The Approaches to the Mathematical-mechanical Modeling Supporting Construction

Rade Vasiljevic[1,*] - Radovan Bulatovic[2] - Mile Savkovic[2]
[1] High Technical School of Vocational Studies, Belgrade
[2] Faculty of Mechanical and Civil Engineering in Kraljevo, University of Kragujevac

In this paper, four approaches to the dynamic analysis of structures are presented on the example of portal crane. The accuracy of the approximate approach compared with correct approach was examined. The appropriate conclusions and guidelines are defined.

Keywords: Portal crane, dynamic analysis, model of distributed masses, the approximated models.

0. INTRODUCTION

The problem regarding oscillation of supporting construction is of importance in civil and mechanical engineering. First of all, in civil engineering, the problem regarding oscillation of construction is related to the structural analysis of bridges and buildings, while in mechanical engineering, it is primarily related to the structural analysis of the cranes.

The procedure of determining eigenfrequencies at complex systems (systems with large number of the freedom degrees) is the most expensive phase of dynamic analysis [1,2]. Previous studies of determining its eigenfrequencies of complex supporting structures were based on the use of approximate expressions and methods [3]. Accurate determination of eigenfrequencies was limited to the simple supporting structure (simple beam and console). Finding out solutions of frequent equation for complex elastic bodies is very difficult, because it contained the trigonometric and hyperbolic functions. Nowadays, mathematical software enables routine solving of frequency equations for complex elastic bodies oscillation. Methodology of solving frequent equation is illustrated by example of portal crane supporting constuction using Mathematica software. Accurate determination of eigenfrequencies is important from the aspect of optimizing supporting structures. But, in the case of too complex supporting structures, using of method of distributed masses is limited. In this case, to determine eigenfrequencies of supporting structure, we opt for the method of consistent masses or method of directy concentrated masses. Methodology of solving frequent equation for both methods is illustrated by identical example. On the other hand, nowadays, modal analysis on the base FEM software is widely used for determining eigenfrequencies of supporting structures.

In this regard, in the third step of this work are solutions of circular frequencies of portal crane supporting construction, provided by accurate method (method of the distributed masses) and approximate methods as well (method of consistent masses and method of directy concentrated masses), verified by a specialized software package SAP2000.

1. SETTING OF THE PROBLEMS

The main parts of the support structure of portal crane, Figure 1, the top grider and two columns. Material support structure is steel S235J2G3. Global dimension crane are: L = 8m - range crane and H = 4m - the height of the supporting column. The grider is made of and profiles - IPE240, while the columns are made of double U profile - 2U140.

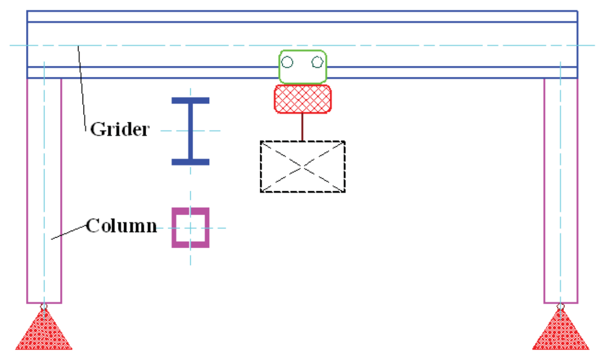

Fig. 1. *Sketch of portal crane*

For getting all eigenvalues and eigenvectors is necessary to perform a number of numerical operations. To reduce the size of the account in the dynamic analysis, by choosing only a certain eigenvectors (oscillation forms). How oscillation forms with a frequency that is close to the frequency of the load most affect the dynamic system response is defined dynamic loads and assumed the dominant form of oscillation.

Given that the crane supporting structure considered symmetrical, to determine the frequency of circular support structure analytical methods sufficient be formed dynamic model for half of the structure.

* Corr. Author's Address: High Technical School, Bul. Zorana Đinđića 152-A, Belgrade, Serbia, rade.vasiljevic@visokatehnicka.edu.rs

2. MATHEMATICAL-MECHANICAL MODEL WITH DISTRIBUTED MASSES

Has been adopted elastic-linear dynamic model for half the supporting structure, Figure 2.

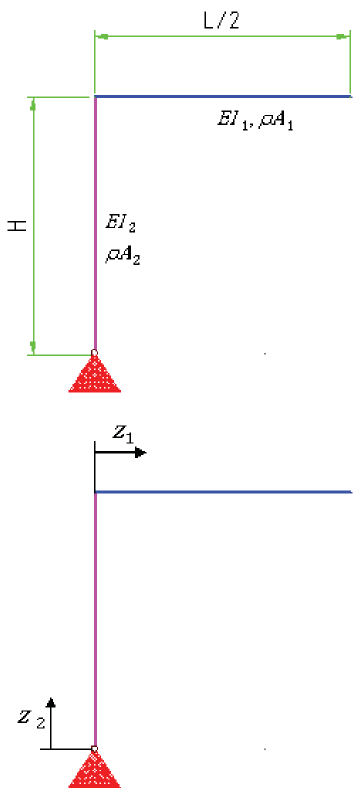

Fig. 2. *Model with distributed masses*

Partial differential equations of free undamped oscillations transverse frame as follows:

