# MINIMIZATION OF JOINT REACTION FORCES OF THE 2-DOF PLANAR SERIAL MANIPULATORS

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# MINIMIZATION OF JOINT REACTION FORCES OF THE 2-DOF PLANAR SERIAL MANIPULATORS

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## Abstract

This paper presents an approach for the minimization of joint reaction forces of a two degrees of freedom (2-DOF) planar serial manipulator. The joint reaction forces due to inertia forces in the manipulator (dynamic joint reaction forces) are minimized using the differential evolution algorithm (DE). The expressions for the dynamic joint reaction forces as well as driving torques in the joints are obtained in a symbolic form by means of the Lagrange equations of motion. The inertial properties of the manipulator links are represented by dynamical equivalent systems of two point masses. A single objective function in the form of the root mean square magnitude of the dynamic joint reactions is used. Three different ways of the minimizing of the dynamic joint reactions are presented. The first way is by attaching counterweights to the links, the second one is based on the lengths of the links as the design variables, and the third one represents the joint reaction forces minimization via the optimal selection of the angular rotations laws of the manipulator links. The effectiveness of the proposed ways of the joint reaction forces minimization is discussed.

**Key words**: serial manipulator, equimomental system, joint reactions, optimization, differential evolution

## 1. Introduction

Determination and optimization of joint reaction forces in various mechanisms in industry represent important tasks. Joint reaction forces directly influence on the stress state and friction forces in joints.

The references, which consider the problem of minimization reaction forces in the joints of manipulators, are quite rare. So in [1] the minimization of joint forces in planar kinematic chains was considered. On the other hand, using Routh's idea [2] for representation of a rigid body by a dynamically equivalent system of point-masses (also known as equimomental system), the minimization of joint reactions in industrial spatial manipulators was studied in the reference [3]. Note that Routh's idea also is applied in solving the problem of balancing of mechanisms (see [4,5,6]).

In this paper, the minimization of joint reaction forces in a two degrees of freedom (2-DOF) planar serial manipulator are considered. In contrary to [1], where the multi-objective approach is used, the optimization problem in this paper is formulated based on the single objective function approach. The derivation of symbolic expressions for joint reactions is based on equimomental system representation of the manipulator links and the approach for the determination of constraint reactions described in [7,8]. For the other methods of determination of joint reactions see [9] and the references cited there. Note that the type of manipulator considered in this paper is also considered in [10] where the input torques minimization was analyzed.

## 2. Dynamics of a two-link planar manipulator using equimomental systems of two point-masses

Let us consider a 2-DOF planar serial manipulator shown in Figure 1. The manipulator links connected to each other and to the base via revolute joints are modeled as homogeneous beams. Masses of the links are $m_1$ and $m_2$, while lengths of the links are $L_1$ and $L_2$. The manipulator is placed in the vertical plane $Oxy$. By the angles $\varphi_1$ and $\varphi_2$ depicted in Figure 1 are denoted the absolute angles of rotation of the manipulator links.

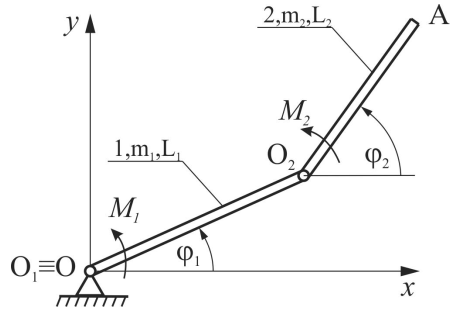


Fig.1. 2-DOF planar serial manipulator

Torques $M_1$ and $M_2$ act in the manipulator joints $O_1$ and $O_2$, respectively. Each of the manipulator links can be, according to [11], represented by a dynamically equivalent system (equimomental system) of two point masses as it is shown in Figure 2. In Figure 2, $J_1$ and $J_2$ are the centroidal moments of inertia in directions normal to the links 1 and 2, respectively. Note that the manipulator link and the two point mass system introduced are dynamically equivalent (equimomental) becose they have the same mass, the same mass center, and the same inertia tensor determined with respect to the mass center of the link (see [2]). For the representation of the manipulator links by equimomental systems containing three point-masses see the reference [12].

