

MINIMIZATION OF DYNAMIC JOINT REACTION FORCES OF THE 2-DOF SERIAL MANIPULATORS BASED ON INTERPOLATING POLYNOMIALS AND COUNTERWEIGHTS

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ABSTRACT. This paper presents two ways for the minimization of joint reaction forces due to inertia forces (dynamic joint reaction forces) in a two degrees of freedom (2-DOF) planar serial manipulator. The first way is based on the optimal selection of the angular rotations laws of the manipulator links and the second one is by attaching counterweights to the manipulator links. The influence of the payload carrying by the manipulator on the dynamic joint reaction forces is also considered. The expressions for the joint reaction forces are obtained in a symbolic form by means of the Lagrange equations of motion. The inertial properties of the manipulator links are represented by dynamical equivalent systems of two point masses. The weighted sum of the root mean squares of the magnitudes of the dynamic joint reactions is used as an objective function. The effectiveness of the two ways mentioned is discussed.

1. Introduction

Determination and optimization of joint reaction forces in various mechanisms in industry represent important tasks. Joint reaction forces directly influence the stress state and friction forces in joints. The references, which consider the problem of minimization reaction forces in the joints of manipulators, are quite rare. So, in [1] the minimization of joint forces in planar kinematic chains was considered. On the other hand, using Routh's idea [2] for representation of a rigid body by a dynamically equivalent system of point masses (also known as equimomental system), the minimization of joint reactions in industrial spatial manipulators was studied in the reference [3]. Note that Routh's idea also is applied in solving the problem of balancing of mechanisms (see [4–6]).

In this paper, the minimization of joint reaction forces in a two degrees of freedom (2-DOF) planar serial manipulator is considered. Note that the type of

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manipulator considered in this paper is also considered in [7–9]. Using the method of the optimal redistribution of links masses, the minimization of the driving torques of the 2-DOF serial manipulator was considered in [7]. The same optimization problem was considered in [8] by using the counterweights method [10,11]. On the other hand, the problem of decoupling of dynamic equations of the manipulator was analyzed in [9].

The objective of our paper is to evaluate two methods for minimization of dynamic joint reaction forces. One method is based on the use of interpolating polynomials [12] and the second one uses counterweights attached to the manipulator links. To the authors' best knowledge of the literature, these two methods were not applied in the available literature in order to reduce dynamic constraint forces in the all joints of the manipulator. Namely, in [3] the method of optimal redistribution of link masses was used, while in [1] the reduction of joint reactions was achieved by a proper choice of the lengths of the manipulator links. For this purpose a new method for the determination of joint constraint forces in a symbolic form is proposed. The method is based on the use of the Lagrange equations with the multipliers [13,14], velocity transformations [15], and the equimomental system representation of the manipulator links. For the other methods of determination of joint reactions see [16] and the references cited there.

2. Dynamics of a two-link planar manipulator using equimomental systems of two point masses and velocity transformation

Let us consider a 2-DOF planar serial manipulator shown in Figure 1. The manipulator links connected to each other and to the base via revolute joints are modeled as homogeneous beams. Masses of the links are m_1 and m_2 , while lengths of the links are L_1 and L_2 . The manipulator is placed in the vertical plane Oxy and carries a payload A of mass m_p . By the angles φ_1 and φ_2 depicted in Figure 1 are denoted the angular displacements of the manipulator links relative to the base.

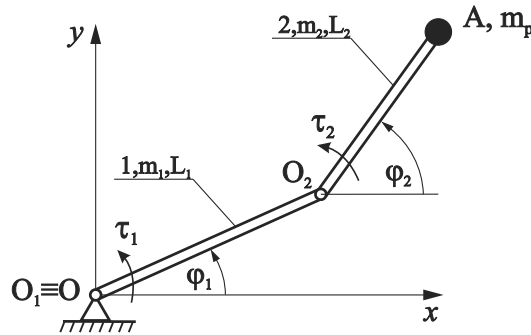


FIGURE 1. 2-DOF planar serial manipulator

The torques τ_1 and τ_2 shown in Figure 1 represent the driving torques acting in joints O_1 and O_2 , respectively. Each of the manipulator links can be, according