$$\frac{\partial^2 Y_i(z,t)}{\partial t^2}+c^2\cdot\frac{\partial^4 x_i(z,t)}{\partial z^4}=0,\quad i=1,2 \tag{1}$$

where:

$$c^2=\frac{E\cdot I_i}{\rho\cdot A_i} \tag{2}$$

If the solution of partial differential equations (1) look of the form:

$$Y_i(z,t)=Z_i(z)\cdot T(t) \tag{3}$$

or individually for each element of the frame:

$$\begin{aligned} Y_1&=Y_1(z,t)=Z_1(z)\cdot T(t),\quad 0\le z\le\frac{L}{2}\\ Y_2&=Y_2(z,t)=Z_2(z)\cdot T(t),\quad 0\le z\le H \end{aligned} \tag{4}$$

we get the following two functions:

$$\begin{aligned} Z(z)&=C_{1i}\,\mathrm{ch}(k_i z)+C_{2i}\,\mathrm{sh}(k_i z)+\\ &+C_{3i}\cos(k_i z)+C_{4i}\sin(k_i z)\\ T(t)&=A_i\cos(\omega t)+B_i\sin(\omega t) \end{aligned} \tag{5}$$

Because of the complexity of the elastic body functions $Z(z)$ will be presented via Krilovljevih function:

$$\begin{aligned} Z_i(z)&=C_{1i}S(k_i z)+C_{2i}T(k_i z)+\\ &+C_{3i}U(k_i z)+C_{4i}V(k_i z) \end{aligned} \tag{6}$$

Circular frequencies of time function in (5) is equal to:

$$\omega=c\cdot k_i^2=k_i^2\cdot\sqrt{\frac{E\cdot I_i}{\rho\cdot A_i}} \tag{7}$$

The frequency of oscillation is calculated by:

$$f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\left(\frac{2\lambda_1}{L}\right)^2\sqrt{\frac{E\cdot I_1}{\rho\cdot A_1}} \tag{8}$$

Given frame is divided into two parts. First, we observe symmetric oscillations, and then asymmetric oscillations.

2.1 Symmetric oscillations

2.1.1 Boundary conditions

According to the geometric boundary conditions, then the following equality [4] to [8]:

$$\begin{aligned} &Z_1'\left(\frac{L}{2},t\right)=0\\ &Z_2(0,t)=0\\ &Z_2(H,t)=0\\ &Z_1'(0,t)=Z_2'(H,t) \end{aligned} \tag{9}$$

According to the dynamic boundary condition, then the following equality [4] to [8]:

$$\begin{aligned} &-E\cdot I_1\cdot Z_1'''\left(\frac{L}{2},t\right)=0\\ &-E\cdot I_2\cdot Z_2''(0,t)=0\\ &-E\cdot I_1\cdot Z_1''(0,t)=-E\cdot I_2\cdot Z_2''(H,t)\\ &\rho\cdot A_1\cdot\frac{L}{2}\cdot\ddot{Z}_1(0,t)=EI_2\cdot Z_2'''(H,t) \end{aligned} \tag{10}$$

2.1.2 Frequency equation

From the geometric and dynamic boundary conditions, (9) and (10), formed a homogeneous system of linear equations, which we conclude frequency equation:

$$\det(F)=0 \tag{11}$$

By introducing for the element frame (half beam girders) shift:

$$\lambda_1=k_1\cdot\frac{L}{2} \tag{12}$$

and introducing following non-dimensional parameters:

$$d=\frac{H}{L},\quad a=\frac{A_1}{A_2}\quad i=\frac{I_1}{I_2},\quad p=\sqrt[4]{i\cdot a} \tag{13}$$

Expression (11) can be written as follows:

$$\det(F)=f(\lambda_1,d,i,p)=0 \tag{14}$$

In the expression (14) F is equal to:

$$F=\begin{pmatrix} 0 & -1 & 0 & 0 & pS(\lambda_1\cdot d\cdot p) & pU(\lambda_1\cdot d\cdot p)\\ 0 & 0 & -1 & 0 & \frac{1}{i}p^2V(\lambda_1\cdot d\cdot p) & \frac{1}{i}p^2T(\lambda_1\cdot d\cdot p)\\ V(\lambda_1) & S(\lambda_1) & T(\lambda_1) & U(\lambda_1) & 0 & 0\\ T(\lambda_1) & U(\lambda_1) & V(\lambda_1) & S(\lambda_1) & 0 & 0\\ \lambda_1 & 0 & 0 & 0 & \frac{1}{i}p^3U(\lambda_1\cdot d\cdot p) & \frac{1}{i}p^3S(\lambda_1\cdot d\cdot p)\\ 0 & 0 & 0 & 0 & T(\lambda_1) & V(\lambda_1) \end{pmatrix} \tag{15}$$

