Harmonic Analysis of a Pneumatic Fixed Orifice

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Orifices, with constant or variable cross-section areas, are essential components in pneumatic circuits. Dynamic and static characteristics of pneumatic systems depend on their flow characteristics. They have a special role in control components in which are used for mass flow control. In this paper the focus is on pneumatic system consisted of fixed orifices and pneumatic chamber. The characteristics of the fixed orifice are presented in frequency domain. For the purpose of analysis, the sinusoidal input describing function is used, obtained by simulation with nonlinear model.

Key words: fixed orifice, mass flow rate, describing function, Hammerstein model, pneumatic chamber

1. INTRODUCTION

Pneumatic servosystems are widely used in industrial applications because of the favourable performances/price ratio. However, high precision control of such systems is difficult due to their complex physical nature. The main causes of that complexity are: air compressibility, friction between the contact surfaces, nonlinear flow-pressure characteristics of the orifice type restriction and parameter variations [1-3]. In order to solve the problem of design and control of such systems, it is necessary to have better understanding of their nonlinear characteristics. A mathematical model which should clarify the most relevant dynamic and nonlinear behaviour in the pneumatic system is used for that purpose.

Fixed orifices are frequently encountered in pneumatic systems. As the nonlinear characteristics of the orifices reflect in the operation of the whole pneumatic system, they are observed and modelled as a separate subsystems. The paper presents and analyzes the nonlinear mass flow rate characteristics of fixed orifice in frequency domain.

One of the methods of analysis of nonlinear-systems the quasi-linearization method [4, 5]. Linearization in the ordinary sense is not valuable in the case when nonlinearity inputs exceed the limits of acceptable linear approximation or when there is discontinuity at the nominal operating point. The advantages of true linearization are kept in the case of quasi-linearization but there is no limit to the range of input signal magnitudes or to the selection of the operating point. The constraint is that linear description of the system depends on some properties of the input signal. The system description thus depends not only on the system itself, but also on the signals passing through the system (which is a property of nonlinear systems). In other words, quasi-linearization is performed for a certain form of input signal. The problem with nonlinear systems with feedback configurations is in difficult determination of the signal form which occurs on entering the nonlinearity. This is the main constraint of the method. It is not always possible to reduce the nonlinearity input signal to a simple form. The practical solution of the problem is to assume the form of the input signal in advance. In practice, three forms of input signals are used in quasi-linearization [4, 5]: bias, sinusoid and Gaussian process.

The quasi-linear function which approximatively describes nonlinearity is called the describing function (DF). As the design of control systems is frequently realized in the frequency domain, the Sinusoidal Input Describing Function (SIDF) is used in this paper. Assuming that the linear part of the system filters high order harmonics (low-pass filter), every periodic signal is reduced to a basic periodic function on entering the nonlinearity. In the case of memoryless nonlinearity, the SIDF represents the gain which is changed depending on the amplitude of the input signal.

The paper determines the SIDF of the nonlinear mass flow rate characteristic of pneumatic fixed orifice.

2. MASS FLOW-RATE CHARACTERISTIC OF PNEUMATIC FIXED ORIFICE

Fixed orifice characteristics depend on the environment in which a fixed orifice is used. In this paper we consider a pneumatic system consisting of a fixed orifice (Or) and a chamber (Ch) of constant volume (V) as shown in Fig. 1. The system is connected to a variable pressure source (Ps). Downstream pressure P depends on chamber dynamic. The chamber represents a generic load (e.g. actuator chamber) and has a role of low-pass filter. Therefore, it should be noted that the following analysis applies when the fixed orifice is connected to a storage type load.