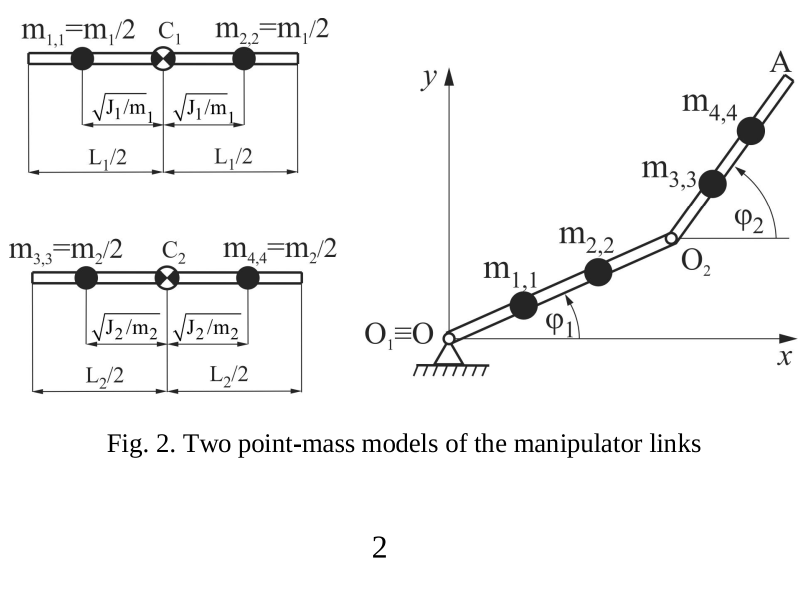


Fig. 2. Two point-mass models of the manipulator links

Using the equimomental system of two point-masses representation, the kinetic energy of the manipulator can be written as:

$$T = \frac{1}{2}\sum_{i=1}^{4} m_{i,i}(\dot{x}_i^2 + \dot{y}_i^2)\,, \tag{1}$$

where $x_i (i = 1,\ldots,4)$ and $y_i (i = 1,\ldots,4)$ are the Cartesian coordinates of point masses with respect to the inertial frame $Oxy$. Here overdot denotes the derivative with respect to time. The coordinates of point masses are expresed in terms of angles $\varphi_1$ and $\varphi_2$ after that, using the Lagrange equations of the second kind [8], the differential equations of motion of the manipulator can be formed as follows:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}_i}\right) - \frac{\partial T}{\partial \varphi_i} = Q_{\varphi_i}\,,\;\; i = 1,2\,, \tag{2}$$

where $Q_{\varphi_i} (i = 1, 2)$ are the generalized forces corresponding to the generalized coordinates $\varphi_i (i = 1,2)$ .

**3. Determination of dynamic joint reaction forces**

Since dynamic joint reaction forces are considered, in the further consideration $g = 0$ it is taken out, where $g$ is the gravitational acceleration. In this paper, the determination of dynamic joint reaction forces of the manipulator considered is based on the general method for determination of constraint reaction forces described in [7, 8]. According to [7, 8], to determine reaction forces in the joints $O_1$ and $O_2$, these joints are cut imaginary (see Figure 3).