to [17], represented by a dynamically equivalent system (equipomental system) of two point masses as it is shown in Figure 2. In Figure 2, J_1 and J_2 are the centroidal moments of inertia in directions normal to the links 1 and 2, respectively. Note that the manipulator link and the two point mass system introduced are dynamically equivalent (equipomental) because they have the same mass, the same mass center, and the same inertia tensor determined with respect to the mass center of the link (see [2]). For one-dimensional links (straight rods) used in this paper, the minimum number of point masses containing in the equipomental systems equals two [6]. Note that the optimization techniques based on interpolating polynomials and counterweights allow the use of equipomental systems containing two point masses. Moreover, the use of the method of the optimal distribution of the link masses requires equipomental systems with three point masses (see [4–7]).

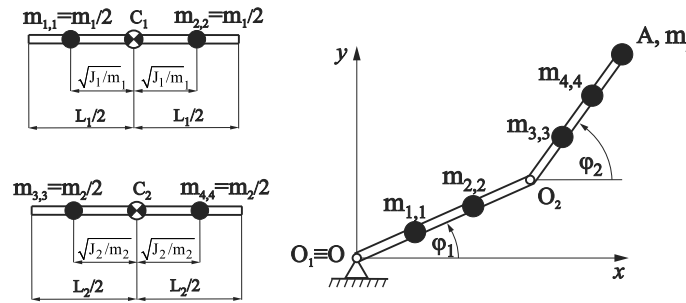


FIGURE 2. Two point masses models of the manipulator links

Using the equipomental system shown in Figure 2, the kinetic energy of the manipulator can be written as:

$$(2.1) \quad T = \frac{1}{2} \dot{\mathbf{y}}^T \mathbf{M} \dot{\mathbf{y}},$$

where $\mathbf{M} = \text{diag}(m_{1,1}, m_{1,1}, \dots, m_{4,4}, m_{4,4}, m_p, m_p)$ is the inertia matrix, $\mathbf{y} = [x_1, y_1, \dots, x_4, y_4, x_p, y_p]^T$ is the vector of the Cartesian coordinates, and x_i ($i = 1, \dots, 4$), y_i ($i = 1, \dots, 4$), x_p , and y_p are the Cartesian coordinates of point masses and payload, respectively, with respect to the inertial frame Oxy . Here an overdot denotes the derivative with respect to time. The coordinates of point masses may be expressed in terms of angles φ_1 and φ_2 after that the following relation can be formed:

$$(2.2) \quad \dot{\mathbf{y}} = \mathbf{B} \dot{\mathbf{q}},$$

where $\mathbf{B} \in R^{10 \times 2}$ is the velocity transformation matrix which is a function of the generalized coordinates φ_1 and φ_2 , and $\dot{\mathbf{q}} = [\dot{\varphi}_1, \dot{\varphi}_2]^T$ is the vector of generalized velocities. The time derivative of Eq. (2.2) yields:

$$(2.3) \quad \ddot{\mathbf{y}} = \mathbf{B} \ddot{\mathbf{q}} + \dot{\mathbf{B}} \dot{\mathbf{q}}.$$

Finally, using the Lagrange equations of the second kind [14, 18], the differential equations of motion of the manipulator can be formed as follows:

$$(2.4) \quad \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\mathbf{q}}} \right)^T - \left(\frac{\partial T}{\partial \mathbf{q}} \right)^T = \mathbf{Q}_\varphi$$

where $\partial T / \partial \dot{\mathbf{q}} = [\partial T / \partial \dot{\varphi}_1, \partial T / \partial \dot{\varphi}_2]$, $\partial T / \partial \mathbf{q} = [\partial T / \partial \varphi_1, \partial T / \partial \varphi_2]$, $\mathbf{Q}_\varphi = [Q_{\varphi_1}, Q_{\varphi_2}]^T$ is the vector of generalized forces associated with the generalized coordinates φ_i ($i = 1, 2$), respectively, and $\mathbf{q} = [\varphi_1, \varphi_2]^T$. Based on Eq. (2.1), the left-hand sides of Eq. (2.4) can be written as (see [15]):

$$(2.5) \quad \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\varphi}_i} \right) - \frac{\partial T}{\partial \varphi_i} = \left(\frac{\partial \dot{\mathbf{y}}}{\partial \dot{\varphi}_i} \right)^T \mathbf{M} \ddot{\mathbf{y}} \quad i = 1, 2.$$