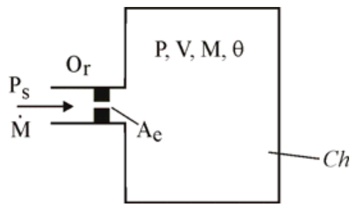

Figure1: Fixed orifice with chamber

The mass flow rate through the restriction can be in sonic or subsonic conditions depending upon the ratio of upstream-downstream pressure. According to the standard theory flow rate through the fixed orifice can be presented

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in the form [6]:

$$\dot{M} = A_e \varphi(P_s, P, \theta_s) \tag{1a}$$

where the function φ is defined as:

$$\varphi(P_s, P, \theta_s) = \begin{cases} C_1 \dfrac{P_s}{\sqrt{\theta_s}} & if \quad \dfrac{P}{P_s} \le P_{cr} \\ C_2 \dfrac{P_s}{\theta_s}\left(\dfrac{P}{P_s}\right)^{\frac{1}{\kappa}} \sqrt{1-\left(\dfrac{P}{P_s}\right)^{\frac{\kappa-1}{\kappa}}} & if \quad P_{cr} < \dfrac{P}{P_s} \le 1 \end{cases} \tag{1b}$$

while the parameters C_1, C_2 and P_{cr} are determined by:

$$C_1 = \sqrt{\frac{\kappa}{R}\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{\kappa-1}}}, \quad C_2 = \sqrt{\frac{2\kappa}{R(\kappa-1)}} \tag{1c}$$

$$P_{cr} = \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}}$$

If the downstream to upstream ratio is smaller than a critical value P_{cr} (0.528 for air), the flow is sonic and the function of upstream pressure is linear. If the pressure ratio is higher than P_{cr}, the flow is subsonic and depends nonlinearly on both pressures.

It can be seen that the mass flow rate, for a given effective area of restriction (A_e), depends on supply pressure (P_s), upstream temperature (θ_s), and working pressure P. Further, we assume that upstream pressure and air temperature in the chamber are constant (isothermal chamber) and equal to the ambient temperature:

$$\theta = \theta_a = \theta_s = const. \tag{2}$$

The relation (1) is graphically shown in Fig.2 for $A_e = 2.063x10^{-7}\ m^2$ and different values of working pressure. The supply pressure, as exogenous quantity, changes in the interval $\left(P_a < P_s \le 2.5P\ \right)$.

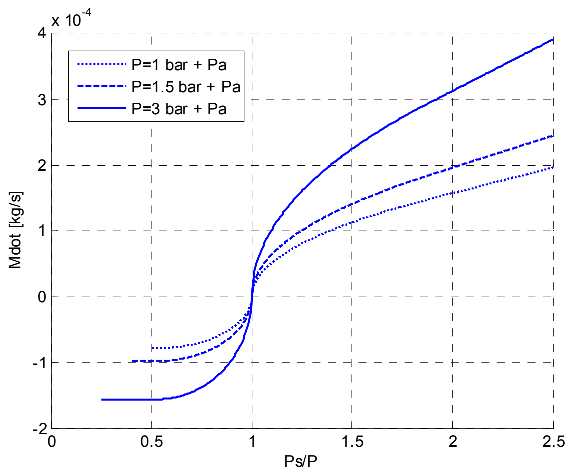

Figure 2: Mass flow rate through fixed orifice

For $P_s > P_a$ and $P_s / P \ge P_{cr}$ (sonic regime) the flow rate through an orifice has a constant value. For $P_{cr} \le P_s / P \le 1/P_{cr}$ (subsonic regime) the flow rate is a nonlinear function of square root-type. For $P_s / P > 1/P_{cr}$ (sonic regime) the flow rate is a linear function of P_s / P. Our objective is to introduce a quasi-linear operator which describe approximately the transfer characteristic of nonlinearity in frequency domain. It should be noted that justification for linearization of the mass flow in a subsonic regime depends on the choice of nominal operating point and the amplitude of the supply pressure P_s. In this paper we determine the describing function of the operating point:

$$P_s / P = 1\,;\ \dot{M} = 0 \tag{3}$$

3. DESCRIBING FUNCTION OF FIXED ORIFICE

Since the descriptive function of fixed orifice depends on the dynamics of the whole system we will first define a mathematical model of the pneumatic system from Fig.1. The dynamics of the pneumatic system can be shown by pseudo bond graph as shown in Fig 3.

