Book of Proceedings

The Ninth International Congress
of the Serbian Society of Mechanics
July 5-7, 2023, Vrnjačka Banja, Serbia

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Dear colleagues,

The Serbian Society of Mechanics organized the 9th International Congress of the Serbian Society of Mechanics from 5th to 7th July 2023 in Vrnjačka Banja, Serbia. The aims of the congress were to bring together leading academic scientists, researchers and research scholars to exchange and share experiences and research results on various aspects of Theoretical and Applied Mechanics. The congress brought an interdisciplinary platform for researchers, practitioners and educators to present and discuss the most recent innovations, theories, algorithms, as well as practical challenges encountered and solutions adopted in the fields of Classical Mechanics, Solid and Fluid Mechanics, Computational Mechanics, Biomechanics, Applied Mathematics and Physics, Structural Mechanics and Engineering. More information on the Conference can be found on the webpage: http://www.ssm.kg.ac.rs/congress_2023/.
Co-organizers of the conference were:
Faculty of Engineering, University of Kragujevac
Faculty of Mechanical Engineering, University of Belgrade
Faculty of Technical Science, University of Novi Sad
Faculty of Mechanical Engineering, University of Niš
Institute for Information Technology, University of Kragujevac
University of Kragujevac
Mathematical Institute of the Serbian Academy of Sciences and Arts
Serbian Academy of Sciences and Arts
Serbian Society of Computational Mechanics.

Financial support was provided by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia.

Plenary speakers were:

1. Assoc. Prof. Dr Miha Brojan – *University of Ljubljana, Slovenia*
   Title: From symmetry breaking to functionality: Examples from nonlinear mechanics of beams, plates and shells
2. Prof. Dr. Dimitri V. Georgievskii – *Institute of Mechanics, Lomonosov Moscow State University, Russia*
   Title: Elements of the Theory of Constitutive Relations and Formulations of the Linearized Problems on Stability
3. Prof. Dr. Stefano Lenci – *Department of Civil and Building Engineering, and Architecture, Polytechnic University of Marche, Ancona, Italy*
   Title: Nonlinear Wave Propagation in Cables and Beams Resting on a Bilinear Foundation
4. Prof. Dr. Parviz Moin – *Center for Turbulence Research, Stanford University, California, USA*
   Title: Large Eddy Simulation at Affordable Cost: Application to a Full Aircraft Configuration
5. Prof. Dr Rafal Rusinek – *Lublin University of Technology, Lublin, Poland*
   Title: Bio-electro-mechanical System of the Human Middle Ear
6. Assoc. Prof. Dr. Jelena M. Svorcan – *Department of Aerospace Engineering, Faculty of Mechanical Engineering, University of Belgrade, Serbia*
   Title: Numerical Investigation of Flows Around Small-Scale Propellers: Possibilities and Challenges

There were also presentations within four Mini-symposia:

- MS1: Mechanical Metamaterials
  Organizers: Milan Cajić, *Mathematical Institute SANU, Serbia*; Danilo Karličić, *Mathematical Institute SANU, Serbia*
- MS2: Turbulence
  Organizer: Đorđe Čantrak, *University of Belgrade, Serbia*
  Plenary speakers:
  1. Prof. Dr. Parviz Moin – *Center for Turbulence Research, Stanford University, California, USA*
  Title: Probing Turbulence Physics Using Numerical Simulation Databases – A Case Study in Predictive Science
  2. Prof. Dr. Nikolay Nikitin – *General Aerodynamics Laboratory, Institute of Mechanics, Lomonosov Moscow State University, Russia*
  Title: Secondary Flows of Prandtl's Second Kind. Mechanism of Formation and Method of Prediction
  3. Prof. Dr. Ji Pei – *National Research Center of Pumps, Jiangsu University, China*
  Title: Research on High Efficiency and High Reliability Pumps in Jiangsu University
- MS3: Biomechanics and Mathematical Biology
  Organizers: Andjelka Hedrih, *Mathematical Institute of the Serbian Academy of Sciences and Arts, Serbia*; Marat Dosaev, *Lomonosov Moscow State University, Moskva, Russian Federation*
  Plenary Speakers:
  1. Prof. Dr. Su Fong-Chin – *Department of Biomedical Engineering, College of Engineering, National Cheng Kung University, Tainan, Taiwan; Medical Device Innovation Center, National Cheng Kung University, Tainan, Taiwan*
  Title: An Exergame-Integrated IoT-Based Ergometer System for Personalized Training of the Elderly
  2. Prof. Dr. Marco Amabili – *Department of Mechanical Engineering, McGill University, Montreal, Canada*
  Title: Effect of Smooth Muscle Activation in the Static and Dynamic Mechanical Characterization of Human Aortas
- MS4: Nonlinear Dynamics
  Organizer: Julijana Simonović, *Faculty of Mechanical Engineering University of Niš, Serbia*