According to Eqs. (2.2), (2.3), and (2.5) the differential equations (2.4) take the form:

$$\mathbf{B}^T \mathbf{M} \mathbf{B} \ddot{\mathbf{q}} + \mathbf{B}^T \mathbf{M} \dot{\mathbf{B}} \dot{\mathbf{q}} = \mathbf{Q}_\varphi.$$

3. Determination of dynamic joint reaction forces

Since dynamic joint reaction forces are considered, in the further consideration $g = 0$ it is taken out, where g is the gravitational acceleration. In this paper, the determination of dynamic joint reaction forces of the manipulator considered is based on the general method for determination of constraint reaction forces described in [13, 14]. According to [13, 14], to determine reaction forces in the joints O_1 and O_2 , these joints are cut imaginary (see Figure 3).

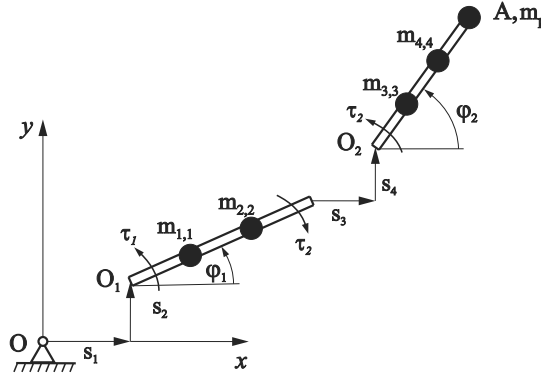


FIGURE 3. Imaginary cutting of the manipulator joints

After the joints have been cut the number of degrees of freedom of the manipulator is increased by four. In regard to this, four new coordinates, s_1 , s_2 , s_3 , and s_4 , are involved that referred to the prohibited motions in the joints as it is shown in Figure 3. Now, the manipulator motion can be observed as the motion with the redundant coordinates s_1, \dots, s_4 subject to the following constraints:

$$(3.1) \quad f_i \equiv s_i = 0, \quad i = 1, \dots, 4.$$

Now the relations (2.2) and (2.3) take the form:

$$\begin{aligned}\dot{\mathbf{y}} &= \mathbf{B}\dot{\mathbf{q}} + \mathbf{B}_1\dot{\mathbf{s}} \\ \ddot{\mathbf{y}} &= \mathbf{B}\ddot{\mathbf{q}} + \dot{\mathbf{B}}\dot{\mathbf{q}} + \mathbf{B}_1\ddot{\mathbf{s}},\end{aligned}$$

where $\mathbf{s} = [s_1, \dots, s_4]^T$ and $\mathbf{B}_1 \in R^{10 \times 4}$ is a constant matrix. After that, using the Lagrange equations with the multipliers [14, 18], the differential equations of motion of the manipulator depicted in Figure 3 read:

$$(3.2) \quad \frac{d}{dt} \left(\frac{\partial T^*}{\partial \dot{\varphi}_i} \right) - \frac{\partial T^*}{\partial \varphi_i} = Q_{\varphi_i}^*, \quad i = 1, 2,$$

$$(3.3) \quad \frac{d}{dt} \left(\frac{\partial T^*}{\partial \dot{s}_j} \right) - \frac{\partial T^*}{\partial s_j} = Q_{s_j}^* + \sum_{r=1}^4 \lambda_r \frac{\partial f_r}{\partial s_j}, \quad j = 1, \dots, 4,$$

where $T^* = T^*(\varphi_i, \dot{\varphi}_i, s_j, \dot{s}_j)$ ($i = 1, 2; j = 1, \dots, 4$) and λ_r ($r = 1, \dots, 4$) are the Lagrange multipliers of constraints. Since it is assumed that $g = 0$, for the manipulator considered one has that:

$$Q_{\varphi_i}^* = \tau_i \quad (i = 1, 2), \quad Q_{s_j}^* = 0 \quad (j = 1, \dots, 4)$$

and, based on Eq. (3.1):

$$\frac{\partial f_r}{\partial s_j} = \delta_{rj}, \quad r, j = 1, \dots, 4$$

where δ_{rj} is the Kronecker delta symbol [14, 18].