2.2 Asymmetrical oscillations

2.2.1 Boundary conditions

According to the geometric boundary conditions, then the following equality:

$$\begin{aligned} & Z_1\left(\frac{L}{2},t\right)=0\\ & Z_2(0,t)=0\\ & Z_1^{'}(0,t)=Z_2^{'}(H,t) \end{aligned} \tag{16}$$

According to the dynamic boundary condition, then the following equality:

$$\begin{aligned} & -E\cdot I_1\cdot Z_1^{''}\left(\frac{L}{2},t\right)=0\\ & -E\cdot I_2\cdot Z_2^{''}(0,t)=0\\ & -E\cdot I_2\cdot Z_2^{'''}(0,t)=-E\cdot I_2\cdot Z_2^{'''}(H,t)\\ & -E\cdot I_1\cdot Z_1^{''}(0,t)=-E\cdot I_2\cdot Z_2^{''}(H,t)\\ & \rho\cdot A_1\cdot\frac{L}{2}\cdot\ddot{Z}_1(0,t)=EI_2\cdot Z_2^{'''}(H,t) \end{aligned} \tag{17}$$

2.2.2 Frequency equation

By analogy, as in symmetric oscillations, F in the expression (14) for asymmetric oscillation is equal to:

$$F=\begin{pmatrix} 0 & -1 & 0 & 0 & pS(\lambda_1\cdot d\cdot p) & pU(\lambda_1\cdot d\cdot p)\\ 0 & 0 & -1 & 0 & \frac{1}{i}p^2V(\lambda_1\cdot d\cdot p) & \frac{1}{i}p^2T(\lambda_1\cdot d\cdot p)\\ S(\lambda_1) & T(\lambda_1) & U(\lambda_1) & V(\lambda_1) & 0 & 0\\ U(\lambda_1) & V(\lambda_1) & S(\lambda_1) & T(\lambda_1) & 0 & 0\\ \lambda_1 & 0 & 0 & 0 & \frac{1}{i}p^3U(\lambda_1\cdot d\cdot p) & \frac{1}{i}p^3S(\lambda_1\cdot d\cdot p)\\ 0 & 0 & 0 & 0 & U(\lambda_1) & S(\lambda_1) \end{pmatrix} \tag{18}$$

2.3 Solving the frequency equation

Frequency equations (15) and (18) are by nature transcendetalne, so that they can not find their solutions in algebraic form. For this particular numerical example will show how solving these equations using Mathematica software.

Frequency solutions of the equation (15) and (18), for the given input data, using Mathematica [9], the diagrams presented in Figures 3 and 4.

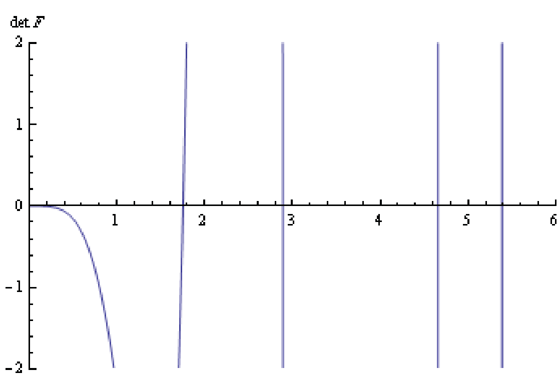

Fig. 3. *Dependence of detF od λ_1 – symmetric oscillations*

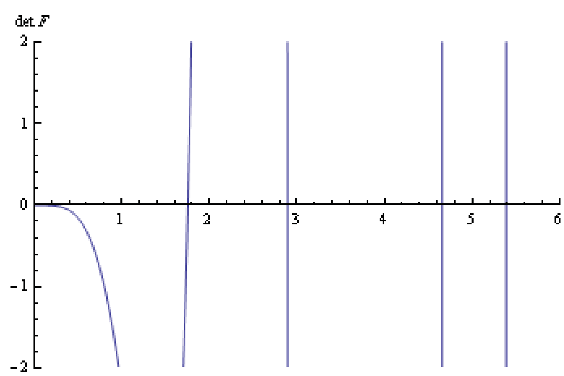

Fig. 4. *Dependence of detF od λ_1 – asymmetrical oscillation*

Solution frequency equations are:

- symmetric oscillations
  $\lambda_1=(1{,}75169;\ 2{,}88979)$
- asymmetrical oscillations
  $\lambda_1=(0{,}889472;\ 3{,}25296)$

Table 1 shows the values of the first four frequency oscillating support structures portal crane for the characteristics.

