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Robust Identification of Time-Varying Stochastic Systems

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Abstract ***- This is a second part for robust parameters estimation. Here we shell consider the case of time-varying parameters. First is considered parameters as deterministic which are modeled as random walk. As an estimator the robust Kalman filter is used. As an input signal is considered 1/f signal with corresponding autocovariance function. This signal is suitable for system identification, especially for the case of robust experiment design. In this part the some modifications for robust Kalman filter, as in part 1, are used. The simulations show good behavior of robust real-time identification algorithms.***

Key words: ***Robust Kalman filter, deterministic parameter variation, random walk, signal with prescribed autocovariance***

I. INTRODUCTION

The standard approach to control system design is to develop a linear model for the process for some operating condition and to design a controller having constant parameters. While linear, time invariant models no doubt form the most common way of a describing a dynamical system, it is also quite, often useful or necessary to employ other descriptions.

A specific, but common, case is when the systems properties vary with time. In this case, the task of the identification algorithm is to adapt itself so that it can approximately track the system dynamics, and it leads to the recursive identification. This is widely studied subject, and among the main references in this area of system identification we find the books [1-4].

One of the problems that appears in the area of identification of industrial processes is the identification in the presence of stochastic disturbance. Practical studies show that disturbance, in general, has non-Gaussian distribution. It is an especially important case when appears inconsistent, in relation to the main part of population, observations (outliers). Probability distributions for this case are approximately normal (ε -contaminated) and they are the subject of intense research in mathematical statistics. For such a case, we consider the robust procedures for parameters estimation.

We used output error (OE) type predictor for parametric identification of the dynamic system with time-varying parameters. Output error predictors can provide good performance when the plant output measurement is disturbed by noise. This can be explained by the fact that the output of this predictor does not directly depend upon the measured variables disturbed by noise, as the case with prediction error type predictor. The output of this predictor depends indirectly upon the measurements through the adaptation algorithm but this dependence can decrease in time by using a decreasing adaptation gain [4-5].

In this paper, we will apply the robust filter theory for solving problem of the robust parameter estimation of time-varying system. The Kalman filter works well, but it assumes that the system model and noise statistics are known. If any of these assumptions are violated then the filter estimates can degrade, [6-7].

Although the Kalman filter is the optimal linear filter, it is not the optimal filter in general for non-Gaussian noise. Noise in nature is often approximately Gaussian but with heavier tails, and the Kalman filter can be modified to accommodate these types of density functions [8-10].

Because of some disadvantages which are associated with the robust Masreliez-Martin's filter, we have decided to make some modification of the robust filter proposed by Masreliez and Martin, [11].

Since modificated Masreliez-Martin's robust filter showed satisfactory properties in the case of constant parameters, we will use this robust Kalman filter as parameter estimator in the case of time-varying system.

The generation of input signal is inspired by recent work on experiment design where it was shown that a bandlimited '1/f' noise has good properties in robust identification, [12]. As in [11], the theory of experiment design is used in order to reduce the time needed for identification. Recall that the aim was to create the input signal for identification (excited signal) through a recursive relation for autocovariance function. Synthesis of autocovariance function is based on the ideas of predictive control, [13].

II. PARAMETER ESTIMATION OF RANDOM WALK PROCESS

If the system parameters vary according to the random walk model, then it can met next assumption, which yields unbounded parameter trajectories.

There exists a constant $0 < c < \infty$ such that

$$E\left(\left\|\theta(k)\right\|^2\right) < c \tag{1}$$

Assumption (1) implies boundedness of the parameter trajectory in the mean square sense.

Let us consider a single-input/single-output linear dynamic system described by:

$$y(k,\theta) = \varphi^T(k)\theta(k) + \nu(k) \quad (2)$$

where $\theta(k)$ represents vector of an unknown time-varying coefficients and $\varphi(k)$ is measurable regression vector. Similarly as in [11] (but now for time-varying case), assume that $\nu(k)$ is observation noise with Non-Gaussian distribution.

