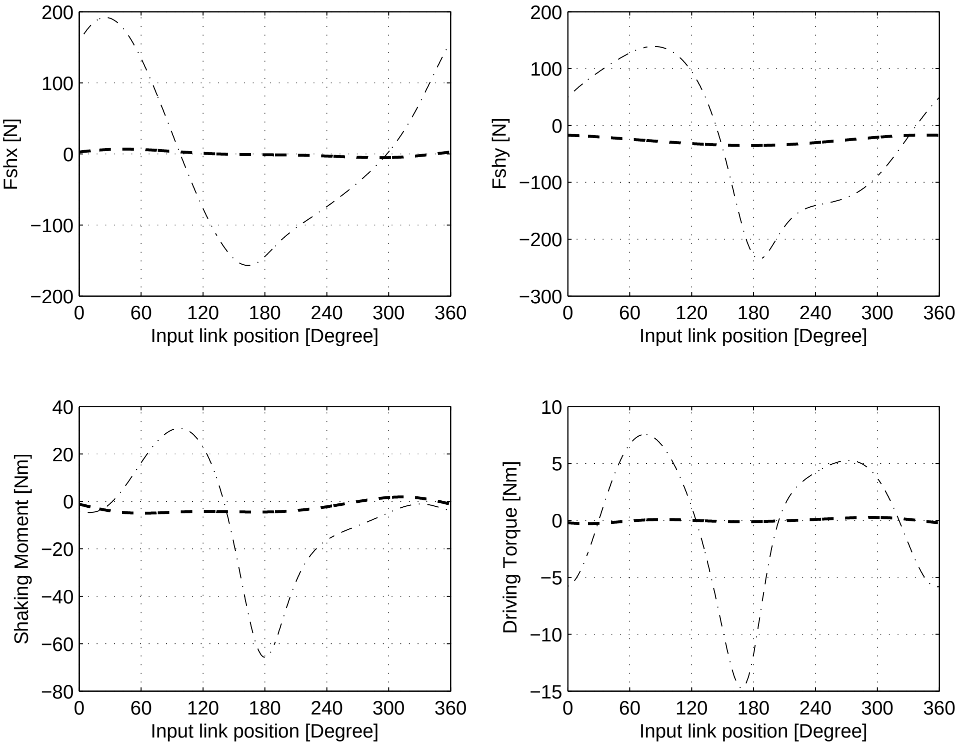


Fig. 4. Original and optimized values of shaking force components, shaking moment and driving torque

## 5. Conclusions

In this paper, the problem of minimizing shaking force and shaking moment of four-bar mechanism is presented. This problem is solved as multi-objective optimization problem. The weighting factors are not used, as in the case in [7]. The eight objective functions that optimize the mass distribution of each link are simultaneously minimized. Improved FA algorithm, named sub-population firefly algorithm, was applied for minimizing objective functions which consist of ground joint reaction forces, shaking force, shaking moment and input torque. The improvement of standard firefly algorithm is reflected in following: avoidance of local minimum and satisfaction of constraints in each iteration step. The obtained results show a certain reduction in values of shaking force and moment, as well as reduction of ground joint reaction forces and driving torque. The proposed algorithm and multi-objective approach can be applied for multi-loop planar mechanisms.

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