The influence of spatial aliasing to experimental determination of dispersion relationship of flexural waves in beams by correlation method

Vladimir Sinđelić[1], Snežana Ćirić Kostić[1], Branko Radičević[1], Zlatan Šoškić[1*]
[1]The Faculty of Mechanical and Civil Engineering in Kraljevo, University of Kragujevac, Kraljevo (Serbia)

The experimental determination of a dispersion relation by measurements of wave properties in different points in space meets the challenge of spatial aliasing, which is a consequence of description of continuous wave by a discrete set of measurement points. The paper presents an analysis of the phenomenon and derives a relationship between the average distance between the measurement points and the frequency range for experimental determination of dispersion relationship of flexural waves in beams by correlation method. The results of the analysis are verified by experiments that comprised determination of the dispersion relationship with variable average distance between the measurement points.

Keywords: Mechanical waves, Dispersion relationship, Frequency range, Spatial aliasing

1. INTRODUCTION

Dispersion relationship (abbreviated as "DR" in the following text) is defined as the relationship between the wavevector of a periodic wave k and its angular frequency ω. Due to the simple linear dependencies, DR also represents, and is sometimes even defined as, the relationship between the wavelength λ ($\lambda = 2\pi/k$) or the wavenumber κ ($\kappa = k/2\pi$) of the wave, on one side, and the period T ($T = 2\pi/\omega$) or the frequency f ($f = \omega/2\pi$) of the wave on the other side of DR. Since the phase velocity c of a wave may be expressed as $c = \omega/k$, and the group velocity of a wave v may be expressed as $v = \mathrm{d}\omega/\mathrm{d}k$, the DR enables calculation of both phase and group velocity of the wave, thus representing an important tool for wave propagation studies. DR of mechanical waves depend on viscoelastic (such as elastic moduli and material loss factors) and inertial such as density) properties of the medium through which the waves propagate.

The usual applications of DR are theoretical and numerical studies of wave propagation in structures, and the recent interest of the Laboratory for advanced design methodologies "3D Impulse" and Laboratory for acoustics of the Faculty of Mechanical and Civil Engineering in Kraljevo for experimental determination of DR [1][2] is raised by studies of dynamic behaviour of 3D printed periodic and aperiodic structures [3]. Besides, DRs are sometimes also used for calculation of elastic properties of viscoelastic properties of matter [4][5].

The DRs may be theoretically derived from partial differential equations of motion, and may be expressed in explicit form only for the cases of wave propagation through media with simple geometries and high symmetry. As a particularly simple geometry, which confines the wave propagation to one dimension, thin beams represent the most convenient object for studies of various applications of DRs. The simplest mechanical waves for excitation and detection are flexural waves, which have a parabolic form of DR [6]:

$$\omega = \sqrt{\frac{EI}{\rho A}} k^2 \qquad (1)$$

with I standing for the area moment of inertia, and A for the area, of the cross-section of the beam.

The main concept for experimental determination of the DR in thin beams consists in measurement of the response (usually acceleration) to an excitation (usually impulse or harmonic) in multiple points along the beam. The responses at these points at a particular excitation frequency (obtained by direct measurement or spectral decomposition) are then used to construct wave fields with known wavevectors that have minimal deviation from the measured responses. There are two main groups of methods for construction of the wavefields with minimal deviation from the measured responses, and they differ in accuracy, mathematical complexity and range of applicability. The first group of methods are based on Prony method for decomposition of a function into a series of damped sinusoids, an idea analogous to use of sinusoids in Fourier analysis [7]-[9]. These methods are computationally efficient, but they have two serious disadvantages: first, their application is limited only to evenly spaced measurement points, and second, they require a large number of measurement points. The other group of methods are based on systematic repetitive testing of different wavefields in search for a set of parameters that provide the best fit between a wavefield and the experimental data. The applicability of the second group of methods is substantially wider, but their drawback is the related substantially longer computation time. Different methods that belong to the second group use different forms of the test wavefields, different criteria selected as measure of the quality of the fit, and the different procedures for finding the optimal wavefield parameters [4][5][10].