Unlike an explicit deterministic model of parameter variation, the statistical filtering approach assumes knowledge of a stochastic model of parameter changes. In general case, it can suppose that the variation of the parameter vector $\theta(k)$ can be described by the random walk model

$$\theta(k+1) = A\theta(k) + \gamma w(k) \quad (3)$$

in which A represents known $n \times n$ stable matrix, $\{w(k)\}$ denotes the parameter driving Gaussian noise with zero mean and the covariance $Q(k)$, independent of the measurement noise $\{\nu(k)\}$. Here, γ serves as a scaling factor.

We will mention agreed dynamic system whose $n \times 1$ dimensional state vector $x(k)$ $(k \geq 0)$ is estimated from scalar observations $y(1),\ldots,y(k)$, in which $x(k)$ and $y(k)$ satisfy the linear state and observation relations

$$x(k+1) = F(k)x(k) + \gamma w(k) \quad (4)$$

$$y(k) = H(k)x(k) + \nu(k) \quad (5)$$

with $n \times n$ state transition matrix $F(k)$, $1 \times n$ observation matrix $H(k)$, process noise with Gaussian distribution $w(k)$ and the scalar measurement noise $\nu(k)$. As for the constant parameter case [11], the density for observation noise $\nu(k)$ is represented by the Gaussian mixture:

$$p(\nu(k)) = (1-\varepsilon)N(\nu|0,\sigma_1^2) + \varepsilon N(\nu|0,\sigma_2^2), \; 0 \leq \varepsilon < 1 \quad (6)$$

Regarding the parameter vector $\theta(k)$ as a "state $x(k)$" of a dynamic system with output governed by (2), one can formulate the problem of parameter estimation as a problem of filtering in the state space. The transposed regression vector $\varphi^T(k)$ in (2) can be interpreted as a vector of time-varying, but known, output (measurement) coefficients $H(k)$, and matrix A in (3) can be interpreted as a matrix of state transition coefficients $F(k)$.

The Kalman filtering approach allows one to incorporate into the process of system identification the prior knowledge about the estimated coefficients. If no such knowledge is available, one can adopt "noninformative" priors by setting

$$\Sigma_0^{-1} = O \quad (7)$$

where $n \times n$ matrix Σ_0 can be interpreted as initial covariance matrix of the filtered state estimate $\hat{\theta}(t)$.

The fact that the output vector in (2) is not only time dependent but also data dependent is clearly a nonstandard feature of the state space description given by (2) and (3). It is known, however, that the Kalman filtering can be extended to the case where system coefficients are functions of past input and output variables, which is true in the case considered.

Finally, we can apply modified robust Masreliez-Martin's filter, which is proposed in [11] , for robust output error parameter estimation of the time-varying system (2-3).

Different from general case of random walk process (3), in this paper we consider the case without Gaussian noise $w(k)$. It means that we solve the robust parameter estimation problem of the time-varying dynamic system in which the system parameters vary according to the deterministic random walk model

$$\theta(k+1) = A\theta(k) \quad (8)$$

Thus, the estimate of $\theta(k)$ can be computed recursively using the robust filtering algorithm:

$$\hat{\theta}(k) = \bar{\theta}(k) + M(k)\phi(k)\psi_p[\nu(k)] \quad (9)$$

$$\bar{\theta}(k) = A\hat{\theta}(k-1) \quad (10)$$

$$\nu(k) = y(k) - \phi^T\bar{\theta}(k) \quad (11)$$

$$M(k) = AP(k-1)A^T \quad (12)$$

where the filter covariance, $P(k)$ now satisfies

$$P(k) = M(k) - M(k)\phi(k)\phi^T(k)M(k)\psi_p^{'}[\nu(k)]. \quad (13)$$

As introduced in [11], $\psi_p[\nu(k)]$ and $\psi^{'}{}_p[\nu(k)]$ represent Huber's function and its derivative, respectively.

Note that the output error adjustable predictor is used:

$$\hat{y}(k) = \phi^T(k)\hat{\theta}(k) \quad (14)$$

where $\hat{y}(k)$ denotes the *a posteriori* output of the predictor, in which:

$$\begin{aligned}\phi(k) &= [-\hat{y}(k-1),\ldots,-\hat{y}(k-n),u(k-1),\ldots,u(k-m)]^T \\ \theta(k) &= [\hat{a}_1(k),\ldots,\hat{a}_n(k),\hat{b}_1(k),\ldots,\hat{b}_m(k)]^T\end{aligned} \quad (15)$$

III. GENERATION OF INPUT SIGNAL

Optimal test signals are frequently specified in terms of their second order properties, e.g. autocovariance or spectrum. This leads to the problem of implementing a real signal with specified second order properties. Moreover, it is usual that the input should also be constrained in its amplitude, i.e. the amplitude must lie in an interval [a, b] $\subset R$.