Tabela 1. *Frequencies – distributed masses*

No	Parameter λ_1	Frequency [Hz]	Circular frequency [rad/s]
1.	0,889472	4,0665	25,413
2.	1,75169	15,6944	98,5611
3.	2,88979	42,7133	268,24
4.	3,25296	54,1239	339,898

3. MATHEMATICAL-MECHANICAL MODEL WITH CONSISTENT MASSES

This approach consists in determining the inertial load along the beam element during motion of the beam, then replace the inertial load equivalent nodal load.

Adopted a dynamic model with consistent masses for half supporting structure, Figure 5.

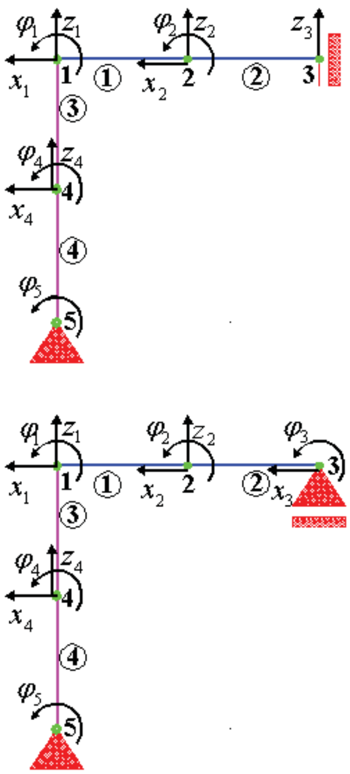

Fig. 5. *Model with consistent masses*

For both sides wedged rod of constant cross-section vector interpolation function following [2], [10] i [11]:

$$N^T = \begin{bmatrix} 1-\xi & 0 \\ 0 & 1-\xi^2+2\xi^3 \\ 0 & l\cdot\left(\xi-2\xi^2+\xi^3\right) \\ \xi & 0 \\ 0 & \xi^2+2\xi^3 \\ 0 & l\cdot\left(2\xi^2+\xi^3\right) \end{bmatrix}, \quad \xi=\frac{x}{l} \tag{19}$$

The corresponding mass matrix and the stiffness of the line element i is defined based on interpolation functions, (19), as follows:

$$M_i = \int_V \rho\cdot N^T\cdot N\cdot dV \tag{20}$$

$$K_i = \int_V E\cdot N^T\cdot N\cdot dV \tag{21}$$

Symmetrically and asymmetric oscillations are discussed separately.

Girder support structure is divided into four finite elements, while the columns are divided into two finite elements. Transformation matrix element as follows:

$$T = \begin{bmatrix} c_\theta & -s_\theta & 0 & 0 & 0 & 0 \\ s_\theta & c_\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_\theta & s_\theta & 0 \\ 0 & 0 & 0 & -s_\theta & c_\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{22}$$

where:

$c_\theta = \cos\theta,\ s_\theta = \sin\theta$

$\theta = 0°, i = 1, 2$

$\theta = 270°, i = 3, 4$

Mass matrix in the global coordinate system is equal to:

$$M_i^G = T_i^T\cdot M_i\cdot T \tag{23}$$

Stiffness matrix in the global coordinate system is equal to:

$$K_i^G = T_i^T\cdot K_i\cdot T \tag{24}$$

Mass matrix of the system:

$$M = \sum_i^4 M_i \tag{25}$$

Stiffness matrix of the system:

$$K = \sum_i^4 K_i \tag{26}$$

How the system mass matrix, and stiffness matrix of the system is decomposed in submatrices.

Eigenfrequencies of the supporting structure are obtained by solving algebraic equations:

$$\det\left(K_{nn} - \omega^2 M_{nn}\right) = 0 \tag{27}$$

3.1 Symmetric oscillations

Submatrix mass matrix of the systems by unknown is obtained by decomposing the matrix (25), while submatrix stiffness matrix of the system by unknown obtained decomposing the matrix (26). These submatrices are represented by (28) and (29):

$$
M_{nn}=\begin{bmatrix}
140+156ad & 0 & 22lad^2 & 70 & 0 & 0 & 0 & 54ad & 0 & -13lad^2 & 0\\
 & 156+140ad & 22l & 0 & 54 & -13l & 0 & 0 & 70ad & 0 & 0\\
 & & 4l^2+4l^2ad^3 & 0 & 13l & -3l^2 & 0 & 13lad^2 & 0 & -3l^2ad^3 & 0\\
 & & & 280 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
 & & & & 312 & 0 & 54 & 0 & 0 & 0 & 0\\
 & & & & & 8l^2 & 13l & 0 & 0 & 0 & 0\\
 & & & & & & 156 & 0 & 0 & 0 & 0\\
 & \text{Symm} & & & & & & 312ad & 0 & 0 & -13lad^3\\
 & & & & & & & & 280ad & 0 & 0\\
 & & & & & & & & & 8l^2ad^3 & -3l^2ad^3\\
 & & & & & & & & & & 4l^2ad^3
\end{bmatrix}\cdot\frac{\rho A_1 l}{420} \qquad (28)
$$

