9th International Congress of the
Serbian Society of Mechanics
July 5-7, 2023, Vrnjacka Banja, Serbia

Book of Proceedings

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Dear colleagues,

The Serbian Society of Mechanics organized the 9th International Congress of the Serbian Society of Mechanics from 5th to 7th July 2023 in Vrnjačka Banja, Serbia. The aims of the congress were to bring together leading academic scientists, researchers and research scholars to exchange and share experiences and research results on various aspects of Theoretical and Applied Mechanics. The congress brought an interdisciplinary platform for researchers, practitioners and educators to present and discuss the most recent innovations, theories, algorithms, as well as practical challenges encountered and solutions adopted in the fields of Classical Mechanics, Solid and Fluid Mechanics, Computational Mechanics, Biomechanics, Applied Mathematics and Physics, Structural Mechanics and Engineering. More information on the Conference can be found on the webpage: http://www.ssm.kg.ac.rs/congress_2023/.
Co-organizers of the conference were:
Faculty of Engineering, University of Kragujevac
Faculty of Mechanical Engineering, University of Belgrade
Faculty of Technical Science, University of Novi Sad
Faculty of Mechanical Engineering, University of Niš
Institute for Information Technology, University of Kragujevac
University of Kragujevac
Mathematical Institute of the Serbian Academy of Sciences and Arts
Serbian Academy of Sciences and Arts
Serbian Society of Computational Mechanics.

Financial support was provided by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia.

Plenary speakers were:

1. Assoc. Prof. Dr Miha Brojan – *University of Ljubljana, Slovenia*
   Title: From symmetry breaking to functionality: Examples from nonlinear mechanics of beams, plates and shells
2. Prof. Dr. Dimitri V. Georgievskii – *Institute of Mechanics, Lomonosov Moscow State University, Russia*
   Title: Elements of the Theory of Constitutive Relations and Formulations of the Linearized Problems on Stability
3. Prof. Dr. Stefano Lenci – *Department of Civil and Building Engineering, and Architecture, Polytechnic University of Marche, Ancona, Italy*
   Title: Nonlinear Wave Propagation in Cables and Beams Resting on a Bilinear Foundation
4. Prof. Dr. Parviz Moin – *Center for Turbulence Research, Stanford University, California, USA*
   Title: Large Eddy Simulation at Affordable Cost: Application to a Full Aircraft Configuration
5. Prof. Dr Rafal Rusinek – *Lublin University of Technology, Lublin, Poland*
   Title: Bio-electro-mechanical System of the Human Middle Ear
6. Assoc. Prof. Dr. Jelena M. Svorcan – *Department of Aerospace Engineering, Faculty of Mechanical Engineering, University of Belgrade, Serbia*
   Title: Numerical Investigation of Flows Around Small-Scale Propellers: Possibilities and Challenges

There were also presentations within four Mini-symposia:

- MS1: Mechanical Metamaterials
  Organizers: Milan Cajić, *Mathematical Institute SANU, Serbia*; Danilo Karličić, *Mathematical Institute SANU, Serbia*
- MS2: Turbulence
  Organizer: Đorđe Čantrak, *University of Belgrade, Serbia*
  Plenary speakers:
  1. Prof. Dr. Parviz Moin – *Center for Turbulence Research, Stanford University, California, USA*
  Title: Probing Turbulence Physics Using Numerical Simulation Databases – A Case Study in Predictive Science
  2. Prof. Dr. Nikolay Nikitin – *General Aerodynamics Laboratory, Institute of Mechanics, Lomonosov Moscow State University, Russia*
  Title: Secondary Flows of Prandtl's Second Kind. Mechanism of Formation and Method of Prediction
  3. Prof. Dr. Ji Pei – *National Research Center of Pumps, Jiangsu University, China*
  Title: Research on High Efficiency and High Reliability Pumps in Jiangsu University
- MS3: Biomechanics and Mathematical Biology
  Organizers: Andjelka Hedrih, *Mathematical Institute of the Serbian Academy of Sciences and Arts, Serbia*; Marat Dosaev, *Lomonosov Moscow State University, Moskva, Russian Federation*
  Plenary Speakers:
  1. Prof. Dr. Su Fong-Chin – *Department of Biomedical Engineering, College of Engineering, National Cheng Kung University, Tainan, Taiwan; Medical Device Innovation Center, National Cheng Kung University, Tainan, Taiwan*
  Title: An Exergame-Integrated IoT-Based Ergometer System for Personalized Training of the Elderly
  2. Prof. Dr. Marco Amabili – *Department of Mechanical Engineering, McGill University, Montreal, Canada*
  Title: Effect of Smooth Muscle Activation in the Static and Dynamic Mechanical Characterization of Human Aortas
- MS4: Nonlinear Dynamics
  Organizer: Julijana Simonović, *Faculty of Mechanical Engineering University of Niš, Serbia*