Similarly to Section 2, the differential equations (3.2) and (3.3) can be written in the following matrix form:

$$(3.4) \quad \mathbf{B}^T \mathbf{M} \mathbf{B} \ddot{\mathbf{q}} + \mathbf{B}^T \mathbf{M} \mathbf{B}_1 \ddot{\mathbf{s}} + \mathbf{B}^T \mathbf{M} \dot{\mathbf{B}} \dot{\mathbf{q}} = \mathbf{Q}_{\varphi}^*,$$

$$(3.5) \quad \mathbf{B}_1^T \mathbf{M} \mathbf{B} \ddot{\mathbf{q}} + \mathbf{B}_1^T \mathbf{M} \mathbf{B}_1 \ddot{\mathbf{s}} + \mathbf{B}_1^T \mathbf{M} \dot{\mathbf{B}} \dot{\mathbf{q}} = \lambda,$$

where $\lambda = [\lambda_1, \dots, \lambda_4]^T$ is a vector of Lagrange multipliers and $\mathbf{Q}_{\varphi}^* = [\tau_1, \tau_2]^T$.

Further, substituting the following relations:

$$\ddot{s}_j(t) \equiv 0, \quad j = 1, \dots, 4,$$

which follow from (3.1), into Eqs. (3.4) and (3.5), Eq. (3.4) yields the equations of motion of the manipulator immediately before cutting of the joints, while Eq. (3.5) yields expressions for the components of dynamic joint reaction forces as follows:

$$[X_{O_1}^d, Y_{O_1}^d, X_{O_2}^d, Y_{O_2}^d]^T \equiv \lambda_0 = \mathbf{B}_1^T \mathbf{M} \mathbf{B} \ddot{\mathbf{q}} + \mathbf{B}_1^T \mathbf{M} \dot{\mathbf{B}} \dot{\mathbf{q}},$$

or in developed form:

$$\begin{aligned}X_{O_1}^d = (\lambda_1)_0 &= \frac{1}{2} \left[-L_1(m_1 + 2m_2 + 2m_p)(\dot{\varphi}_1^2 \cos \varphi_1 + \ddot{\varphi}_1 \sin \varphi_1) \right. \\ &\quad \left. - L_2(m_2 + 2m_p)(\ddot{\varphi}_2 \sin \varphi_2 + \dot{\varphi}_2^2 \cos \varphi_2) \right],\end{aligned}$$

$$\begin{aligned}Y_{O_1}^d = (\lambda_2)_0 &= \frac{1}{2} \left[-L_1(m_1 + 2m_2 + 2m_p)(\dot{\varphi}_1^2 \sin \varphi_1 - \ddot{\varphi}_1 \cos \varphi_1) \right. \\ &\quad \left. - L_2(m_2 + 2m_p)(\dot{\varphi}_2^2 \sin \varphi_2 - \ddot{\varphi}_2 \cos \varphi_2) \right],\end{aligned}$$

$$X_{O_2}^d = (\lambda_3)_0 = -\frac{1}{2} [2(m_2 + m_p)L_1(\dot{\varphi}_1^2 \cos \varphi_1 + \ddot{\varphi}_1 \sin \varphi_1) + (m_2 + 2m_p)L_2(\dot{\varphi}_2^2 \cos \varphi_2 + \ddot{\varphi}_2 \sin \varphi_2)],$$

$$Y_{O_2}^d = (\lambda_4)_0 = \frac{1}{2} [-2L_1(m_2 + m_p)(\dot{\varphi}_1^2 \sin \varphi_1 - \ddot{\varphi}_1 \cos \varphi_1) - L_2(m_2 + 2m_p)(\dot{\varphi}_2^2 \sin \varphi_2 - \ddot{\varphi}_2 \cos \varphi_2)],$$

where the notation $(\bullet)_0$ means that the quantity (\bullet) is calculated for $s_i(t) \equiv 0$ ($i = 1, \dots, 4$). Now, the magnitudes of dynamic reaction forces in joints O_1 and O_2 are $R_{O_1}^d = \sqrt{(X_{O_1}^d)^2 + (Y_{O_1}^d)^2}$ and $R_{O_2}^d = \sqrt{(X_{O_2}^d)^2 + (Y_{O_2}^d)^2}$, respectively.