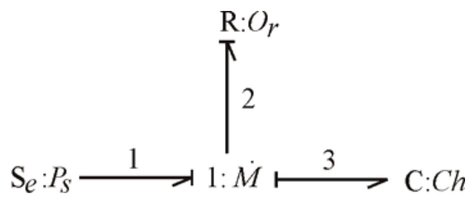

Figure 3: Pseudo bond graph of pneumatic system

A pressure and mass flow are used as energy values. The energy that comes from a constant pressure source (*E*-type source) is partly converted into heat in dissipator *R* and partly goes to the chamber, which represents the storage of *C*-type. The constitutive relation of the dissipator *R* is determined by flow characteristics of fixed orifice. For causality shown in Fig 3 the nonlinear block diagram of the pneumatic system is shown in Fig.4.

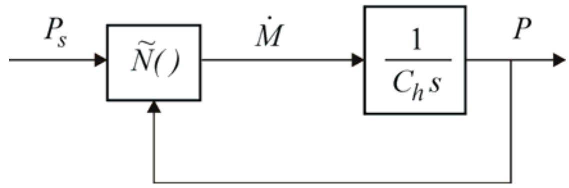

Figure 4: Nonlinear block diagram of pneumatic system

The nonlinearity $\tilde{N}()$ is determined by equations (1). We will now find the parameter C_h. Neglecting the kinetic and potential energy of the gas, based on the first law of thermodynamics, for the chamber we can write:

$$\frac{d}{dt}\left(C_v M\theta\right) = C_p \dot{M}\theta_s - A_h h\left(\theta - \theta_a\right) = C_p \dot{M}\theta \tag{4}$$

Using the state equation for a perfect gas:

$$PV = MR\theta \tag{5}$$

Based on (2), (4) it can be written:

$$C_h \frac{dP}{dt} = \dot{M} \tag{6}$$

where C_h:

$$C_h = \frac{V}{\kappa R\theta} \tag{7}$$

Our goal is that, in frequency domain, we find the appropriate Hammerstein model of the system, as shown in Fig.5.

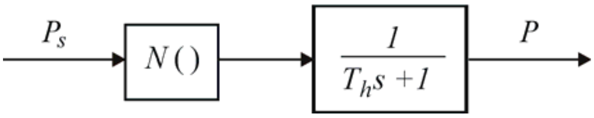

Figure 5: Hammerstein model of pneumatic szstem

Models from Fig.4 and Fig.5 are equivalent in the sense that the second model sufficiently accurate approximates the first model in frequency domain. The model shown in Fig.5 has separated linear dynamics and nonlinearity $N()$ whose describing function we need to determine. The linear part is the first order and the unity-gain. The time constant has the value:

$$T_h = \frac{C_h}{K_L} = \frac{1}{\omega_c} \tag{8}$$

where ω_c is cutoff frequency of linearized frequency characteristics of the pneumatic system.

The amplitude and phase characteristics of the linear part are given by the following equations:

$$A(\omega) = \frac{1}{\sqrt{1 + (\omega T_h)^2}} \tag{9a}$$

$$\varphi(\omega) = -\tan^{-1}(\omega T_h) \tag{9b}$$

Now, let us suppose that for a change of exogenous pressure:

$$P_s = P_{sA} \sin(\omega t) \tag{10}$$

a change of the operating pressure P in a stationary regime, is a periodic function that can be approximated by basic harmonics of the Fourier series:

$$P = a_0 + a_1 \cos(\omega t) + b_1 \sin(\omega t) \tag{11a}$$

or

$$P = a_0 + \sqrt{a_1^2 + b_1^2} \sin(\omega t + \varphi) \tag{11b}$$

where

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} P \, d(\omega t) \tag{12a}$$

$$a_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} P \cos(\omega t) \, d(\omega t) \tag{12b}$$

$$b_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} P \sin(\omega t) \, d(\omega t) \tag{12c}$$

$$\tan(\varphi) = a_1 / b_1 \tag{12d}$$

Similarly, based on (6) the mass flow through the orifice, in the frequency domain, can be approximated by:

$$\begin{aligned} \dot{M} &= C_h \omega [b_1 \cos(\omega t) - a_1 \sin(\omega t)] \\ &= C_h \omega \sqrt{a_1^2 + b_1^2} \cos(\omega t + \varphi) \end{aligned} \tag{13}$$

For many mechanical systems, due to the inertial nature, previois assumption is valid, i.e. they have necessary low-pas characteristics. For the system from Fig.4, accuracy of this assumption depends on the values of the time constant C_h of linear dynamics. This assumption is a fundamental condition for the application of SIDF technique, which requires that the input signal to the nonlinear element be essentially in sinusoidal form. The reason is that a limit of all periodic functions after propagation through law-pass linear filter is a sinusoid.