As mentioned in introduction section, to generate a binary signals whose sampled autocovariance is as close as possible to some prescribed autocovariance, we apply a simple procedure, based on the use of the receding horizon concept commonly employed in Model Predictive Control, see [13]. The idea is to solve, for each time instant, a finite horizon optimisation problem to find the optimal set of the next, say, T values of the sequence such that the sampled autocovariance sequence so obtained is as close as possible (in a prescribed sense) to the desired autocovariance. One then takes the first term of this optimal set for the sequence, advances time by one step and repeats the procedure.

The generation of input signal is inspired by recent work on experiment design where it was shown that a bandlimited ‘1/f’ noise has good properties in robust identification, [12].

Bandlimited ‘1/f’ noise is defined by the following spectrum:

$$\phi^{1/f}(\omega) := \begin{cases} \dfrac{1/\omega}{\ln \bar{\omega} - \underline{\omega}} & \omega \in [\underline{\omega}, \bar{\omega}] \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

where $\underline{\omega}, \bar{\omega} \in R$, $(\underline{\omega} < \bar{\omega})$. The autocovariance sequence of this signal is given by

$$r_k^{1/f} := \frac{1}{\ln \bar{\omega} - \underline{\omega}} \int_{\underline{\omega}}^{\bar{\omega}} \frac{\cos kx}{x} dx, \; k \in N_0 \quad (17)$$

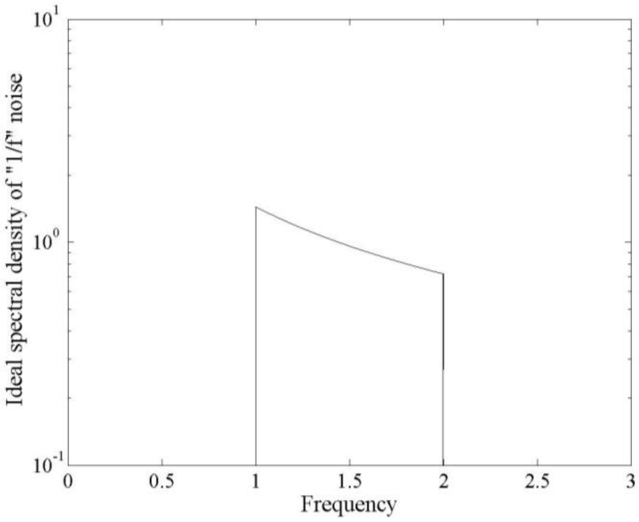

Fig. 1. Power spectral density of bandlimited ‘1/f’ noise signal for $\underline{\omega}$ =1 and $\bar{\omega}$ = 2

Figure 1 shows the ideal spectral density of bandlimited ‘1/f’ noise signal for $\underline{\omega}$ =1 and $\bar{\omega}$ = 2. In Figure 2 we present the results obtained from the receding horizon algorithm for $\underline{\omega} = 1$, $\bar{\omega} = 2$, m=1, N=10^6 and n=50. This last figure verifies the ability of the algorithm to generate a binary non-white noise signal. The discrepancies between the desired and the achieved autocovariances seem to be due to the impossibility of generating a binary signal with a true bandlimited ‘1/f’ spectrum, as the results do not appear to improve significantly by increasing m and n. For more details see [13].