We have received more than 130 high-quality research papers. As a result of the strict review process and evaluation, the committee selected 91 papers for publishing in this Book of proceedings.
Special gratitude goes to the members of the program and scientific review committee as well as to all chairs, organizers and committee members for their dedication and support.

On behalf of the Organizing Committee

Prof. Dr Nenad Filipović
Chair of ICSSM2023

Book of Proceedings

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PHASE-FIELD MODELING OF LOW CYCLIC FATIGUE IN DUCTILE MATERIALS

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Abstract

Plastic strains in engineering structures occur if the loading conditions exceed the designed values. The loading above the yield strength of the material leads to fatigue of the structure and fracture. The fracture occurs after a lower number of cycles, in comparison to the high cyclic fatigue when the material operates in the elastic range. However, the fatigue behavior of structures made of ductile materials in the range above the yield strength can be simulated by Phase-Field Damage Modeling (PFDM) approach, which includes a fatigue degradation function. In this paper, the cyclic tensile loading-unloading behavior is simulated by PAK finite element method software with Von Mises plasticity constitutive model and Simo's hardening function. The material parameters are proposed for generic metal with the phase-field constants set to show the phenomena which can be simulated. The results show that the proposed theory and implementation should be further investigated and applied to simulate the experimental testing of ductile materials under cyclic loading and fatigue conditions.

Keywords: cyclic loading, fatigue, plasticity, phase-field modeling, damage, ductile materials.

1. Introduction

Low cyclic fatigue (LCF) can be defined as repeated loading-unloading of material that exhibits plastic strains. The loading amplitude is above the yield strength, leading to fatigue and fracture after fewer cycles compared to the high cyclic fatigue (HCF) when the structures operate in an elastic range.

There are various constitutive models for the plasticity of engineering materials, such as metals, alloys, polymers, etc., but steel and aluminum are the most used mechanical engineering materials. The well-known Von Mises plasticity constitutive model can be used for that purpose. Besides the constitutive model, the Phase-Field Damage Model (PFDM) can be used to create multifield finite elements with an additional degree of freedom to compute the damage field in the structure. The PFDM is based on Griffith's thermodynamical framework, which defines that the crack will increase if the released energy exceeds the material toughness, explained in detail in Francfort and Marigo [1] and Bourdin et al. [2].

The fatigue life is estimated as the relationship between cyclic stress or strain range and the number of cycles in the form of Wöhler diagrams or S-N curves. In LCF, the material is subjected to high-stress amplitude in a plastic regime, leading to fewer cycles until fracture. Various researchers are working on developing PFDM and implementing fatigue features into FEM software, mainly for HCF [3, 4]. However, the extension of this approach to the ductile behavior and the LCF need further investigation. A fatigue degradation function describing the fatigue history is also used to reduce the material toughness [4-6]. The PFDM approach has already been used by the authors [7,8] for the simulation of damage in metals and aluminum. In this paper, the authors present the PFDM implementation for modeling the LCF behavior of metallic ductile materials, and the first simulation results are shown.

The theoretical background of the phase field fatigue framework is described in Section 2. Then, the obtained results and discussion are provided in Section 3. Finally, the article ends with conclusions in Section 4.