4. Formulation of the optimization problem

Similarly to [3], an objective function is defined as follows:

$$(4.1) \quad F = \frac{w_1}{\delta} \sqrt{\sum_{i=0}^{\delta} f_1^2(t_i)} + \frac{w_2}{\delta} \sqrt{\sum_{i=0}^{\delta} f_2^2(t_i)},$$

where w_1 and w_2 are the weighting factors whose values are $w_1 = w_2 = 0.5$, δ is the number of discrete positions of the manipulator over the interval $(0, t_f]$, t_f represents time for which the motion has to be completed, $f_1(t_i) = \sqrt{(X_{O_1}^d(t_i))^2 + (Y_{O_1}^d(t_i))^2}$,

$f_2(t_i) = \sqrt{(X_{O_2}^d(t_i))^2 + (Y_{O_2}^d(t_i))^2}$, and $t_i = (it_f)/\delta$ is the instant corresponding to the i th discrete position of the manipulator. Note that if objective is to minimize reaction $R_{O_1}^d$ only, then it is $w_1 = 1.0$ and $w_2 = 0$, while for the minimization of $R_{O_2}^d$ only one has $w_1 = 0$ and $w_2 = 1.0$.

Now, the optimization problem consists in finding design variables b_i ($i = 1, \dots, p$) collected in the column matrix $\mathbf{b} = [b_1, \dots, b_p]^T$ that minimize the objective function (4.1) and subject to equality constraints:

$$(4.2) \quad g_j(\mathbf{b}) = 0, \quad j = 1, \dots, p_1,$$

as well as inequality constraints:

$$(4.3) \quad b^* \leq h_k(\mathbf{b}) \leq b^{**}, \quad k = 1, \dots, p_2$$

where b^* and b^{**} are constants. The choice of design variables as well as the constraints (4.2) and (4.3) depends on the choice of optimization methods. In this paper, the differential evolution [19] is applied to minimize the objective function (4.1) subject to the constraints (4.2) and (4.3). The differential evolution represents an evolutionary optimization technique, which is simple for using, fast in convergence to the global minimum solution and has many other advantages as compared to the conventional optimization algorithm (for more details see [19]). In the further numerical computations the following values of the control variables of the differential evolution are used: crossover probability constant $CR = 0.5$, scaling factor $MF = 0.6$.

5. Numerical examples

Numerical simulations in this section concerning to the unoptimized (original) manipulator were carried out for the combination of the manipulator parameters given in [9, 20], that is: $m_1 = 12$ kg, $m_2 = 6$ kg, $m_p = 4$ kg, $L_1 = L_2 = 0.5$ m, $t_f = 10$ s, $\varphi_1(0) = \varphi_2(0) = 0$, $\varphi_1(t_f) = \pi$, $\varphi_2(t_f) = 3\pi/2$,

$$\varphi_i(t) = \varphi_i(0) + (\varphi_i(t_f) - \varphi_i(0))[t/t_f - (1/(2\pi)) \sin(2\pi t/t_f)], \quad i = 1, 2.$$

Also, it is taken that $\delta = 200$. For these values of the parameters, the function F has the value $F^* = 0.220271$ N. Note that at the initial and terminal instants of motion the angular speeds and the angular accelerations of links are equal to zero.

5.1. Optimal selection of the angular rotations laws of the manipulator links. Let us assume the angular rotations laws in forms of interpolating polynomials [12] as follows:

$$\begin{aligned} \varphi_1(t) &= \varphi_1(0) + (\varphi_1(t_f) - \varphi_1(0)) \sum_{k=0}^6 a_k^{(1)} \left(\frac{t}{t_f}\right)^k, \\ \varphi_2(t) &= \varphi_2(0) + (\varphi_2(t_f) - \varphi_2(0)) \sum_{k=0}^6 a_k^{(2)} \left(\frac{t}{t_f}\right)^k. \end{aligned}$$