We have received more than 130 high-quality research papers. As a result of the strict review process and evaluation, the committee selected 91 papers for publishing in this Book of proceedings.
Special gratitude goes to the members of the program and scientific review committee as well as to all chairs, organizers and committee members for their dedication and support.

On behalf of the Organizing Committee

Prof. Dr Nenad Filipović
Chair of ICSSM2023

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PHASE-FIELD MODELING OF HIGH CYCLIC FATIGUE IN BRITTLE MATERIALS

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Abstract

The engineering structures are designed to operate in an elastic domain, and the plastic deformations are not permissible. Therefore, the safety of structures can be violated if the structure is subjected to cyclic loading conditions due to the accumulation of damage in the material. The damage of the material is usually considered as the appearance of micro-cracks and dislocation. These phenomena can be modeled by the Phase-Field Damage Modeling (PFDM) approach. The PFDM in brittle materials considers the elastic strain energy the main and the only source of material damage. By introducing the fatigue degradation function, the PFDM can be extended to predict the structure's fatigue under cyclic loading-unloading conditions. The PFDM framework is implemented into Finite Element Software, and the first simulation results on one element example are presented. The results show good qualitative simulation results, which follow the phenomenology of structures' behavior under cyclic loading and fatigue conditions.

Keywords: cyclic loading, fatigue, elastic strain energy, phase-field modeling, damage.

1. Introduction

The fracture of structures subjected to cyclic loading is one of the primary failure mechanisms. So, developing the correct simulation models to predict fatigue behavior is essential. In the material, the crack is first initiated by micro-voids and micro-cracks, while in further loading, the cracks grow and propagate, leading to the structure's fracture.

In the experimental investigation of fatigue, the fatigue life is estimated as the relationship between cyclic stress or strain range and the number of cycles in the form of Wöhler diagrams or S-N curves. In high-cycle fatigue (HCF), the material is subjected to low-stress amplitude in an elastic regime, leading to many fracture cycles.

The phase-field damage model (PFDM) is based on Griffith's thermodynamic framework. The crack will increase if the released energy exceeds the material toughness Francfort and Marigo [1] and Bourdin et al. [2].

Many researchers tried implementing fatigue into PFDM [3, 4]. Therefore, a fatigue degradation function describing the fatigue history is introduced to reduce the material toughness [4-6].

This paper presents the PFDM implementation for modeling the fatigue behavior of metallic materials, and the first simulation results are shown. The application of the implementation is proposed for HCF based on Golahmar et al. [4] and Carrara et al. [5]

The theoretical background of the phase field fatigue framework is described in Section 2. Then, the obtained results and discussion are provided in Section 3. Finally, the article ends with conclusions in Section 4.

2. Methods

2.1 A phase field model for fatigue damage

By the formulation of cracks in one-dimensional solids, and the extension of the regularized crack functional to multi-dimensional problems [7,8], the crack surface is defined as follows:

$$S(d)=\int_V \gamma(d,\nabla d)dV \tag{1}$$

where γ is the crack surface density function per unit volume. The internal potential energy density ψ is considered as the sum of mechanical $\psi^M(\boldsymbol{\varepsilon},d)$ and fracture surface energy density $\varphi^S(d,\nabla d)$ [7,8]:

$$\psi=\psi^M(\boldsymbol{\varepsilon},d)+\varphi^S(d,\nabla d) \tag{2}$$

where $\boldsymbol{\varepsilon}$ is the total strain tensor and d is the damage variable. The mechanical strain energy density of virgin material $\psi_0(\boldsymbol{\varepsilon})$ is multiplied by degradation function $g(d)$ to define the mechanical strain energy density ψ^M as [7,8]:

$$\psi^M(\boldsymbol{\varepsilon},d)=g(d)\psi_0(\boldsymbol{\varepsilon})=g(d)\frac{1}{2}\boldsymbol{\varepsilon}^T:\boldsymbol{\sigma}_0 \tag{3}$$

where $\boldsymbol{\sigma}_0$ is the Cauchy stress tensor of an undamaged solid. Similarly, the "damaged" Cauchy stress $\boldsymbol{\sigma}$ is given in the following form [7,8]:

$$\boldsymbol{\sigma}=g(d)\boldsymbol{\sigma}_0=g(d)\mathbf{C}_0:\boldsymbol{\varepsilon} \tag{4}$$

where $\mathbf{C}_0$ is the fourth-order elastic constitutive matrix. The fracture surface energy Φ^S at the crack surface S is defined as [7,8]:

$$\Phi^S=\int_S G_c dS\approx\int_V G_c\gamma(d,\nabla d)dV=\int_V \varphi^S(d,\nabla d)dV \tag{5}$$

where the fracture surface energy density dissipated by the formation of the crack is:

$$\varphi^S(d,\nabla d)=G_c\gamma(d,\nabla d) \tag{6}$$

and G_c is the Griffith-type critical fracture energy release rate. For a cumulative history variable $\bar{\alpha}\geq 0$, and a fatigue degradation function $f(\bar{\alpha})$, the fracture surface energy density can be re-formulated as follows [4,5]:

$$\bar{\varphi}^S(d,\nabla d,\bar{\alpha})=f(\bar{\alpha})G_c\gamma(d,\nabla d) \tag{7}$$

The total internal potential energy W_{int} functional is defined as [6-8]

$$W_{\text{int}} = \int_V \psi dV = \int_V \left\{ g\left(d\right)\frac{1}{2}\boldsymbol{\varepsilon}^T : \boldsymbol{\sigma}_0 + f\left(\bar{\alpha}\right) G_V \left[\frac{d^2}{2} + \frac{l_c^2}{2}\left|\nabla d\right|^2 \right] \right\} dV \tag{8}$$

where a critical fracture energy release rate per unit volume is $G_V = G_c / l_c$ and l_c is the characteristic length. The variation of the internal potential energy over the total strain and damage is given as [4-8]:

$$\delta W_{int} = \int_V \left\{ \boldsymbol{\sigma} : \delta\boldsymbol{\varepsilon} + f\left(\bar{\alpha}\right) G_V \left[d\delta d + l_c^2 \nabla d \nabla \delta d \right] \right\} dV \tag{9}$$

A variation of the external potential energy W_{ext} is known as [7,8]:

$$\delta W_{ext} = \int_V \mathbf{b} \cdot \delta\mathbf{u} dV + \int_A \mathbf{h} \cdot \delta\mathbf{u} dA \tag{10}$$

where b is a body force field per unit volume, h is a boundary traction per unit area, and u is the displacements vector. The equilibrium of the internal and external potential energy [4-8]:

$$\begin{aligned} &\int_V \left\{ \boldsymbol{\sigma} : \delta\boldsymbol{\varepsilon} + \frac{1}{2} g'\left(d\right) \boldsymbol{\varepsilon}_E^T : \boldsymbol{\sigma}_0 \delta d + f\left(\bar{\alpha}\right) G_V \left[d\delta d + l_c^2 \nabla d \cdot \nabla \delta d \right] \right\} dV \\ &= \int_V \mathbf{b} \cdot \delta\mathbf{u} dV + \int_A \mathbf{h} \cdot \delta\mathbf{u} dA \end{aligned} \tag{11}$$