The method that is the subject of this paper is the correlation method [10], which uses sets of progressive waves as the test wavefields, and the simple sequential search of wavevector space to find the wavevector value that provides maximum of correlation between the measured data and the test wavefield at certain frequency, which is used as the measure of deviation of the test wavefield from experimental data. In its simplest form, used to study the waves propagating along a beam, the correlation method for experimental determination of DR (in further

*Corresponding author: Zlatan Šoškić, Dositejeva 19, Kraljevo, soskic.z@mfkv.kg.ac.rs

text just *"correlation method"*) uses wavefields constructed on the basis of a harmonic progressive waves with selected test wavevector k_t and angular frequency ω, and the longitudinal axis of the beam as the x-axis. If a harmonic progressive wave is generated by force acting at one end of the beam, expressed in a complex form as $\underline{F}(t) = \underline{F}_\omega \cdot \exp(-\omega t)$, the accelerations of beam points due to the wavefield may be expressed as $\underline{a}_t(x,t) = \underline{a}_\omega \cdot \exp(k_t x - \omega t)$, and the wavefield of the response, the accelerances $\underline{w}_t(x,t) = \underline{a}_t(x,t)/F(t)$ may be described by its complex amplitudes:

$$\underline{w}_t(x,\omega) = \underline{w}_{t\omega} \cdot \exp(k_t x), \tag{2}$$

which represent the *frequency response function* (FRF) of the measurement system represented by the experiment. The complex amplitudes of the accelerances of the wavefield for a selected wavevector k_t are compared to complex amplitudes of accelerances $\underline{w}(x,\omega)$ measured in L measurement points x_l (l = 1, 2,…, L) along the beam, and the measure of the fit between two datasets is their correlation defined in the form:

$$\begin{aligned} \underline{W}_\omega(k_t) &= \int_{-\infty}^{+\infty} \underline{w}(x,\omega)\,\underline{w}_t^*(x,\omega)\,dx = \\ &= \underline{w}_{t\omega}^* \int_{-\infty}^{+\infty} \underline{w}(x,\omega)\,e^{-ik_t x}\,dx \end{aligned} \tag{3}$$

which is, due to the discrete nature of input data, estimated as:

$$\hat{\underline{W}}_\omega(k_t) = \underline{w}_{t\omega}^* \sum_{l=1}^{L} \underline{w}(x_l,\omega)\,e^{-ik_t x_l}\,\Delta x_l = \underline{w}_{t\omega}^* \sum_{l=1}^{L} \underline{w}_{l\omega}\,e^{-ik_t x_l}\,\Delta x_l \tag{4}$$

where Δx_l represents the space interval around the point x_l, and introducing the notation $\underline{w}(x_l,\omega) = \underline{w}_{l\omega}$. The expression for correlation between the two datasets given by (3) leads to the conclusion that, if the wave propagating along the beam is a progressive harmonic wave, then the modulus of the estimated correlation will have maximal value when the wavevector of the test wavefield k_t is equal to the wavevector of the propagating wave k. In order to determine the DR of waves propagating along the beam, the wavevector space is scanned in the range $k_{min} < k_t < k_{max}$ for each angular frequency ω, and the value of k_t that leads to maximal modulus of the estimated correlation is assumed to be the wavevector that corresponds to the angular frequency ω, denoted as $k(\omega)$. Since the correlation is proportional to the complex amplitude factor $\underline{w}_{t\omega}^*$, which does not affect the position of the maxima of the estimated correlation (3), and the DR may be determined by finding the wavevectors that maximize the expression:

$$Y_\omega = \left| \sum_{l=1}^{L} \underline{w}_{l\omega} \exp(-ik_t x_l)\,\Delta x_l \right|, \tag{5}$$

which will be called "*the correlation function"* in further text, for each of the angular frequencies ω in the range $\omega^{min} < \omega < \omega^{max}$ where DR is determined. As it may be seen, the correlation method does not make any assumption, nor imposes any limitation, on the number and the positions of data measurement points x_l, which, together with its simplicity, represents the fundamental advantage of the method. In the case of equidistant measurement points $x_l = l \cdot d$, the correlation function takes the form

$$Y_\omega = \left| \sum_{l=1}^{L} \underline{w}_{l\omega} \exp(-i \cdot l \cdot d \cdot k_t) \right|, \tag{6}$$

with d being the distance between the measurement points.