From the conditions $\varphi_1(0) = \varphi_2(0) = 0$, $\dot{\varphi}_1(0) = \dot{\varphi}_2(0) = 0$, and $\ddot{\varphi}_1(0) = \ddot{\varphi}_2(0) = 0$ it follows that $a_j^{(1)} = 0$ ($j = 0, \dots, 2$) and $a_j^{(2)} = 0$ ($j = 0, \dots, 2$), while the conditions $\varphi_1(t_f) = \pi$, $\varphi_2(t_f) = 3\pi/2$, $\dot{\varphi}_1(t_f) = \dot{\varphi}_2(t_f) = 0$, and $\ddot{\varphi}_1(t_f) = \ddot{\varphi}_2(t_f) = 0$ imply the following constraints on the polynomials coefficients:

$$(5.1) \quad a_3^{(1)} + a_4^{(1)} + a_5^{(1)} + a_6^{(1)} = 1,$$

$$(5.2) \quad a_3^{(2)} + a_4^{(2)} + a_5^{(2)} + a_6^{(2)} = 1,$$

$$(5.3) \quad 3a_3^{(1)} + 4a_4^{(1)} + 5a_5^{(1)} + 6a_6^{(1)} = 0,$$

$$(5.4) \quad 3a_3^{(2)} + 4a_4^{(2)} + 5a_5^{(2)} + 6a_6^{(2)} = 0,$$

$$(5.5) \quad 6a_3^{(1)} + 12a_4^{(1)} + 20a_5^{(1)} + 30a_6^{(1)} = 0,$$

$$(5.6) \quad 6a_3^{(2)} + 12a_4^{(2)} + 20a_5^{(2)} + 30a_6^{(2)} = 0.$$

The design variables are defined as:

$$\mathbf{b} = [a_3^{(1)}, \dots, a_6^{(1)}, a_3^{(2)}, \dots, a_6^{(2)}]^T.$$

The optimization problem is solved under the equality constraints (5.1)-(5.6) and the following inequality constraints:

$$-100 \leq a_i^{(1)} \leq 100, \quad -100 \leq a_i^{(2)} \leq 100, \quad i = 3, \dots, 6.$$

In this case, the solution of the minimization problem reads:

$$\begin{aligned} a_6^{(1)} &= 15.885, & a_5^{(1)} &= -41.6551, & a_4^{(1)} &= 32.6551, \\ a_3^{(1)} &= -5.885, & a_6^{(2)} &= -8.7111, & a_5^{(2)} &= 32.1333, \end{aligned}$$

$$a_4^{(2)} = -41.1333, \quad a_3^{(2)} = 18.7111, \quad F_{\min} = 0.117662 \text{ N.}$$

On this way the reduction of 46.6% in the value of the objective function is achieved. The magnitudes $R_{O_1}^d$ and $R_{O_2}^d$ versus time are shown in Figure 4.

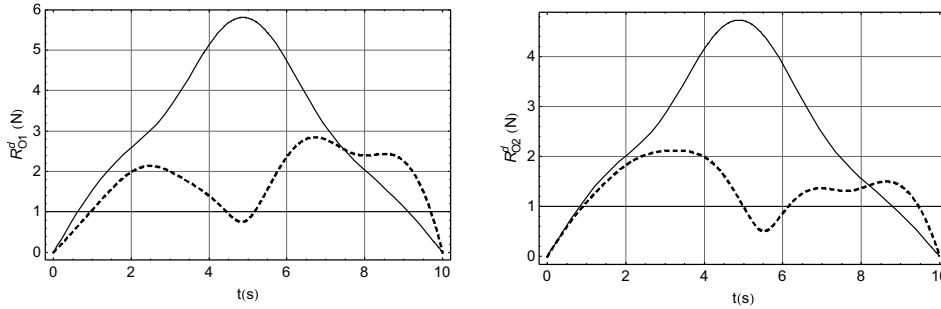


FIGURE 4. Magnitudes of dynamic joint reaction forces (method of interpolating polynomials): Original (solid line) and minimized (dashing line)

5.2. Using counterweights to minimize dynamic joint reaction forces.

In this section, in order to minimize dynamic joint reaction forces, the counterweight method [10, 11] is used. The counterweights are attached to the manipulator links in the same way as in [8, 9, 21] (see Figure 5). Let us denote by m_{1d} and m_{2d} the masses of counterweights. The locations of the counterweights with respect to the manipulator links are specified by the distances r_i ($i = 1, 2$) between the mass centers of counterweights and the joints (see Figure 5).