$$
K_{nn}=\begin{bmatrix}
75l^2+\frac{12i}{d^3} & 0 & \frac{6li}{d^2} & -75l^2 & 0 & 0 & 0 & -12\frac{i}{d^3} & 0 & 6\frac{li}{d^2} & 0\\
 & 12+\frac{75l^2i}{d} & 6l & 0 & -12 & 6l & 0 & 0 & -75\frac{l^2i}{d} & 0 & 0\\
 & & 4l^2+\frac{4l^2i}{d} & 0 & -6l & 2l^2 & 0 & -6\frac{li}{d^2} & 0 & 2\frac{l^2i}{d} & 0\\
 & & & 150l^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
 & & & & 24 & 0 & -12 & 0 & 0 & 0 & 0\\
 & & & & & 8l^2 & -6l & 0 & 0 & 0 & 0\\
 & & & & & & 12 & 0 & 0 & 0 & 0\\
 & \text{Symm} & & & & & & 24\frac{i}{d^3} & 0 & 0 & 6\frac{li}{d^2}\\
 & & & & & & & & 150\frac{l^2i}{d} & 0 & 0\\
 & & & & & & & & & 8\frac{l^2i}{d} & 2\frac{l^2i}{d}\\
 & & & & & & & & & & 4\frac{l^2i}{d}
\end{bmatrix}\frac{EI_1}{l^3} \qquad (29)
$$

Solving the equation (27), by inserting matrix (28) and (29) circular frequencies are:
$\omega=(97{,}7225;\ 253{,}964)[s^{-1}]$

3.2 Asymmetrical oscillations

Analogously for symmetric oscillation are obtained submatrix mass matrix systems with unknown (30) and submatrix stiffness matrix systems with unknown (31):

$$
M_{nn}=\begin{bmatrix}
140+156ad & 0 & 22lad^2 & 70 & 0 & 0 & 0 & 0 & 54ad & 0 & -13lad^2 & 0\\
 & 156+140ad & 22l & 0 & 54 & -13l & 0 & 0 & 13lad^2 & 70ad & 0 & 0\\
 & & 8l^2a^3 & 0 & 13l & -3l^2 & 0 & 0 & 0 & 0 & -3l^2ad^3 & 0\\
 & & & 280 & 0 & 0 & 70 & 0 & 0 & 0 & 0 & 0\\
 & & & & 312 & 0 & 0 & -13l & 0 & 0 & 0 & 0\\
 & & & & & 8l^2 & 0 & -3l^2 & 0 & 0 & 0 & 0\\
 & & & & & & 140 & 0 & 0 & 0 & 0 & 0\\
 & & & & & & & 4l^2 & 0 & 0 & 0 & 0\\
 & & \text{Symm} & & & & & & 312ad & 0 & 0 & -13lad^2\\
 & & & & & & & & & 280ad & 0 & 0\\
 & & & & & & & & & & 8l^2ad^3 & -3l^2ad^3\\
 & & & & & & & & & & & 4l^2ad^3
\end{bmatrix}\cdot\frac{\rho A_1 l}{420} \qquad (30)
$$

$$K_{nn}=\begin{bmatrix} 75l^2+\frac{12i}{d^3} & 0 & \frac{6li}{d^2} & -75l^2 & 0 & 0 & 0 & 0 & -12\frac{i}{d^3} & 0 & 6\frac{li}{d^2} & 0 \\ & 12+\frac{75l^2i}{d} & 6l & 0 & -12 & 6l & 0 & 0 & 0 & -75\frac{l^2i}{d} & 0 & 0 \\ & & 4l^2+\frac{4l^2i}{d} & 0 & -6l & 2l^2 & 0 & 0 & -6\frac{li}{d^2} & 0 & 2\frac{l^2i}{d} & 0 \\ & & & 150l^2 & 0 & 0 & -75l^2 & 0 & 0 & 0 & 0 & 0 \\ & & & & 24 & 0 & 0 & 6l & 0 & 0 & 0 & 0 \\ & & & & & 8l^2 & 0 & 2l^2 & 0 & 0 & 0 & 0 \\ & & & & & & 75l^2 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & 4l^2 & 0 & 0 & 0 & 0 \\ & & \text{Symm} & & & & & & 24\frac{i}{d^3} & 0 & 0 & 0 \\ & & & & & & & & & 150\frac{l^2i}{d} & 0 & 0 \\ & & & & & & & & & & 8\frac{l^2i}{d} & 2\frac{l^2i}{d} \\ & & & & & & & & & & & 8\frac{l^2i}{d} \end{bmatrix}\cdot\frac{EI_1}{l^3} \qquad (31)$$

Solving the equation (27), by inserting matrix (30) and (31) circular frequencies are:

$\omega=(22{,}697;\ 244{,}074)[s^{-1}]$

3.3 Frequency

Table 2 shows the values of the first four frequency oscillating support structures portal crane for the characteristics.