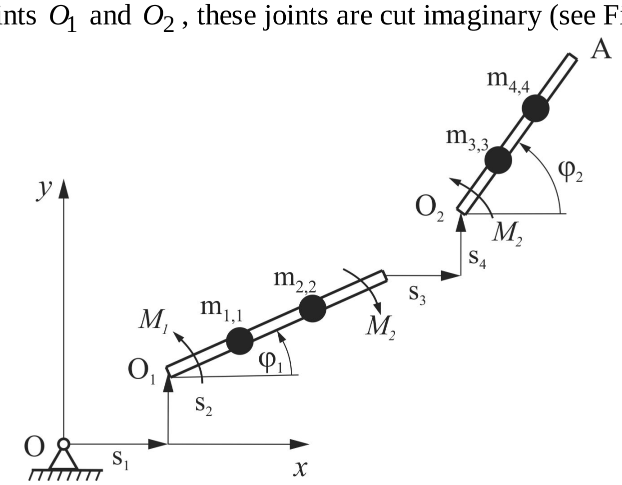


Fig. 3. Imaginary cutting of the manipulator joints

After the joints have been cut the number of degrees of freedom of the manipulator is increased by four. In regard to this, four new coordinates, $s_1, s_2, s_3$, and $s_4$, are involved that referred to the prohibited motions in the joints as it is shown in Figure 3. Now, the manipulator motion can be observed as the motion with the redundant coordinates $s_1,\ldots,s_4$ subject to the following constraints:

$$f_i \equiv s_i = 0,\ i = 1,\ldots,4\ . \tag{3}$$

After that, using the Lagrange equations with the multipliers [8], the differential equations of motion of the manipulator dipected in Figure 3 read:

$$\frac{d}{dt}\left(\frac{\partial T^*}{\partial \dot{\varphi}_i}\right) - \frac{\partial T^*}{\partial \varphi_i} = Q^*_{\varphi_i},\ i = 1,2, \tag{4}$$

$$\frac{d}{dt}\left(\frac{\partial T^*}{\partial \dot{s}_j}\right) - \frac{\partial T^*}{\partial s_j} = Q^*_{s_j} + \sum_{r=1}^{4} \lambda_r \frac{\partial f_r}{\partial s_j},\ j = 1,\ldots,4\ , \tag{5}$$

where $T^* = T^*(\varphi_i, \dot{\varphi}_i, s_j, \dot{s}_j)(i = 1,2; j = 1,\ldots,4)$ and $\lambda_r (r = 1,\ldots,4)$ are the Lagrange multipliers of constraints. Since it is assumed that $g = 0$, for the manipulator considered one have that $Q^*_{\varphi_i} = M_i$ and $Q^*_{s_j} = 0$. Further, substituting the following relations:

$$s_j(t) \equiv 0,\quad \dot{s}_j(t) \equiv 0,\quad \ddot{s}_j(t) \equiv 0,\ j = 1,\ldots,4\ , \tag{6}$$

which follow from (3), into Eqs. (4) and (5), Eq. (4) yields the equations of motion of the manipulator immediately before cutting of the joints, while Eq. (5) yields expressions for the components of dynamic joint reaction forces as follows:

$$X_{O_1} = (\lambda_1)_0 = \frac{1}{2}\left[-L_2 m_2 \dot{\varphi}_2^2 \cos\varphi_2 - L_1(m_1 + 2m_2)\left(\dot{\varphi}_1^2 \cos\varphi_1 + \ddot{\varphi}_1 \sin\varphi_1\right) - L_2 m_2 \ddot{\varphi}_2 \sin\varphi_2\right], \tag{7}$$

$$Y_{O_1} = (\lambda_2)_0 = \frac{1}{2}\left[-L_2 m_2 \dot{\varphi}_2^2 \sin\varphi_2 - L_1(m_1 + 2m_2)\left(\dot{\varphi}_1^2 \sin\varphi_1 - \ddot{\varphi}_1 \cos\varphi_1\right) + L_2 m_2 \ddot{\varphi}_2 \cos\varphi_2\right], \tag{8}$$

$$X_{O_2} = (\lambda_3)_0 = -\frac{1}{2} m_2 \left(2L_1\dot{\varphi}_1^2 \cos\varphi_1 + L_2\dot{\varphi}_2^2 \cos\varphi_2 + 2L_1\ddot{\varphi}_1 \sin\varphi_1 + L_2\ddot{\varphi}_2 \sin\varphi_2\right), \tag{9}$$

$$Y_{O_2} = (\lambda_4)_0 = \frac{1}{2} m_2 \left(-2L_1\dot{\varphi}_1^2 \sin\varphi_1 - L_2\dot{\varphi}_2^2 \sin\varphi_2 + 2L_1\ddot{\varphi}_1 \cos\varphi_1 + L_2\ddot{\varphi}_2 \cos\varphi_2\right), \tag{10}$$

where the notation $(\bullet)_0$ means that the quantity $(\bullet)$ is calculated for $s_i(t) \equiv 0 (i = 1,\ldots,4)$. Now, the magnitudes of reaction forces in joints $O_1$ and $O_2$ are $R_{O_1} = \sqrt{X^2_{O_1} + Y^2_{O_1}}$ and $R_{O_2} = \sqrt{X^2_{O_2} + Y^2_{O_2}}$, respectively.