2. Methods

2.1 A phase field model for LCF

By the formulation of cracks in one-dimensional solids, and the extension of the regularized crack functional to multi-dimensional problems [7,8], the crack surface is defined as follows:

$$S(d)=\int_V \gamma(d,\nabla d)dV \tag{1}$$

where γ is the crack surface density function per unit volume. The internal potential energy density ψ is considered as the sum of elastic $\psi^E(\boldsymbol{\varepsilon}_E,d)$, plastic $\psi^P(\bar{\varepsilon}_P)$, plastic dissipation $\varphi^P(\bar{\varepsilon}_P)$ and fracture surface energy density $\varphi^S(d,\nabla d)$ [7,8]:

$$\psi=\psi^E(\boldsymbol{\varepsilon}_E,d)+\psi^P(\bar{\varepsilon}_P)+\varphi^S(d,\nabla d)+\varphi^P(\bar{\varepsilon}_P) \tag{2}$$

where $\boldsymbol{\varepsilon}_E$ is the elastic strain tensor, $\bar{\varepsilon}_P$ is the equivalent plastic strain, and d is the damage variable. The elastic strain energy density of virgin material $\psi_0^E(\boldsymbol{\varepsilon}_E)$ is multiplied by degradation function $g(d)$ to define the elastic strain energy density $\psi^E(\boldsymbol{\varepsilon}_E,d)$ as [7,8]:

$$\psi^E(\boldsymbol{\varepsilon}_E,d)=g(d)\psi_0^E(\boldsymbol{\varepsilon}_E)=g(d)\frac{1}{2}\boldsymbol{\varepsilon}_E^T:\boldsymbol{\sigma}_0 \tag{3}$$

where $\boldsymbol{\sigma}_0$ is the Cauchy stress tensor of an undamaged solid. Similarly, the "damaged" Cauchy stress $\boldsymbol{\sigma}$ is given in the following form [7,8]:

$$\boldsymbol{\sigma}=g(d)\boldsymbol{\sigma}_0=g(d)\mathbf{C}_0:\boldsymbol{\varepsilon}_E \tag{4}$$

where $\mathbf{C}_0$ is the fourth-order elastic constitutive matrix. The fracture surface energy Φ^S at the crack surface S is defined as [7,8]:

$$\Phi^S=\int_S G_c dS\approx\int_V G_c\gamma(d,\nabla d)dV=\int_V \varphi^S(d,\nabla d)dV \tag{5}$$

where the fracture surface energy density dissipated by the formation of the crack is:

$$\varphi^S(d,\nabla d)=G_c\gamma(d,\nabla d) \tag{6}$$

and G_c is the Griffith-type critical fracture energy release rate. For a cumulative history variable $\bar{\alpha}\geq 0$, and a fatigue degradation function $f(\bar{\alpha})$, the fracture surface energy density can be re-formulated as follows [4,5]:

$$\bar{\varphi}^{S}\left(d,\nabla d,\bar{\alpha}\right)=f\left(\bar{\alpha}\right)G_{c}\gamma\left(d,\nabla d\right) \tag{7}$$

The plastic energy density for Simo hardening function is [7]:

$$\psi^{P}\left(\bar{\varepsilon}_{P}\right)=\left(\sigma_{y0,\infty}-\sigma_{yv}\right)\left(\bar{\varepsilon}_{P}+\frac{1}{n}e^{-n\bar{\varepsilon}_{P}}\right)+\frac{1}{2}H\bar{\varepsilon}_{P}^{2} \tag{8}$$

while the plastic dissipated energy density is [7]:

$$\varphi^{P}\left(\bar{\varepsilon}_{P}\right)=\sigma_{yv}\bar{\varepsilon}_{P} \tag{9}$$