Table 2. *Frequencies – consistent masses*

No	Circular frequency [rad/s]	Frequency [Hz]
1.	22,697	3,6142
2.	97,7225	15,5609
3.	244,074	38,8653
4.	253,964	40,4401

4. MATHEMATICAL-MECHANICAL MODEL WITH DIRECT LUMPED MASSES

This approach consists in concentrating mass members in nodes girder. Equivalent nodal load is equal to zero due to the assumption that mass is only the nodal points.

Adopted a mathematical model with lumped mass directly for half the supporting structure, Figure 6.

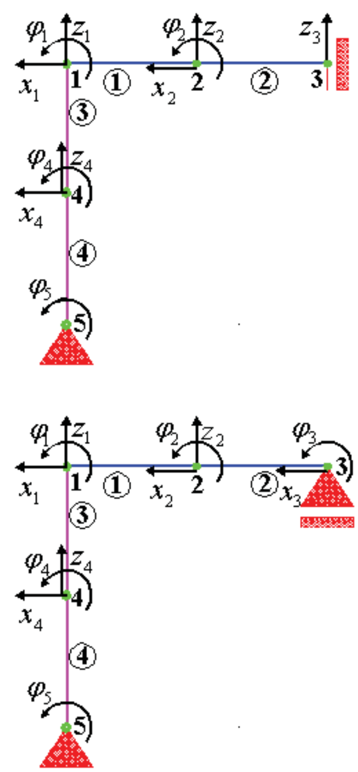

Fig. 6. *Model with direct lumped masses*

Eigenfrequencies of the supporting structure are obtained by solving algebraic equations:

$$\det\left(K_C-\omega^2M_{11}\right)=0 \qquad (32)$$

Reduced masses have the following values:

$$m_1 = \frac{1}{4}\rho \cdot A_1 \cdot L(1 + 2 \cdot a \cdot d)$$
$$m_2 = \frac{1}{4}\rho \cdot A_1 \cdot L$$
$$m_3 = \frac{1}{8}\rho \cdot A_1 \cdot L \qquad (33)$$
$$m_4 = \frac{1}{2}\rho \cdot A_1 \cdot L \cdot a \cdot d$$

4.1 Symmetric oscillations

Inertia matrix M_{nn} is singular. Rearrange matrix is obtained M_{11}:

$$M_{11} = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ & m_1 & 0 & 0 & 0 & 0 & 0 \\ & & m_2 & 0 & 0 & 0 & 0 \\ & & & m_2 & 0 & 0 & 0 \\ & & & & m_3 & 0 & 0 \\ & & \text{Symm} & & & m_4 & 0 \\ & & & & & & m_4 \end{bmatrix} \qquad (34)$$

Analogously, rearranging the system stiffness matrix K_{nn} is obtained matrix K_{11}, $K_{12} = K_{21}^T$ i K_{22}:

$$K_{11} = \begin{bmatrix} 75l^2 + \frac{12i}{d^3} & 0 & -75l^2 & 0 & 0 & -12\frac{i}{d^3} & 0 \\ & 12 + \frac{75l^2 i}{d} & 0 & -12 & 0 & 0 & -75\frac{l^2 i}{d} \\ & & 150l^2 & 0 & 0 & 0 & 0 \\ & & & 24 & -12 & 0 & 0 \\ & & & & 12 & 0 & 0 \\ & & \text{Symm} & & & 24\frac{i}{d^3} & 0 \\ & & & & & & 150\frac{l^2 i}{d} \end{bmatrix} \frac{EI_1}{l^3} \qquad (35)$$

$$K_{21} = \begin{bmatrix} \frac{6li}{d^2} & 6l & 0 & -6l & 0 & -6\frac{li}{d^2} & 0 \\ 0 & 6l & 0 & 0 & -6l & 0 & 0 \\ 6\frac{li}{d^2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6\frac{li}{d^2} & 0 \end{bmatrix} \frac{E}{l} \qquad (36)$$

$$K_{22} = \begin{bmatrix} 4l^2 + \frac{4l^2 i}{d} & 2l^2 & 2\frac{l^2 i}{d} & 0 \\ & 8l^2 & 0 & 0 \\ & & 8\frac{l^2 i}{d} & 2\frac{l^2 i}{d} \\ \text{Symm} & & & 4\frac{l^2 i}{d} \end{bmatrix} \frac{EI_1}{l^3} \qquad (37)$$

Condensed stiffness matrix is equal to:

$$K_C = K_{11} - K_{12} \cdot K_{22}^{-1} \cdot K_{21} \qquad (38)$$

Solving the equation (32), by inserting matrix (34) and (38), circular frequencies are:

$\omega = (92{,}4762;\ 249{,}741)[s^{-1}]$

4.2 Asymmetrical oscillations

In the same way as in the symmetric oscillations we obtain the following matrix:

$$M_{11} = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ & m_1 & 0 & 0 & 0 & 0 & 0 \\ & & m_2 & 0 & 0 & 0 & 0 \\ & & & m_2 & 0 & 0 & 0 \\ & & & & m_3 & 0 & 0 \\ & & \text{Symm} & & & m_4 & 0 \\ & & & & & & m_4 \end{bmatrix} \qquad (39)$$

$$K_{11}=\begin{bmatrix} 75l^2+\frac{12i}{d^3} & 0 & -75l^2 & 0 & 0 & -12\frac{i}{d^3} & 0 \\ & 12+\frac{75l^2 i}{d} & 0 & -12 & 0 & 0 & -75\frac{l^2 i}{d} \\ & & 150l^2 & 0 & -75l^2 & 0 & 0 \\ & & & 24 & 0 & 0 & 0 \\ & & & & 75l^2 & 0 & 0 \\ & & \text{Symm} & & & 24\frac{i}{d^3} & 0 \\ & & & & & & 150\frac{l^2 i}{d} \end{bmatrix}\frac{EI_1}{l^3} \qquad (40)$$

$$K_{21}=\begin{bmatrix} \frac{6li}{d^2} & 6l & 0 & -6l & 0 & -6\frac{li}{d^2} & 0 \\ 0 & 6l & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6l & 0 & 0 & 0 \\ \frac{6li}{d^2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6\frac{li}{d^2} & 0 \end{bmatrix}\frac{EI_1}{l^3} \qquad (41)$$

$$K_{22}=\begin{bmatrix} 4l^2+\frac{4l^2 i}{d} & 2l^2 & 0 & 2\frac{l^2 i}{d} & 0 \\ & 8l^2 & 2l^2 & 0 & 0 \\ & & 4l^2 & 0 & 0 \\ & & & \frac{8l^2 i}{d} & \frac{2l^2 i}{d} \\ & \text{Symm} & & & \frac{4l^2 i}{d} \end{bmatrix}\frac{EI_1}{l^3} \qquad (42)$$

Condensed stiffness matrix is equal to:

$$K_C = K_{11} - K_{12}\cdot K_{22}^{-1}\cdot K_{21} \qquad (43)$$

Solving the equation (32), by inserting matrix (39) and (43), natural frequencies are:

$\omega = (22{,}6283;\ 233{,}49)[s^{-1}]$

4.3 Frequency

Table 3 shows the values of the first four frequency oscillating support structures portal crane for the characteristics.

Table 3. *Frequencies – direct lumped masses*

No	Circular frequency [rad/s]	Frequency [Hz]
1.	22,6283	3,6032
2.	92,4762	14,7255
3.	233,49	37,1799
4.	249,741	39,7677

5. FEM MODEL IN SAP2000 SOFTWARE

In order to verify on analytical method of determining the frequency of software SAP2000 is formed and solved Finite element model of crane support structures [10]. Finite element model of crane support structure is shown in Figure 7.

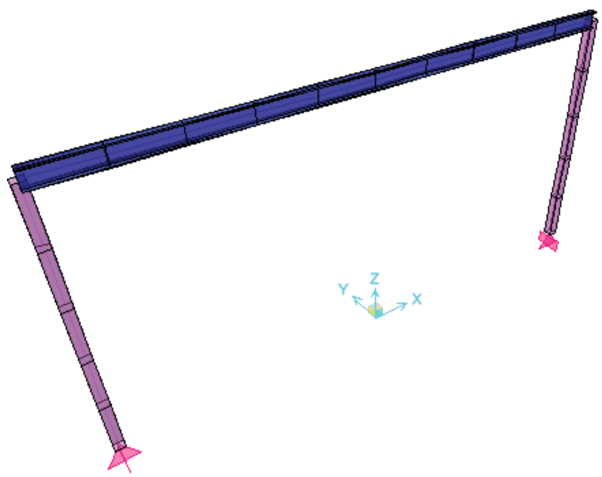

Fig. 7. *Finite element model*

The first four modes oscillating support structures are shown in Figure 8.

By implementation modal analysis software SAP2000, the values obtained frequencies for the first four modes of oscillation considered supporting structure, table 4.

Table 4. *Frequencies – SAP2000*

No	Period [s]	Frequency [Hz]	Circular frequency [rad/s]
1.	0,27818	3,595	22,58
2.	0,06433	15,545	97,62
3.	0,02720	36,7647	230,882
4.	0,02602	38,4320	241,353