## 4. Formulation of the optimization problem

In contrary to [1], where the multi-objective functions approach is used, the single objective function approach is applied in this paper. An objective function is defined as follows:

$$F = \frac{1}{\delta}\sqrt{\sum_{i=0}^{\delta} f^2(t_i)}\ , \tag{11}$$

where $\delta$ is the number of discrete positions of the manipulator over the interval $(0, t_f]$, $t_f$ represents time for which the motion has to be completed, $f(t_i) = \sqrt{X_{O_1}^2(t_i) + Y_{O_1}^2(t_i) + X_{O_2}^2(t_i) + Y_{O_2}^2(t_i)}$, and $t_i = (i\, t_f) / \delta$ is the instant corresponding to the *i*th discrete position of the manipulator. Now, the optimization problem consists in finding design variables $b_i\,(i = 1, \ldots, p)$ collecccted in the column matrix $\mathbf{b} = [b_1, \ldots, b_p]^T$ that minimize the objective function (11) and subject to equality constraints:

$$g_j(\mathbf{b}) = 0,\ j = 1, \ldots, p_1, \tag{12}$$

as well as inequality constraints:

$$b^* \le h_k(\mathbf{b}) \le b^{**},\ k = 1, \ldots, p_2 \tag{13}$$

where $b^*$ and $b^{**}$ are constants. The differential evolution [13] is applied to minimize the objective function (11) subject to the constraints (12) and (13). The differential evolution represents an evolutionary optimization technique, which is simple for using, fast in convergence to the global minimum solution and has many other advantages as compared to the conventional optimization algorithm (for more details see [13]). In the further numerical computations the following values of the control variables of the differential evolution are used: crossover probability constant $CR = 0.5$, scaling factor $MF = 0.6$, starting value for the random number generator is equal to zero.

## 5. Analysis of some ways for the minimization of joint reaction forces

Numerical simulations in this section concerning to the unoptimized (original) manipulator were carried out for the same values of the manipulator parameters as in [12], that is: $m_1 = m_2 = 1\,\mathrm{kg}$, $L_1 = L_2 = 1\,\mathrm{m}$, $t_f = 10\,\mathrm{s}$, $\varphi_1(0) = \varphi_2(0) = 0$, $\varphi_1(t_f) = \pi$, $\varphi_2(t_f) = 3\pi/2$, $\varphi_i(t) = \varphi_i(0) + \left(\varphi_i(t_f) - \varphi_i(0)\right)\left[t - \left(t_f/(2\pi)\right)\sin\left(2\pi t/t_f\right)\right]/t_f$, $i = 1,2$. Also, it is taken that $\delta = 200$. The initial and final positions of the endpoint $A$ of the link 2 are determined, respectively, by the coordinates $(2, 0)$ and $(-1, -1)$. These positions are the same for both the optimized and unoptimized manipulators. For these values of the parameters, the function $F$ has the value $F^* = 0.052178\ \mathrm{N}$. Note that at the initial and terminal instants of motion the angular speed and the angular acceleration of links are equal to zero.

### *5.1 Using counterweights to minimize joint reaction forces*

In the counterweight method, counterweights are attached to the manipulator links in order to minimize joint reaction forces. Let us denote by $m_{1d}$ and $m_{2d}$ the masses of counterweights. The locations of the counterweights with respect to the manipulator links are specified by the distances $r_i\,(i = 1, 2)$ between the mass centers of counterweights and the joints as well as by the angles $\theta_i\,(i = 1, 2)$ (see Figure 4).