By the application of total derivatives and by using the Gauss theorem, the following can be obtained [4-8]:

$$\begin{aligned} &\int_V \left\{ -\left[g'\left(d\right)\psi_0 + f\left(\bar{\alpha}\right) G_V \left[d - l_c^2 \nabla^2 d \right] \right] \delta d - \left[Div\left[\boldsymbol{\sigma}\right] + \mathbf{b} \right] \cdot \delta\mathbf{u} \right\} dV \\ &+ \int_A \left\{ \left[\boldsymbol{\sigma} \cdot \mathbf{n} - \mathbf{h} \right] \cdot \delta\mathbf{u} \right\} dA + \int_A \left\{ \left[G_V l_c^2 \nabla d \cdot \mathbf{n} \right] \delta d \right\} dA = 0 \end{aligned} \tag{12}$$

where $\mathbf{n}$ is the unit outer normal to the surface A. The Neumann-type boundary conditions are

$$\boldsymbol{\sigma} \cdot \mathbf{n} - \mathbf{h} = 0 \tag{13}$$

$$\nabla d \cdot \mathbf{n} = 0 \tag{14}$$

what leads to the governing balance equations [4-8]:

$$Div\left[\boldsymbol{\sigma}\right] + \mathbf{b} = 0 \tag{15}$$

$$f\left(\bar{\alpha}\right) G_V \left[d - l_c^2 \nabla^2 d \right] + g'\left(d\right)\psi_0 = 0 \tag{16}$$

2.2 Fatigue damage

The cyclic loading produces damage increase what is captured by a fatigue degradation function $f\left(\bar{\alpha}\right)$ [3-6]. This function degrades the material toughness concerning the fatigue history variable. For the pseudo time, the history variable can be defined as [3-6]:

$$\bar{\alpha}\left(t\right) = \int_0^t H\left(\alpha\dot{\alpha}\right)\left|\dot{\alpha}\right| d\tau \tag{17}$$

where $H(\alpha\dot{\alpha})$ is the Heaviside step function [6]:

$$H(\alpha\dot{\alpha}) = \begin{cases} 1, & \alpha\dot{\alpha} \geq 0 \text{ for loading} \\ 0, & \text{otherwise for unloading} \end{cases} \tag{18}$$

The fatigue history variable α is defined as [3-6]:

$$\alpha = g(d)\psi \tag{19}$$

while the fatigue degradation function is [6]:

$$f(\bar{\alpha}) = \begin{cases} 1 & i\text{f} \quad \bar{\alpha} \leq \alpha_T \\ \left(\dfrac{2\alpha_T}{\bar{\alpha} + \alpha_T}\right)^2 & \text{if} \quad \bar{\alpha} > \alpha_T \end{cases} \tag{20}$$

Here, α_T represents a threshold value, below which the fracture energy remains unaffected, which should be determined experimentally and in this case is adopted according to the literature [6]:

$$\alpha_T = \frac{G_V}{12} \tag{21}$$

3. Results and discussion

The uniaxial loading example of the unit cube is created to simulate the phenomenology of the damage evolution under High Cyclic Fatigue (HCF) loading conditions using the PFDM approach extended by the fatigue degradation function described in the previous section.

The unit cube is loaded on one side by prescribed displacements while the other directions are restrained. The opposite side is fully restrained [9]. Only the elastic material parameters of general metallic material are used as data from the literature [9] and are given in Table 1. According to the literature, the additional material parameters, such as fracture energy, characteristic length, and tolerance of convergence, are set to show the material damaging and fatigue phenomena [9].

Firstly, the monotonic loading is performed to the total strain of 1% to show the stress-strain response of the material. Then, two simulations were performed - one with the influence of the fatigue degradation function and the one without. The examples are solved in 1000 loading steps.

After that, the three loading and unloading cycles are performed for both cases. The results are given in Figure 1. The examples are solved in 1800 loading steps up to a total strain of 0.3%.

As can be noticed in Figure 1, the fatigue degradation function has a significant influence on the material response. The damage value for the total strain is given in Figure 2, where we can see how it increases for each simulation case.