The total internal potential energy W_{int} functional is defined as [6,7,8]

$$W_{\text{int}}=\int_{V}\psi dV=\int_{V}\left\{g\left(d\right)\frac{1}{2}\boldsymbol{\varepsilon}_{E}^{T}:\boldsymbol{\sigma}_{0}+\psi^{P}\left(\bar{\varepsilon}_{P}\right)+f\left(\bar{\alpha}\right)G_{V}\left[\frac{d^{2}}{2}+\frac{l_{c}^{2}}{2}\left|\nabla d\right|^{2}\right]+\sigma_{yv}\bar{\varepsilon}_{P}\right\}dV \tag{10}$$

where a critical fracture energy release rate per unit volume is $G_V=G_c/l_c$ and l_c is the characteristic length. The variation of the internal potential energy over the total strain, equivalent plastic strain and damage is given as [4-8]:

$$\begin{aligned}\delta W_{int}=\int_{V}\Bigg\{&\boldsymbol{\sigma}:\delta\boldsymbol{\varepsilon}_{E}+\frac{1}{2}g'\left(d\right)\boldsymbol{\varepsilon}_{E}^{T}:\boldsymbol{\sigma}_{0}\delta d+f\left(\bar{\alpha}\right)G_{V}\left[d\delta d+l_{c}^{2}\nabla d\nabla\delta d\right]+\\&+\left(-\boldsymbol{\sigma}:\frac{\partial\boldsymbol{\varepsilon}_{P}}{\partial\bar{\varepsilon}_{P}}+\left(\sigma_{y0,\infty}-\sigma_{yv}\right)\left(1-e^{-n\bar{\varepsilon}_{P}}\right)+H\bar{\varepsilon}_{P}+\sigma_{yv}\right)\delta\bar{\varepsilon}_{P}\Bigg\}dV\end{aligned} \tag{11}$$

A variation of the external potential energy W_{ext} is known as [7,8]:

$$\delta W_{ext}=\int_{V}\mathbf{b}\cdot\delta\mathbf{u}dV+\int_{A}\mathbf{h}\cdot\delta\mathbf{u}dA \tag{12}$$

where b is a body force field per unit volume, h is a boundary traction per unit area, and u is the displacements vector. The equilibrium of the internal and external potential energy can be transformed by the application of total derivatives and by using the Gauss theorem, as [4-8]:

$$\begin{aligned}&\int_{V}\Bigg\{-\left[g'(d)\psi_{0}+f\left(\bar{\alpha}\right)G_{V}\left[d-l_{c}^{2}\nabla^{2}d\right]\right]\delta d-\left[Div\left[\boldsymbol{\sigma}\right]+\mathbf{b}\right]\cdot\delta\mathbf{u}+\\&+\left(-\boldsymbol{\sigma}:\frac{\partial\boldsymbol{\varepsilon}_{P}}{\partial\bar{\varepsilon}_{P}}+\left(\sigma_{y0,\infty}-\sigma_{yv}\right)\left(1-e^{-n\bar{\varepsilon}_{P}}\right)+H\bar{\varepsilon}_{P}+\sigma_{yv}\right)\delta\bar{\varepsilon}_{P}\Bigg\}dV\\&+\int_{A}\left\{\left[\boldsymbol{\sigma}\cdot\mathbf{n}-\mathbf{h}\right]\cdot\delta\mathbf{u}\right\}dA+\int_{A}\left\{\left[G_{V}l_{c}^{2}\nabla d\cdot\mathbf{n}\right]\delta d\right\}dA=0\end{aligned} \tag{14}$$

where $\mathbf{n}$ is the unit outer normal to the surface A. The Neumann-type boundary conditions are

$$\boldsymbol{\sigma}\cdot\mathbf{n}-\mathbf{h}=0 \tag{15}$$

$$\nabla d\cdot\mathbf{n}=0 \tag{16}$$

what leads to the governing balance equations [4-8]:

$$Div\left[\boldsymbol{\sigma}\right]+\mathbf{b}=0 \tag{17}$$

$$f\left(\bar{\alpha}\right)G_{V}\left[d-l_{c}^{2}\nabla^{2}d\right]+g'\left(d\right)\psi_{0}=0 \tag{18}$$

$$\bar{\sigma}_{eq} - \sigma_{yv} - \left(\sigma_{y0,\infty} - \sigma_{yv}\right)\left(1 - e^{-n\bar{\varepsilon}_P}\right) - H\bar{\varepsilon}_P = 0 \tag{19}$$