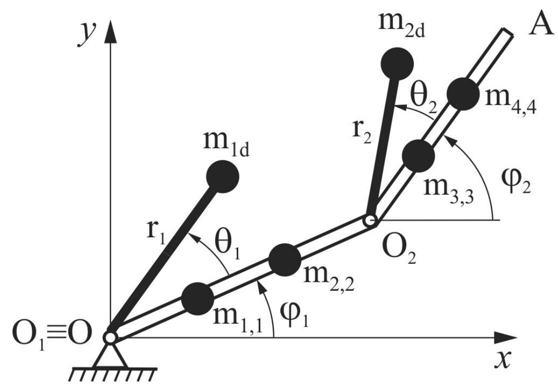


Fig. 4. Counterweights added to the links to achieve minimization of joint reaction forces

Note that the problem of balancing of robot mechanisms in [14] as well as the torque minimization of the 2-DOF manipulators in [10] were solved under the assumption that $\theta_i = 0 (i = 1, 2)$. For the considered way of the minimization, the components of the joint reaction forces (7)-(10) are slightly modified as follows:

$$\begin{aligned} X_{O_1} &= \frac{1}{2}\Big\{-\left[L_1(m_1+2m_2+2m_{2d})\cos\varphi_1 - 2r_1m_{1d}\cos(\theta_1+\varphi_1)\right]\dot{\varphi}_1^2 \\ &+\left[-L_2m_2\cos\varphi_2 + 2r_2m_{2d}\cos(\theta_2+\varphi_2)\right]\dot{\varphi}_2^2 \\ &-\left[L_1(m_1+2m_2+2m_{2d})\sin\varphi_1 - 2r_1m_{1d}\sin(\theta_1+\varphi_1)\right]\ddot{\varphi}_1 \\ &+\left[-L_2m_2\sin\varphi_2 + 2r_2m_{2d}\sin(\theta_2+\varphi_2)\right]\ddot{\varphi}_2\Big\}, \end{aligned} \tag{14}$$

$$\begin{aligned} Y_{O_1} &= \frac{1}{2}\Big\{-\left[L_1(m_1+2m_2+2m_{2d})\sin\varphi_1 - 2r_1m_{1d}\sin(\theta_1+\varphi_1)\right]\dot{\varphi}_1^2 \\ &+\left[-L_2m_2\sin\varphi_2 + 2r_2m_{2d}\sin(\theta_2+\varphi_2)\right]\dot{\varphi}_2^2 \\ &+\left[L_1(m_1+2m_2+2m_{2d})\cos\varphi_1 - 2r_1m_{1d}\cos(\theta_1+\varphi_1)\right]\ddot{\varphi}_1 \\ &+\left[L_2m_2\cos\varphi_2 - 2r_2m_{2d}\cos(\theta_2+\varphi_2)\right]\ddot{\varphi}_2\Big\}, \end{aligned} \tag{15}$$

$$\begin{aligned} X_{O_2} &= \frac{1}{2}\Big\{-2L_1(m_2+m_{2d})\dot{\varphi}_1^2\cos\varphi_1 + \left[-L_2m_2\cos\varphi_2 + 2r_2m_{2d}\cos(\theta_2+\varphi_2)\right]\dot{\varphi}_2^2 \\ &-2L_1(m_2+m_{2d})\ddot{\varphi}_1\sin\varphi_1 + \left[-L_2m_2\sin\varphi_2 + 2r_2m_{2d}\sin(\theta_2+\varphi_2)\right]\ddot{\varphi}_2\Big\}, \end{aligned} \tag{16}$$

$$\begin{aligned} Y_{O_2} &= -L_1(m_2+m_{2d})\dot{\varphi}_1^2\sin\varphi_1 + \left[-\frac{1}{2}L_2m_2\sin\varphi_2 + r_2m_{2d}\sin(\theta_2+\varphi_2)\right]\dot{\varphi}_2^2 \\ &+(m_2+m_{2d})\ddot{\varphi}_1L_1\cos\varphi_1 + \frac{1}{2}\left[L_2m_2\cos\varphi_2 - 2r_2m_{2d}\cos(\theta_2+\varphi_2)\right]\ddot{\varphi}_2\,. \end{aligned} \tag{17}$$