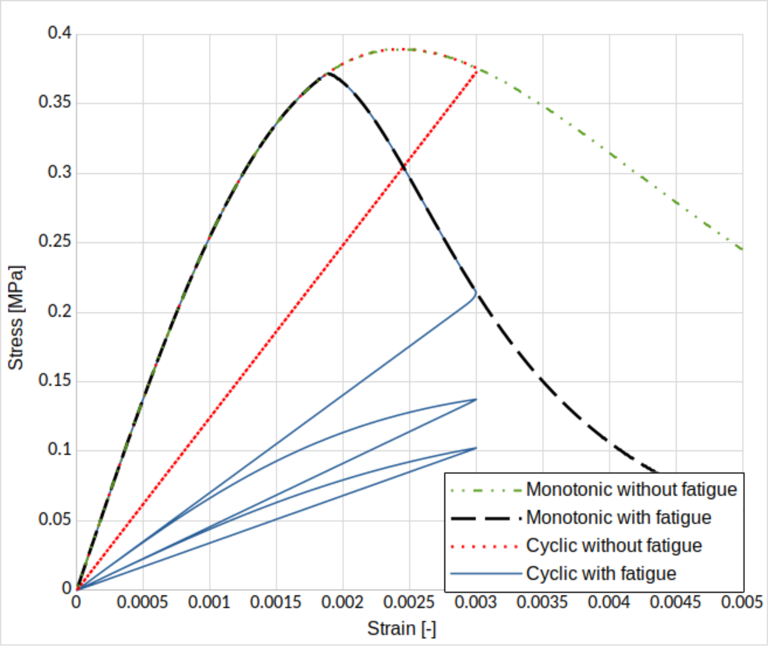

Fig. 1. Stress-strain response for monotonic and cyclic loading conditions with and without fatigue function

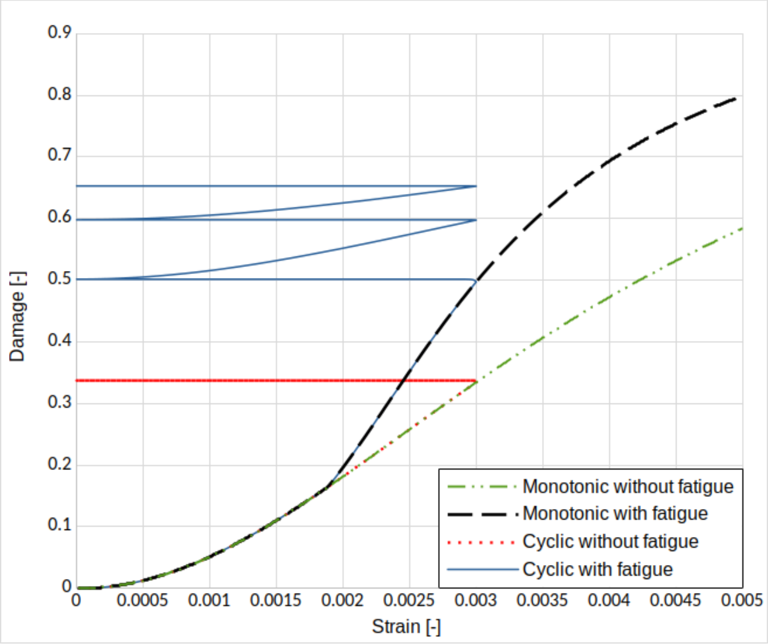

Fig. 2. Damage-strain response for monotonic and cyclic loading conditions with and without fatigue function

E [MPa]	ν [-]	G_V [MPa]	l_c [mm]	tol
210000	0.3	5	0.1	1.e-7

Table 1. Material parameters used in simulation [9]

4. Conclusions

The PFDM is a cutting-edge technique that is nowadays often implemented into FEM software for the prediction of damage and fracture in structures, but also for the evaluation of the structures' safety. By introducing the fatigue degradation function, the PFDM features are enhanced with the capability of simulation fatigue in materials.

In this paper, the authors presented the necessary modifications of the PFDM for that purpose, and the behavior's phenomenology is presented on a simple unit cube model modeled by one finite element. The stress-strain diagram, as well as damage value-strain dependence, show that with the included fatigue degradation function, it is possible to simulate fatigue and that the damage value increases while, at the same time, the stress-strain relationship decreases.

These results are the first step that will be extended by the application on more complex structures and the experimental investigation of the threshold value of the cumulative history variable.

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