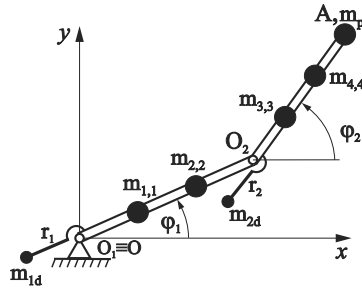


FIGURE 5. Counterweights added to the links to achieve minimization of dynamic joint reaction forces

In this case, applying the method described in Section 3, the following expressions for the components of dynamic joint reaction forces are obtained:

$$(5.7) \quad X_{O_1}^d = \frac{1}{2} \left[- (m_2 L_2 + 2m_p L_2 - 2r_2 m_{2d}) (\dot{\varphi}_2^2 \cos \varphi_2 + \ddot{\varphi}_2 \sin \varphi_2) \right. \\ \left. - (L_1 m_1 + 2m_2 L_1 + 2m_{2d} L_1 + 2m_p L_1 - 2r_1 m_{1d}) \right]$$

$$(5.8) \quad \begin{aligned} & (\dot{\varphi}_1^2 \cos \varphi_1 + \ddot{\varphi}_1 \sin \varphi_1) \Big], \\ Y_{O_1}^d = & \frac{1}{2} \Big[- (m_2 L_2 + 2m_p L_2 - 2r_2 m_{2d}) (\dot{\varphi}_2^2 \sin \varphi_2 - \ddot{\varphi}_2 \cos \varphi_2) \\ & - (L_1 m_1 + 2m_2 L_1 + 2m_{2d} L_1 + 2m_p L_1 - 2r_1 m_{1d}) \\ & (\dot{\varphi}_1^2 \sin \varphi_1 - \ddot{\varphi}_1 \cos \varphi_1) \Big], \end{aligned}$$

$$(5.9) \quad \begin{aligned} X_{O_2}^d = & -L_1 (m_2 + m_{2d} + m_p) (\dot{\varphi}_1^2 \cos \varphi_1 + \ddot{\varphi}_1 \sin \varphi_1) \\ & - \frac{1}{2} (L_2 m_2 + 2L_2 m_p - 2r_2 m_{2d}) (\dot{\varphi}_2^2 \cos \varphi_2 + \ddot{\varphi}_2 \sin \varphi_2), \end{aligned}$$

$$(5.10) \quad \begin{aligned} Y_{O_2}^d = & -L_1 (m_2 + m_{2d} + m_p) (\dot{\varphi}_1^2 \sin \varphi_1 - \ddot{\varphi}_1 \cos \varphi_1) \\ & - \frac{1}{2} (L_2 m_2 + 2L_2 m_p - 2r_2 m_{2d}) (\dot{\varphi}_2^2 \sin \varphi_2 - \ddot{\varphi}_2 \cos \varphi_2). \end{aligned}$$

Here the set of design variables is defined as:

$$\mathbf{b} = [m_{1d} r_1, m_{2d}, r_2]^T,$$

and only the inequality constraints (4.3) exist, that is:

$$0 \leq m_{1d} r_1 \leq 12, \quad 0 \leq m_{2d} \leq 12, \quad 0 \leq r_2 \leq 0.5.$$

Solving the optimization problem (4.1)-(4.3) by means of the differential evolution yields the following optimal values of the design variables:

$$(5.11) \quad m_{1d} r_1 = 11.5 \text{ kgm}, \quad m_{2d} = 7 \text{ kg}, \quad r_2 = 0.5 \text{ m}.$$

For these values, the minimum of the objective function equals 0.074888 N. In this manner one has reduction of 66% in the value of the objective function with respect to the value F^* . The magnitudes $R_{O_1}^d$ and $R_{O_2}^d$ versus time are shown in Figure 6.