The method was successfully used for determination of DR in steel and composite plates [10], and a later modification of the method [11], which uses damped plane waves for construction of the test wavefields, was successfully used for materials with high material losses.

2. THEORY

The key conceptual problems of the implementation of the correlation method lie in the fact that the concept of the method is based on the **existence of a unique value of test wavevector k_t that maximizes the correlation, given by the integral (3)**. There are two aspects of the concept that represent the problem for implementation:

A1: An integral is a mathematical operator defined for a continuous function as an integrand, so calculation of the integral in (3) requires knowledge of a continuous FRF $\underline{w}(x,t)$;

A2: The correlation $\underline{W}_\omega(k_t)$, as defined in (3), is a continuous function of the test wavenumber k_t;

However, the discrete nature of the digital computer calculations prevents complete implementation of the concept of continuity of mathematical functions, reducing the continuous functions to a representation by discrete arrays of pairs of arguments and values of functions in the process known as *discretization of continuous mathematical functions*.

The discretization of FRF in space is made by its measurement in finite number of measurement points measurement points x_l (l = 1, 2,…, L), and its further implementation using the equations (4)-(6) leads to "picket-fence" effect, the well-known phenomenon that two signals become indistinguishable if they are sampled in a finite number of measurement points where they have equal values. Omnipresent in discrete data analysis [12], the "picket-fence" effect is best known as the cause of the *temporal aliasing* in frequency analysis, while in this case, since the discretized variable is space, it causes *spatial aliasing*. The spatial aliasing arises because for any given value of wavevector k_t, there is an infinite number of wavevectors $k_t' \neq k_t$ such that for each measurement point x_l holds:

$$\exp(-ik_t' x_l) = \exp(-ik_t x_l) \quad (l = 1,2,...,L), \tag{7}$$

As a consequence, the wavefields with complex amplitudes given by equation (2), constructed using wavevectors k_t and k_t' are indistinguishable in the measurement points, and, therefore, they have equal estimations for correlation with experimental data. It further means that in an interval $k_t^{min} < k_t < k_t^{max}$ there may be several wavevector values with equal correlation with the measurement data as any selected k_t, and, in particular, that **there may exist more than one value of test wavevector k_t that maximizes the correlation function given by (5)**, which opposes to the concept of the method expressed at the beginning of the chapter.

When the measurement points are equidistant, hence when $x_l = l \cdot d$, then the condition of indistinguishability of two test wavefields, expressed by the equation (7), reduces to

$$\exp(-ik_t' ld) = \exp(-ik_t ld) \quad (l = 1,2,...,L), \tag{9}$$

which may be expressed as

$$\exp\left(i\left[k_t'-k_t\right]d\right)=1, \tag{10}$$

resulting in

$$k_t'-k_t = z\frac{2\pi}{d}. \tag{11}$$

Therefore, if the measurement data are equidistant, with the distance between the measurement points being d, then the correlation function (5) is periodic, with the period $2\pi/d$, as it is shown in Figure 1.

If the measurement points are not equidistant, then the correlation function is not periodic in wavevector space, but the system of equations (7) still has an infinite number of solutions for any selected value of k_t, and the average difference between the solutions is close to $(2\pi/d)$, where d is the average distance between the measurement points [13].