Values of coefficients a_1 and b_1 cannot be found in the analytical form due to the complexity of expressions (1) and (10). For the determination it is used the simulation of *A.C. potentiometer method* [2] for the nonlinear model which is shown in Fig.4. A shematic diagram of this method is shown in figure Fig.6.

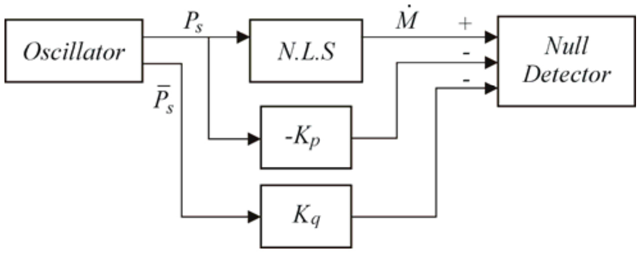

Figure 6: A.C. potentimeter method schematic view

At the oscillator output two signals of frequency ω are generated. One is P_s which is defined by (10) and the other is $\overline{P}_s$ and has the form:

$$\overline{P}_s = P_{sA} \cos(\omega t) \tag{14}$$

The signal P_s is used as the exogenous signal for the nonlinear system (N.L.S). It is recorded the change of variable $\dot{M}$ at the output. Variable gains K_p and K_q are set thus, in a steady state, it holds:

$$\dot{M} \approx K_q \cos(\omega t) - K_p \sin(\omega t) \tag{15}$$

For the determination of optimal values of parameters K_p and K_q the particle swarm optimization method is used [7]. The objective function has the form:

$$OF = (1/2\pi)\int_{0}^{2\pi} e^2 d(\omega t) \qquad (16a)$$

where is:

$$e = \dot{M} - \left[K_q \cos(\omega t) - K_p \sin(\omega t)\right] \qquad (16b)$$

After that, we can determine the value of coefficients in (11):

$$a_1 = K_p P_{sA} / (C_h \omega) \qquad (17a)$$

$$b_1 = K_q P_{sA} / (C_h \omega) \qquad (17b)$$

In Fig.7 the values of coefficients a_1 and b_1 are shown for different frequency and amplitude values of the input signal P_{sA}.

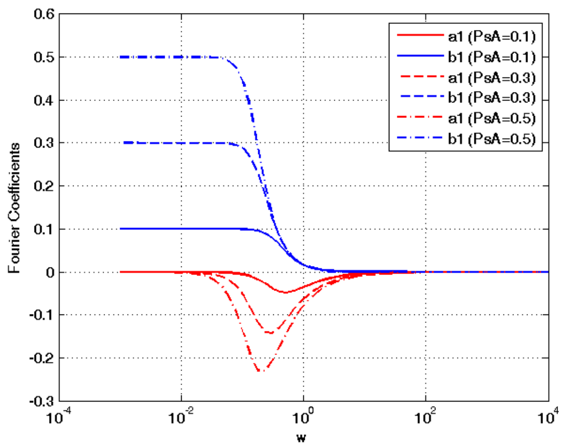

Figure 7:Fourier coefficients

It can be seen that the coefficient a_1 has a negative value in the whole interval for ω and all values of the input signal amplitude. The coefficient b_1 has a positive value. For small values of frequencies ($\omega < 10^{-2}$) we have:

$$a_1 = 0 \text{ i } b_1 = P_{sA} \qquad (18a)$$

which means that a phase shift of the system is equal to zero and a gain is equal to one. For large frequency values ($\omega > 10$) a completely signal attenuation is obtained:

$$a_1 = b_1 = 0 \qquad (18b)$$

The change of pressure P of nonlinear model (Fig.4) and its corresponding approximation (P_{apr}) which is defined by (11a) for $P_{sA} = 0.2$ and $\omega = 0.5$ are shown in Fig.8a.