Here the set of design variables is defined as:

$$\mathbf{b} = \left[m_{1d}, m_{2d}, r_1, r_2, \theta_1, \theta_2\right]^T \tag{18}$$

and only the inequality constraints (13) exist, that is:

$$0.3 \le m_{1d} \le 5.0,\ \ 0.3 \le m_{2d} \le 5.0,\ \ 0 \le r_1 \le 1.0,\ \ 0 \le r_2 \le 1.0,$$
$$0 \le \theta_1 \le 2\pi,\ \ 0 \le \theta_2 \le 2\pi. \tag{19}$$

Solving the optimization problem (11)-(13) by means of the differential evolution yields the following optimal values of the design variables:

$$m_{1d} = 1.77153 \text{ kg},\quad m_{2d} = 1.1734 \text{ kg},\quad r_1 = 0.659974 \text{ m},$$
$$r_2 = 1.0 \text{ m},\quad \theta_1 = 0.0645383,\quad \theta_2 = 5.76928. \tag{20}$$

For these optimal values, the minimum of the objective function equals $0.017238\,\text{N}$ what means that one has the reduction of $66.96\%$ in the value of the objective function with respect to the value $F^*$. The magnitudes $R_{O_1}$ and $R_{O_2}$ versus time are shown in Figure 5.

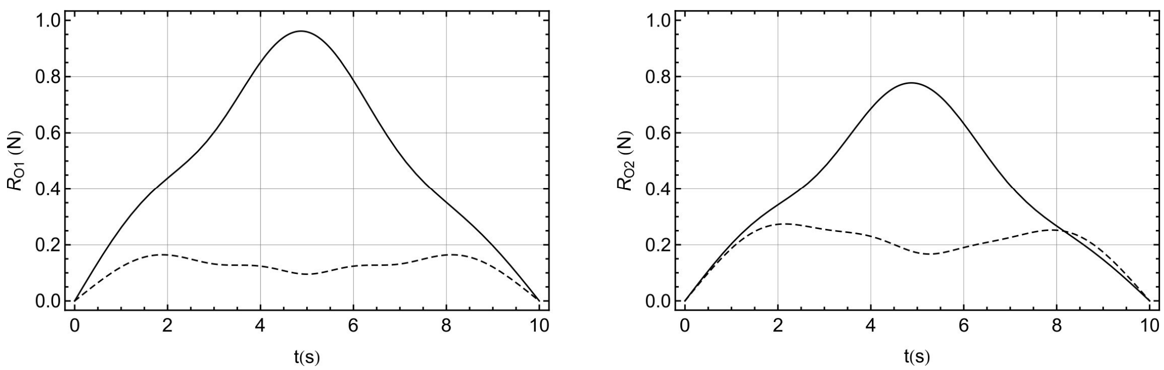


Fig. 5. Magnitudes of joint reaction forces: original (solid line) and minimized (dashing line)

*5.2 Lengths of the manipulator links as design variables*

In this case, the design variables are:

$$\mathbf{b} = \left[L_1,\, L_2,\, \varphi_{10},\, \varphi_{20},\, \varphi_{1f},\, \varphi_{2f}\right]^T \tag{21}$$

where $\varphi_{10} = \varphi_1(0)$, $\varphi_{20} = \varphi_2(0)$, $\varphi_{1f} = \varphi_1(t_f)$, and $\varphi_{2f} = \varphi_2(t_f)$. Now, the masses of the links can be expressed as $m_1 = \mu_1 L_1$ and $m_2 = \mu_2 L_2$ where $\mu_1$ and $\mu_2$ are the masses per unit length of the links 1 and 2, respectively. In the further computations it is taken that $\mu_1 = \mu_2 = 1\,\text{kg/m}$. Also, the following constraints are proposed:

$$0.2 \le L_1 \le 2.0, \qquad 0.2 \le L_2 \le 2.0,$$
$$0 \le \varphi_{10} \le 2\pi, \quad 0 \le \varphi_{20} \le 2\pi, \quad 0 \le \varphi_{1f} \le 2\pi, \quad 0 \le \varphi_{2f} \le 2\pi,$$
$$L_1 \cos\varphi_{10} + L_2 \cos\varphi_{20} - 2 = 0, \qquad L_1 \sin\varphi_{10} + L_2 \sin\varphi_{20} = 0, \tag{22}$$
$$L_1 \cos\varphi_{1f} + L_2 \cos\varphi_{2f} + 1 = 0, \qquad L_1 \sin\varphi_{1f} + L_2 \sin\varphi_{2f} + 1 = 0.$$