2.2 Fatigue degradation function

The cyclic loading produces damage increasing what is captured by a fatigue degradation function $f\left(\bar{\alpha}\right)$ [3-6]. This function degrades the material toughness concerning the fatigue history variable. For the pseudo time, the history variable can be defined as [3-6]:

$$\bar{\alpha}\left(t\right) = \int_0^t H\left(\alpha\dot{\alpha}\right)\left|\dot{\alpha}\right| d\tau \tag{20}$$

where $H\left(\alpha\dot{\alpha}\right)$ is the Heaviside step function [6], where $\alpha = g\left(d\right)\psi_0$ is the fatigue history variable and $\dot{\alpha}$ is its derivative. The fatigue degradation function is [6]:

$$f\left(\bar{\alpha}\right) = \begin{cases} 1 & if \quad \bar{\alpha} \le \alpha_T \\ \left(\dfrac{2\alpha_T}{\bar{\alpha} + \alpha_T}\right)^2 & \text{if} \quad \bar{\alpha} > \alpha_T \end{cases} \tag{21}$$

Here, α_T represents a threshold value, below which the fracture energy remains unaffected, which should be determined experimentally and in this case is adopted according to literature [6]:

$$\alpha_T = \frac{G_V}{12} \tag{22}$$

3. Results and discussion

The uniaxial loading example of the unit cube is created to simulate the phenomenology of the damage evolution under Low Cyclic Fatigue (HCF) loading conditions for ductile materials using the PFDM approach extended by the fatigue degradation function described in the previous section.

The unit cube is loaded on one side by prescribed displacements while the other directions are restrained. The opposite side is restrained in the direction of prescribed displacements [9]. Elasticity and plasticity parameters of arbitrary metallic material are used and given in Table 1. The additional material parameters, such as fracture energy, characteristic length, and tolerance of convergence, are set to show the material damaging and fatigue phenomena [9].

Simulations are performed for the case of Von Mises plasticity and Simo hardening function without PFDM, only with PFDM, and with PFDM and fatigue. For each case, three loading and unloading cycles are performed. The results are given in Fig. 1. The examples are solved in 500 loading steps up to a different strain for each cycle: first cycle - 2%, second cycle – 3%, and third cycle – 4%.

As can be noticed in Fig. 1, the fatigue degradation function has a significant influence on the material response. The damage value for the total strain is given in Fig. 2, where we can see how it increases for each simulation case, except for the plasticity without PFDM because there is no damage.

E [MPa]	ν [-]	G_V [MPa]	σ_{y0} [MPa]	$\sigma_{y\infty}$ [MPa]	H [MPa]	n [-]	l_c [mm]	tol
199000	0.29	3.09	345	635	9	18	0.01	1.e-7

Table 1. Material parameters used in simulation.