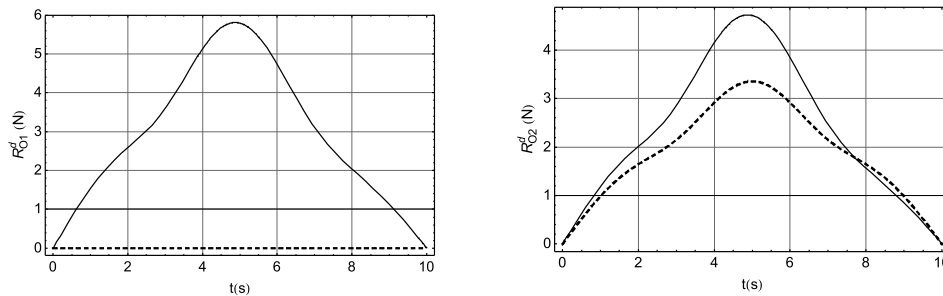


FIGURE 6. Magnitudes of dynamic joint reaction forces (method of counterweights): Original (solid line) and minimized (dashed line)

From Figure 6 it can be observed that using the method of counterweights the dynamic reaction force in joint O_1 is canceled. Analytic conditions for this can be

obtained from Eqs. (5.7)-(5.10) as follows:

$$(5.12) \quad m_2 L_2 + 2m_p L_2 - 2r_2 m_{2d} = 0,$$

$$(5.13) \quad L_1 m_1 + 2m_2 L_1 + 2m_{2d} L_1 + 2m_p L_1 - 2r_1 m_{1d} = 0.$$

Note that the values (5.11) satisfy the conditions (5.12) and (5.13). Similarly to [9], the practical realization of the considered counterweights method can be achieved by means of movable inertias m_{1d} and m_{2d} connected to the links 1 and 2, respectively, by prismatic joints.

6. Conclusions

In this paper, a method for the determination of joint reaction forces in 2-DOF planar serial manipulators in a symbolic form based on the use of the Lagrange equations with multipliers, velocity transformation technique, and equimomental systems consisting of two point masses has been presented. The extension of the method to systems consisting of n planar links interconnected by revolute joints is straightforward. The proposed method of determination of joint reaction forces is computationally more efficient than the method from [16] because our method uses equimomental system representations of the manipulator links and does not require the computation of the following kinematic characteristics of the links: angular velocities, angular accelerations, velocities and accelerations of the links mass centers. The method has been applied to solve the problem of minimization of dynamic joint reaction forces. The evaluation of both the interpolating polynomials method and the counterweights method has been conducted. It has been shown that complete canceling of the dynamic joint reaction force in joint O_1 is achieved by means of the counterweights method. Note that the method of formulation of equations of motion described in Section 2 can be used as an alternative to [4–6] (the method based on the Newton–Euler equations) as well as to [7, 20] (the method based on Newton’s equations of linear motion of particles). The obtained results are valuable for the improvement of the dynamic performance of the considered type of manipulators.

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**МИНИМИЗАЦИЈА ДИНАМИЧКИХ РЕАКЦИЈА У
ЗГЛОБОВИМА СЕРИЈСКОГ МАНИПУЛАТОРА СА
ДВА СТЕПЕНА СЛОБОДЕ КОРИШЋЕЊЕМ
ИНТЕРПОЛАЦИОНИХ ПОЛИНОМА И ПРОТИВТЕГОВА**

РЕЗИМЕ. У раду су представљена два начина за минимизацију реакција у зглобовима раванског серијског манипулатора са два степена слободе које потичу од инерцијалних сила (динамичке реакције). Први начин представља оптимални избор закона обртања сегмената манипулатора док се други начин састоји у везивању противтегова за сегменте манипулатора. Такође је узет у обзир и утицај масе терета, који се преноси манипулатором, на динамичке реакције у зглобовима. Коришћењем Лагранжевих једначина кретања добијени су изрази за динамичке реакције у зглобовима у сумболичком облику. Инерцијалне карактеристике сегмената манипулатора представљене су динамички еквивалентним системима од две везане материјалне тачке. Коришћена је функција циља у облику збира квадратних коренова од збира квадрата дискретних вредности интензитета динамичких реакција у одговарајућим зглобовима. Дата је анализа ефикасности разматраних начина за минимизацију динамичких реакција у зглобовима.

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