For the case considered, the following solution is obtained:

$$L_1 = 0.592853\,\text{m}, \; L_2 = 1.58075\,\text{m}, \; \varphi_{10} = 0.674097,$$
$$\varphi_{20} = 6.04689, \; \varphi_{1f} = 2.26825, \; \varphi_{2f} = 4.30987, \; F_{\min} = 0.017008\ \text{N}. \tag{23}$$

Using this way of the minimization one has the reduction of $67.4\%$ in the value of the objective function with respect to the value $F^*$ and the magnitudes $R_{O_1}$ and $R_{O_2}$ versus time are shown in Figure 6.

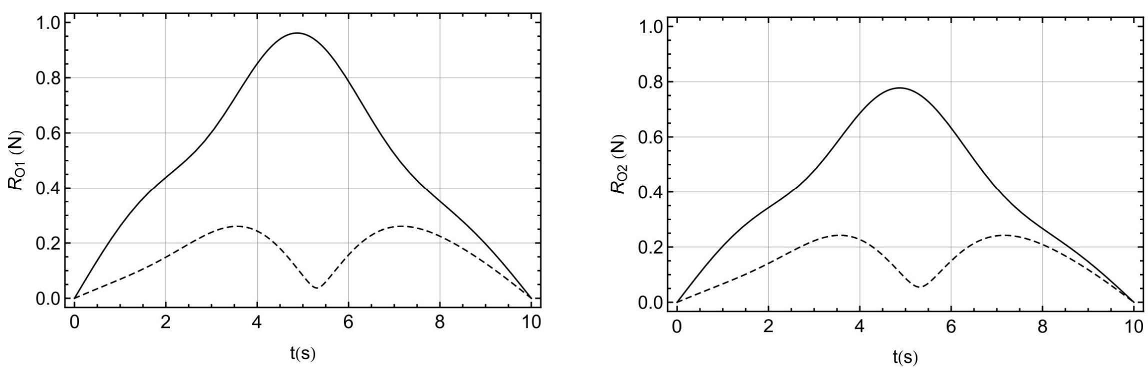


Fig. 6. Magnitudes of joint reaction forces: original (solid line) and minimized (dashing line)

*5.3 Optimal selection of the angular rotations laws of the manipulator links*

Similarly as in [15], let us assumed the angular rotations laws in forms of polynomials with constant coefficients as follows:

$$\varphi_1(t) = a_6 t^6 + a_5 t^5 + a_4 t^4 + a_3 t^3 + a_2 t^2 + a_1 t + a_0, \tag{24}$$

$$\varphi_2(t) = b_6 t^6 + b_5 t^5 + b_4 t^4 + b_3 t^3 + b_2 t^2 + b_1 t + b_0. \tag{25}$$

From the conditions $\varphi_1(0) = \varphi_2(0) = 0$, $\dot{\varphi}_1(0) = \dot{\varphi}_2(0) = 0$, and $\ddot{\varphi}_1(0) = \ddot{\varphi}_2(0) = 0$ it follows that $a_j = 0 (j = 0,\dots,2)$ and $b_j = 0 (j = 0,\dots,2)$, while the conditions $\varphi_1(t_f) = \pi$, $\varphi_2(t_f) = 3\pi / 2$, $\dot{\varphi}_1(t_f) = \dot{\varphi}_2(t_f) = 0$, and $\ddot{\varphi}_1(t_f) = \ddot{\varphi}_2(t_f) = 0$ implye the following constraints on the polynomials coefficients:

$$10^3 a_3 + 10^4 a_4 + 10^5 a_5 + 10^6 a_6 = \pi, \tag{26}$$

$$10^3 b_3 + 10^4 b_4 + 10^5 b_5 + 10^6 b_6 = 3\pi / 2, \tag{27}$$

$$3 \cdot 10^2 a_3 + 4 \cdot 10^3 a_4 + 5 \cdot 10^4 a_5 + 6 \cdot 10^5 a_6 = 0, \tag{28}$$

$$3 \cdot 10^2 b_3 + 4 \cdot 10^3 b_4 + 5 \cdot 10^4 b_5 + 6 \cdot 10^5 b_6 = 0, \tag{29}$$

$$60 a_3 + 1200 a_4 + 2 \cdot 10^4 a_5 + 3 \cdot 10^5 a_6 = 0, \tag{30}$$

$$60 b_3 + 1200 b_4 + 2 \cdot 10^4 b_5 + 3 \cdot 10^5 b_6 = 0. \tag{31}$$

The design variables are defined as:

$$\mathbf{b} = [a_3, \ldots, a_6, b_3, \ldots, b_6]^T . \tag{32}$$

The optimization problem is solved under the equivality constraints (26)-(31) and the following inequality constraints:

$$-2 \le a_i \le 2, \qquad -2 \le b_i \le 2, \qquad i = 3, \ldots, 6. \tag{33}$$

In this case, the solution of the minimizatio problem reads:

$$\begin{aligned} &a_6 = 0.0000361118, \quad a_5 = \text{-}0.000894858, \quad a_4 = 0.00612114, \\ &a_3 = \text{-}0.00469584, \quad b_6 = \text{-}0.0000581389, \quad b_5 = 0.00202691, \\ &b_4 = \text{-}0.0245103, \quad b_3 = 0.105263, \quad F_{\min} = 0.0292009. \end{aligned} \tag{34}$$

On this way the reduction of $44.14\%$ in the value of the objective function is achieved.

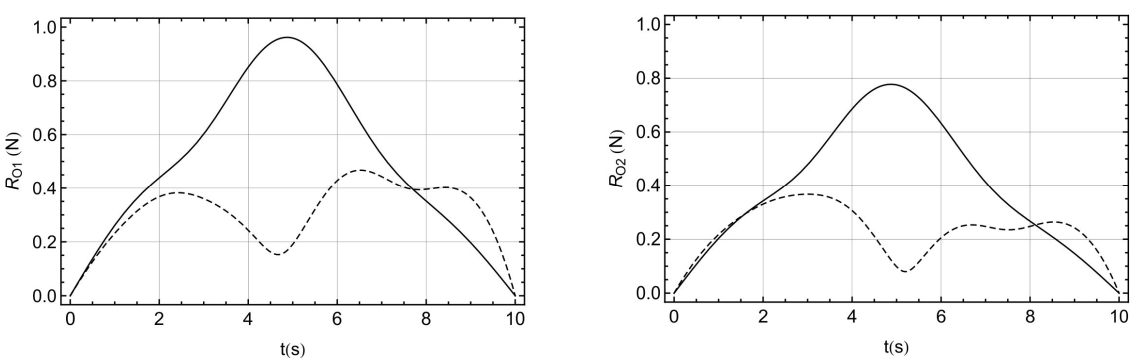


Fig. 7. Magnitudes of joint reaction forces: Original (solid line) and minimized (dashing line)

## 6. Conclusions

This paper presents a method for the determination of joint reaction forces in 2-DOF planar serial manipulators in a symbolic form based on the use of the Lagrange equations with multipliers and an equimomental system consisting of two point masses. The method is applied to solve the problem of minimization of dynamic joint reaction forces. Using the differential evolution algorithm, the optimum solutions corresponding to various kind of design variables are obtained. Depending on the choice of design variables, the evaluation of three methods for minimization of joint reaction forces has been conducted. The results of the evaluation show that all three ways of the minimization have high level of the reduction of the magnitude of joint reaction forces. The proposed method of determination of joint reaction forces is computationally more efficient than the method from [9] because of the use of equimomental system representation of the manipulator. The obtained results are valuable for the improvement of the dynamic performance of the considered type of manipulators.

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