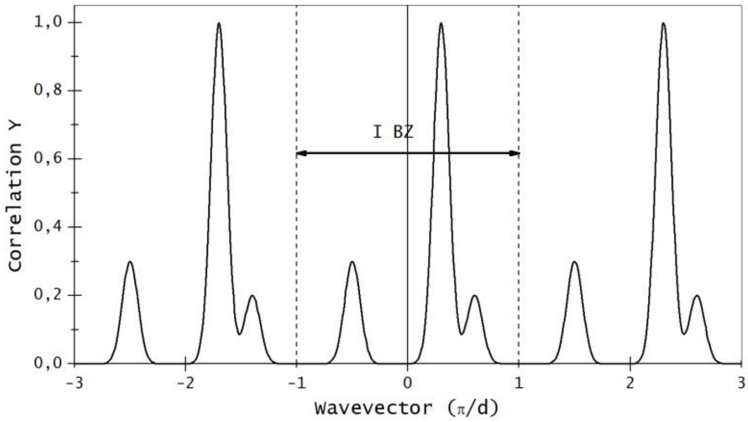

Figure 1: Periodic correlation function in the case of equidistant measurement points and the limits of the IBZ

One way to ensure that the correlation method leads to correct results is to reduce the scanned wavevector range $k_{min}<k_t<k_{max}$ to one period of the correlation function (5). In its famous treatise of propagation of waves in periodic structures [14], Leon Brillouin has shown that two one-dimensional progression waves with wavevectors k_t and k_t', which propagate through periodic structures with period d, cannot be distinguished by both amplitude and direction if the wavevectors satisfy the equation (11). For those reasons, he argued that the periodic structures act as low-frequency filters that enable propagation of waves in both directions, so that only the waves with wavevectors in the range $-\pi/d < k < +\pi/d$ may propagate through such structures. The wavevector range is called *the first Brillouin zone* (abbreviated as IBZ) and it is of fundamental importance in quantum mechanics of solids. While the theoretical model used to study the flexural waves propagation in thin beams, and for derivation of the equation (1), considers a thin beam to be a continuous medium, so that the periodicity of the correlation function (5) is only an artefact introduced by the finite number of measurement points L, the obtained experimental data are exactly the same as they would be in the case of a periodic structure, so that the obtained results may be properly interpreted only by limiting to the wavevectors within the first Brillouin zone.

The limitation of the wavevectors range for experimental determination of DR to

$$|k| < k_{BZ} = \frac{\pi}{d} \tag{12}$$

means, according to the equation (1), the respective reduction of the frequency range where the DR is determined to

$$f < f_{BZ} = \frac{1}{2\pi}\sqrt{\frac{EI}{\rho A}}\left(\frac{\pi}{d}\right)^2 = \frac{\pi}{2d^2}\sqrt{\frac{EI}{\rho A}} \tag{13}$$

This relationship between the frequency range for experimental determination of the DR and the average distance between the measurement points may be used to determine the maximal distance d_{max} between the measurement points that may be used to determine DR by correlation method in a frequency range $f < f_{max}$,

$$d_{max} = \sqrt[4]{\frac{EI}{\rho A}}\cdot\sqrt{\frac{\pi}{2 f_{max}}} \tag{14}$$

3. EXPERIMENT

In order to experimentally verify the derived relationship (14), two experiments were carried out. The average distance between the measurement points in one experiment was satisfying the condition (14), while in the other experiment the condition (14) was not met. Each of the experiments comprised a series of subsequent measurements of time history of acceleration $a_x(t)$ at a single point of a beam excited by impact hammer.

The measurement object was a steel rod with length $D \approx 1{,}65$ m, and roughly square cross-section with side $b \approx 1$ cm. Therefore, the area of the cross-section of the rod was $A = b^2 \approx 1\cdot10^{-4}$ m^2, while the moment of inertia of the cross-section of the rod for flexion was $I = b^4/12 \approx \approx 8{,}33\cdot10^{-10}$ m^4.

The vibrations were excited by the impact hammer B&K 8204 with sensitivity 30.89 mV/N, and the excitation was measured by the B&K Pulse system with CCLD input with range ±10V, so that force measurement range was around 300 N. The impact hammer is light and small, and it is equipped with a light and hard impact head, so that the bandwidth of the impact hammer is broad, up to 10 kHz.

Automatic double-hit detection was not provided, so that each excitation hit was monitored by inspecting the time history of impacting force $F(t)$, and the double-hits were discarded.