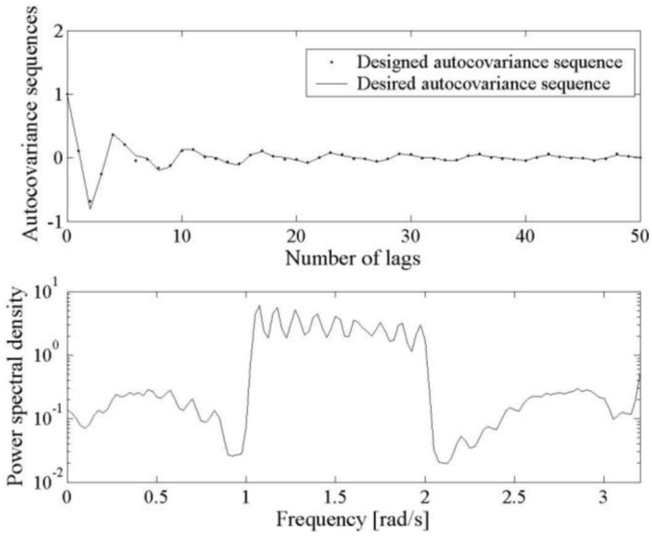

Fig. 2. Characteristics of the generated pseudo bandlimited ‘1/f’ noise signal for m = 1, N = 10^6 and n = 50.

IV. SIMULATION RESULTS

To demonstrate the performance of proposed robust procedure for parameters estimation, we consider a single-input/single-output (SISO) linear dynamic system, described by:

$$y(k,\theta) = \varphi^T(k)\theta(k) + \nu(k) \quad (18)$$

$$\theta(k+1) = A\theta(k) \quad (19)$$

where square matrix A is Schur stable:

$$A = \begin{bmatrix} 0.99 & -0.012 & 0.001 & -0.001 \\ -0.005 & 0.98 & -0.001 & -0.001 \\ -0.001 & 0.01 & 1 & -0.001 \\ 0.01 & 0.001 & -0.002 & 0.99 \end{bmatrix} \quad (20)$$

From (18), can be seen the dynamics of the time-varying process is involved in the noise process. We consider the case when the density for observation noise $\nu(k)$ is Non-Gaussian, with contamination ε =0.1 and variances σ_1^2 =1 and σ_2^2 =100, see (6). As input signal we utilize the generated bandlimited ‘1/f’ noise, which frequencies lie in interval $[1, 2]$.

Figures 3 to 5 show identification results obtained using proposed modified robust Masreliez-Martin’s filter as parameter estimator, for the time-varying output error process model.