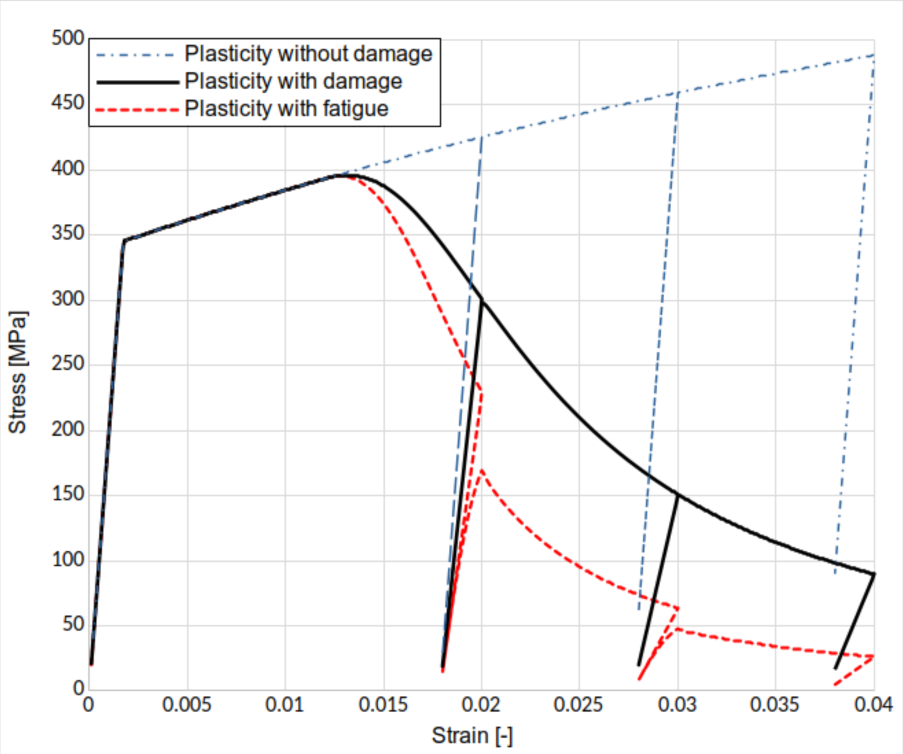

Fig. 1. Stress-strain response for monotonic and cyclic loading conditions with and without fatigue function.

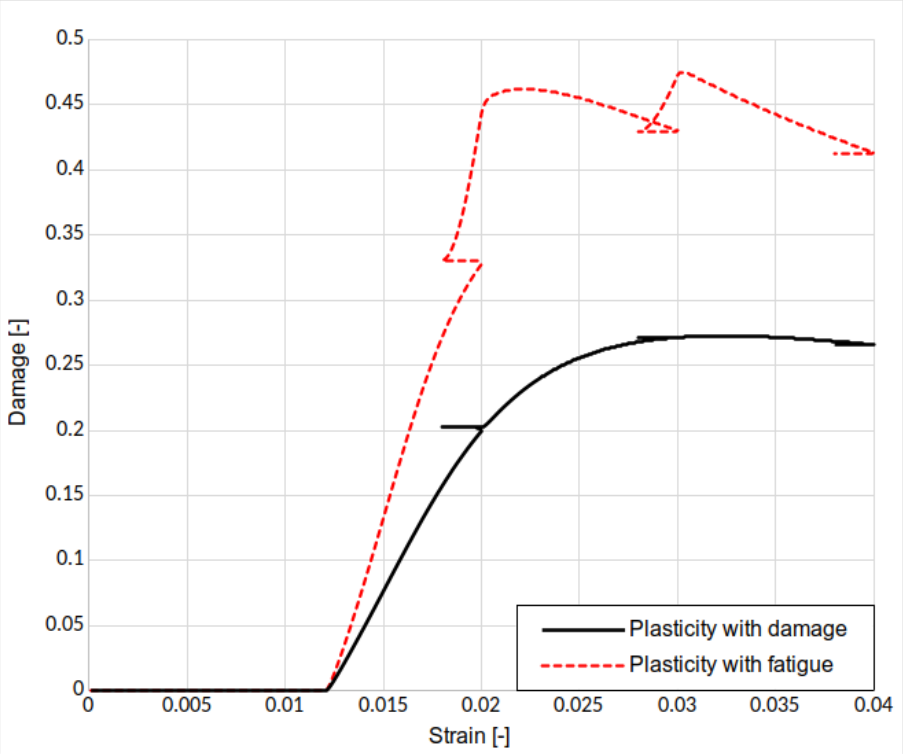

Fig. 2. Damage-strain response for monotonic and cyclic loading conditions with and without fatigue function.

4. Conclusions

The PFDM is a technique widely used for various simulations of damage in materials. For example, it is often implemented into FEM software to create multifield finite elements for predicting damage and fracture in structures. Recently, the fatigue degradation function opened the possibility of simulation of damage accumulation and fatigue.

In this paper, the authors presented the extension of Von Mises plasticity constitutive model with Simo hardening function with the PFDM and fatigue degradation function to show how it influences the simulation of ductile materials behavior under the cyclic loading. A simple unit cube model is used to show the phenomenology of the implementation. The stress-strain diagram, as well as damage value-strain dependence, show that it is possible to simulate fatigue and that the damage value increases while, at the same time, the stress-strain relationship decreases.

These results are the first step that will be extended by application on more complex structures.

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