The mass flow rate ($\dot{M}$) and its corresponding approximation ($\dot{M}_{apr}$) for the pressure change from previous figure are shown in Fig.8b.

The most significant changes of a_1 and b_1 occur on a mean interval. In Fig.9 it is shown the ratio a_1/b_1 on the interval $10^{-1} < \omega < 1$ for different values of P_{sA}.

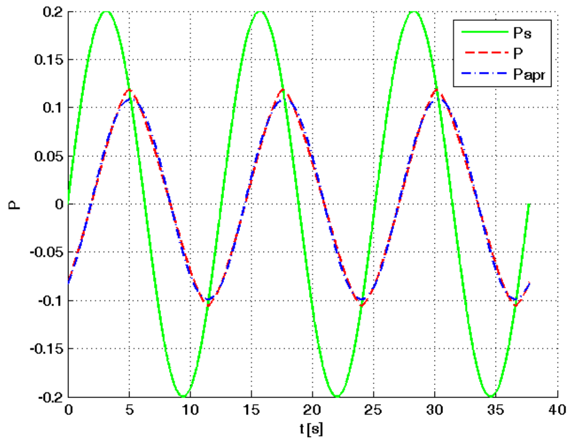

Figure 8a: Simulated pressure and approximation

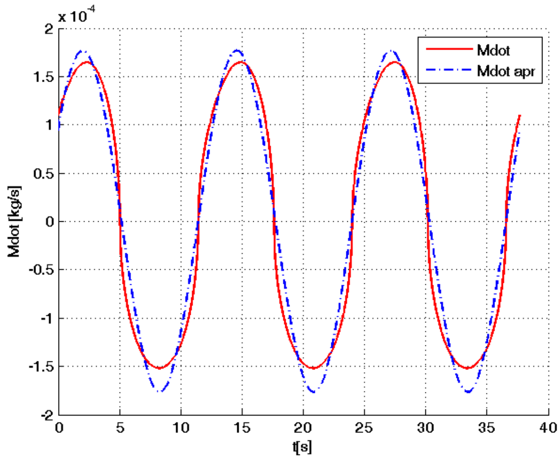

Figure 8b: Simulated mass flow rate and approximation

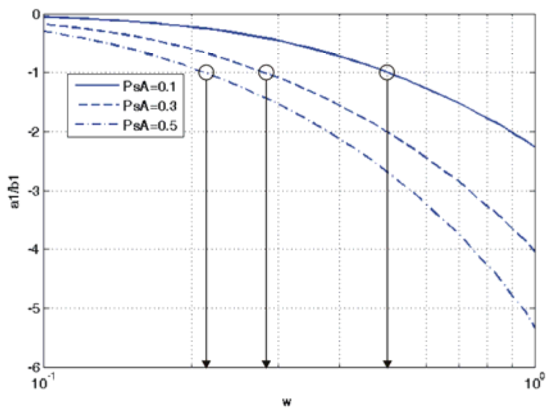

Figure 9: Relationship a_1/b_1

The picture shows the values for ω in which $a_1 = -b_1$. Based on (12d), at these frequencies a phase delay of the system is $-\pi/4$. For $C_h = 8.49x10^{-9}$ [ms^2] the values are obtained:

$$\omega^* = [0.5; 0.28; 0.21] \qquad (19)$$

Figs. 10a and 10b show the frequency characteristics of the whole system.