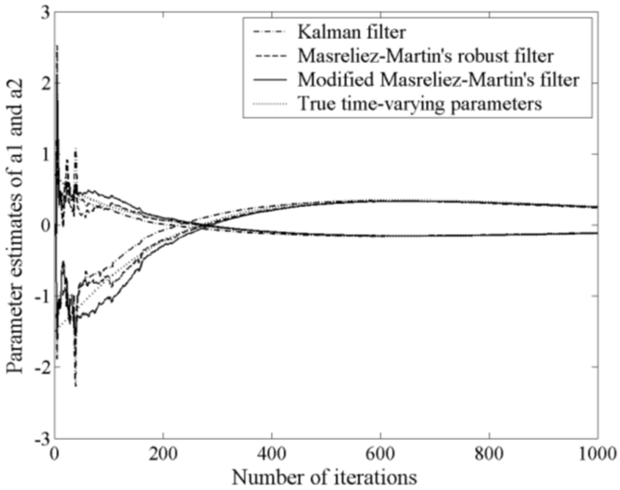

Fig. 3. Estimates of parameters a_1 and a_2

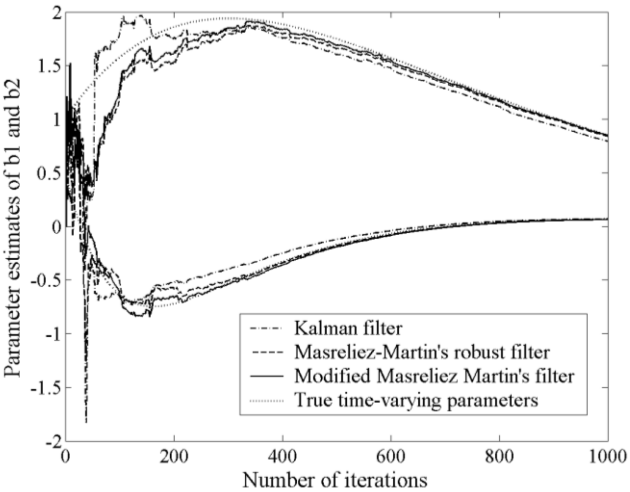

Fig. 4. Estimates of parameters b_1 and b_2

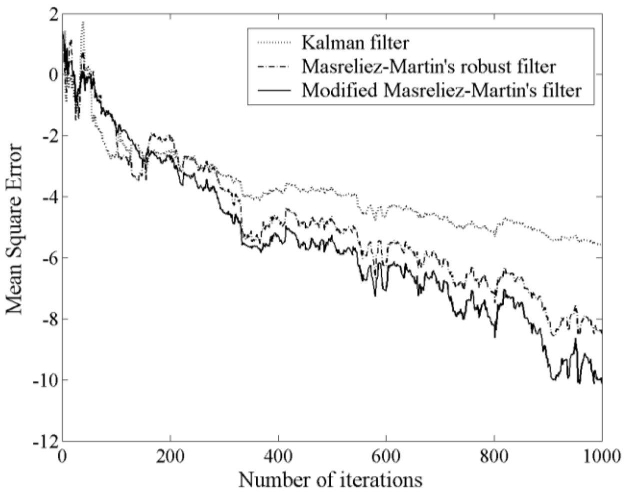

Fig. 5. Mean Square Error

V. CONCLUSION

The basic objective of this paper is to consider how the proposed robust filter theory copes with the problem of the robust parameter estimation of the time-varying process. It assumed that the plant output measurement is disturbed by Non-Gaussian noise. We chose the output error predictor because it has naturally a better rejection of the effect of disturbances then prediction error type predictor. Because of its good properties in robust identification, bandlimited white noise is generated by proposed algorithm based on ideas from predictive control.

Simulation results have demonstrated the efficiency of the proposed robust filtering method for solving problem of the parameter estimation of time-varying system in the presence of outliers.

REFERENCES

[1] L.Ljung and T. Söderström, *Theory and Practice of Recursive Identification*, MIT Press, Cambridge, Massachusetts, 1983

[2] L. Ljung, *System Identification*, 2nd ed., New Jersey: Prentice Hall, 1999.

[3] G.C. Goodwin and K.S. Sin, *Adaptive Filtering, Prediction and Control*, Prentice-Hall, Englewood Cliffs, New Jersey, 1984

[4] I.D. Landau, R. Lozano and M. M'Saad, *Adaptive Control*, 1st ed., Springer, 1998

[5] L. Dugard and I.D. Landau, "Recursive output error identification algorithms theory and evaluation", *Automatica*, Vol. 16, Iss. 5, pp. 443-462, 1980

[6] T. Nishimura, "On the a priori information in sequential estimation problems", *IEEE Transactions on Automatic Control*, AC–11, pp. 197-204, 1966.

[7] H. Heffes, "The effect of erroneous models on the Kalman filter response", *IEEE Transactions on Automatic Control*, AC-11, pp. 541-543, 1966.

[8] C. J. Masreliez and R. D. Martin, "Robust Bayesian estimation for the linear model and robustifying the Kalman filter", *IEEE Transactions on Automatic Control*, Vol. 22, Iss. 3, pp. 361–371, 1977.

[9] C. J. Masreliez, "Approximate non-Gaussian filtering with linear state and observation relations", *IEEE Transactions on Automatic Control*, Vol. 20, Iss. 1, pp. 107–110, 1975.

[10] C. Tsai and L. Kurz, "An adaptive robustizing approach to Kalman filtering", *Automatica*, Vol.19, Iss.3, pp. 278-288, May 1983.

[11] V. Filipovic and V. Stojanovic, "Robust Kalman filter as parameter estimator for output error models ", *X Triennial International SAUM Conference on Systems, Automatic Control and Measurements*, Niš, Serbia, November 10th-12th, 2010

[12] C. R. Rojas, G. C. Goodwin, J. S. Welsh, and A. Feuer, "Robust optimal experiment design for system identification", *Automatica*, Vol. 43, pp. 993-1008, 2007.

[13] R. Rojas, J. Welsh and G. Goodwin, "A receding horizon algorithm to generate binary signals with a prescribed autocovariance", *American Control Conference*, pp. 122-127, New Jork, 2007