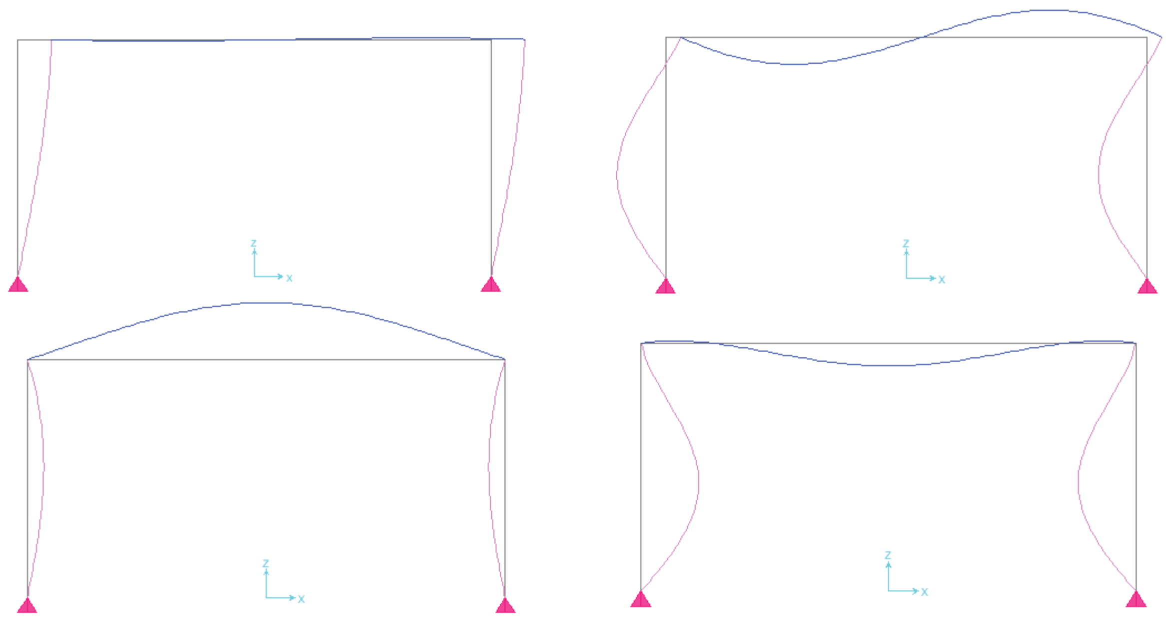

Fig. 8. *The first four modes oscillating support structures*

6. ANALYSIS OF RESULTS

Based on the results in subtitles, 2, 3, 4 and 5, Table 5 shows the comparative results for the first four circular frequencies. The analysis of the results showed that solutions for the circular frequency are approximately exact but just for the first and second mode, which means that for higher accuracy at higher modes it is necessary to apply a larger number sticks (finite element).

Comparing the values of eigenfrequencies obtained by approximate methods with exact method is concluded that the maximum relative error for the first two modes is 10,69% and 10,96%.

Furthermore, comparative results show that the approximation of the consistent masses is better than the approximation of the direct concentrated masses.

Table 5. *Frequencies – comparative results*

Circular frequency [rad/s]	Distributed masses	Consistent masses	Direct lumped masses	Software SAP2000
ω_1	25,413	22,697 (Δ=10,69%)	22,6283(Δ=10,96%)	22,58 (Δ=11,15%)
ω_2	98,5611	97,7225 (Δ=0,85%)	92,4762 (Δ= 6,17%)	97,62 (Δ=0,95%)
ω_3	268,24	244,074 (Δ=9,01%)	233,49(Δ=12,95%)	230,882(Δ=13,93%)
ω_4	339,898	253,964(Δ=25,28%)	249,741(Δ=26,52%)	241,353(Δ=28,99%)

7. CONCLUSION

The problem of oscillation in the plane is present in lifting engineering. It is particularly strong in crane high performance (high range and high speed trolley).

This paper presents three approaches to determining the characteristic frequency of bearing construction portal crane. At first step, the accurate approach to the dynamic analysis or mathematical-mechanical model with distributed masses is presented. In accordance to this, in the second step, two approximate approaches or mathematical model with consistent masses and the mathematic model with directly concentrated masses are presented. All three mentioned approaches are described by non-dimensional parameters, so they have universal character for symmetrical portal crane supporting constructions. This has special important in the first phase of crane designing.

By changing the parameters in the algorithm, the optimum solution is easy to be achieved, and resonance region can be avoided. The third step is formed and solved finite element model in the software SAP2000. This was roughly verified the results accurate and approximate approaches to dynamic analysis. It is shown that a consistent approximation to the masses better than direct lumped mass approximation.

Furthermore, the results of analysis which has been done have a direct importance for the problem of moving loads. From the aspect of determining the dynamic response of supporting construction, excited by moving load, approximation by consistently masses is better. In this sense, this work provides a good basis for examining the impact of the moving load on the dynamic behavior of the load-bearing structures. The paper also leaves room for optimization of these types of structures, based on the limited value intrinsic frequencies.

8. REFERENCES

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NOMENCLATURE

E	Young’s modulus
I	moment of inertia of cross-section
A	cross-sectional are
ρ	mass density of material
L	lenght of girder
H	height of colums
Y	transversal displacement
$Z(z)$	mode shape
$T(t)$	time function
z	spatial coordinate
t	time
ω	circular frequency
f	frequency
c	speed of wave propagation
l	length of finite element