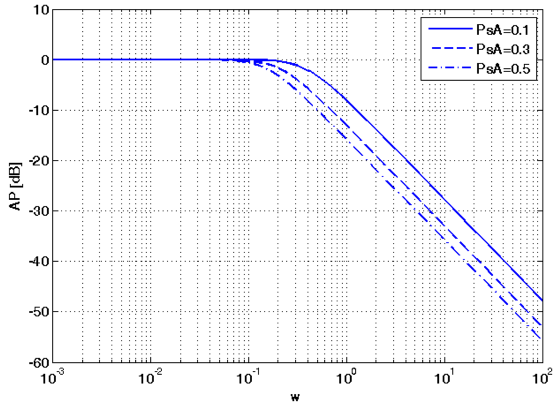

Figure 10a: Amplitude response

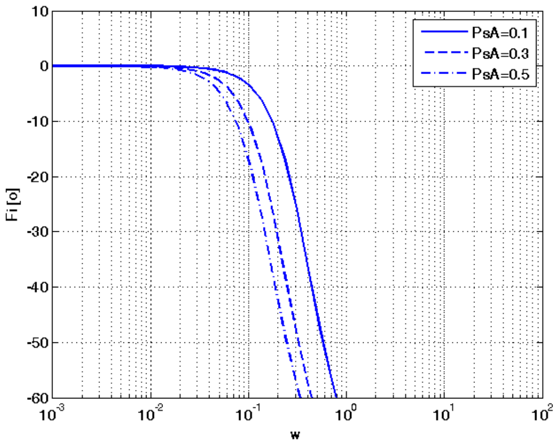

Figure 10b: Frequency response

With amplitude changing of the input signal the time constant of the system is also changed. At higher input amplitude the system becomes slower. It can be seen that the cutoff frequency of the whole system is determined by the cutoff frequency of the linear part (9). This means that the time constant for given amplitudes of the input signal is determined by frequencies (19)

$$T_h = [2;\ 3.57;\ 4.76]\ [s] \qquad (20)$$

For these time constants and the model which is shown in Fig.5 it can be numerically calculated the describing function of nonlinearity $N()$. Fig. 11a shows the module of describing function and the argument depending on the frequency of the input signal is shown in Fig. 11b.

It can be seen that the describing function depends on a frequency of the input signal and its amplitude. Around an inflection frequency the gain decreases and move in the interval [0.8 - 1]. It is also decreased the phase at the interval from 10^0 (phase sequencing) to -3^0 (phase delay).

Nomenclature

$\dot{M}$ - mass flow rate through orifice kg/s

A_e - effective area of restriction m^2

κ - specific heat ratio [.]

R - gas constant $J/(kgK)$

P - absolute pressure Pa

θ - temperature K

Subscripts

a - atmosphere

u - upstream

d - downstream

N - nominal operating regime

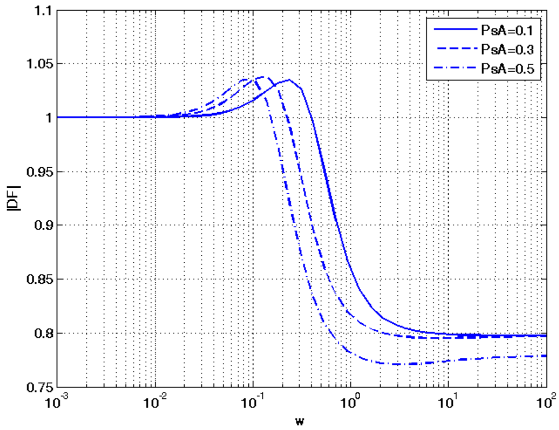

Figure 11a: |DF| - ω dependency

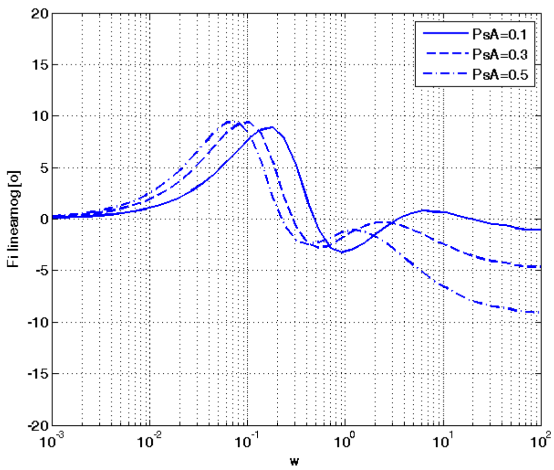

Figure 11b: arg(DF) - ω dependency

4. CONCLUSION

The presented system is a frequent configuration in pneumatic systems. In order to simplify the analysis the initial nonlinear model is transformed into an equivalent Hammerstein model. Instead of nonlinearity of two inputs it is introduced the nonlinearity with a single input. New nonlinearity is described by the describing function. For the determination of describing function simulation results are used. Describing function is dependent on the amplitude and frequencies of the input signal. In addition, the time constant of the linear part is changed with amplitude changing of the input signal.

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