The rod was hit close to one of its ends, at the area x = 0-1 cm from the beginning of the rod. The hitting area was covered by a piece of scotch-tape that provided a bit of dampening to the hit with the aim to reduce excessive accelerations, which prevented recording of the response by the software used for data acquisition, but it also reduced the bandwidth to around 6 kHz. The maximal impact force was in the range 8-12 N.

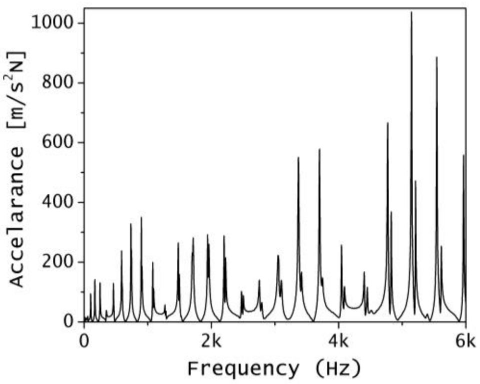

Figure 3: FRF of accelerance at point x = 90 cm *from beginning of the rod*

The response was measured using the accelerometer B&K 4507Bx with sensitivity 10.055 mV/ms^{-2} and B&K Pulse system with CCLD input with range ±10V, so that so that acceleration measurement range was around ±1000 m/s^2.

For each of the measurement points the measurements of acceleration were taken 10 times. During each of the measurements were recorded time histories of excitation force and acceleration. After each of the measurements, the data acquisition software calculated power spectral density of force and acceleration, FRF of accelerance, and the respective coherence.

The measurements were organized in two series – experiments, with the aim to test both the case when the average distance between the measurement points satisfy the equation (14) and the case when they do not satisfy it. Since the frequency range determined by the experimental setup is limited to f_{max} = 6 kHz, the maximal average distance between the measurement points that would, by the equation (14), allow to experimentally determine DR in the range $f < f_{max}$ is $d_{max} \approx 6{,}1$ cm, which is calculated using the values $E = 200$ GPa and $\rho = 7800$ kg/m^3 for Young modulus and density of steel.

In the first experiment, the measurement points were uniformly distributed at 10 equidistant positions with distances $d \approx 15$ cm, so that $d > d_{max}$, and the corresponding frequency of the border of the IBZ was $f_{BZ} \approx 1010$ Hz.

In the second experiment, 31 non-equidistant measurement points were selected to have the distances to the beginning of the rod (the excited end) being 7.5 cm, 8.7 cm, 11.1 cm, 13.5 cm, 18.3 cm, 20.7 cm, 25.5 cm, 27.9 cm, 32.7 cm, 39.9 cm, 42.3 cm, 49.5 cm, 54.3 cm, 56.7 cm, 61.5 cm, 68.7 cm, 75.9 cm, 78.3 cm, 85.5 cm, 90.3 cm, 92.7 cm, 99.9 cm, 104.7 cm, 111.9 cm, 121.5 cm, 126.3 cm, 128.7 cm, 133.5 cm, 135.9 cm, 140.7 cm and 157.5 cm, which is proportional to the first 31 prime numbers sequence 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107 109, 113 and 127. The average distance between the points was $d \approx 5$ cm, so that $d < d_{max}$, and the corresponding frequency of the border of the IBZ was $f_{BZ} \approx 9090$ Hz.

4. RESULTS

4.1. Frequency response spectra

The accelerance amplitude spectra revealed the expected resonant behaviour (Figure 3). The resonant frequencies present in all spectra are the same, but the amplitudes of the FRFs at different resonant points show different behaviours at different points. The frequency with maximal amplitude shows dependence on the measurement point. The resonant peaks at low frequencies (14 Hz – 590 Hz) are singlets, while at higher frequencies the resonant peaks appear to be doublets, with the frequency split of the doublets increasing with resonant frequency.

4.2. Correlation function

The calculated correlation functions show the predicted behaviour. In the first case, with equidistant measurement points, the correlation function is periodic, with period $2\pi/d \approx 21$ m^{-1}. In Figure 4 are shown correlation function values calculated for the case of equidistant measurement points at frequency $f = 200$ Hz. The theoretical equation (1) for that frequency predicts the corresponding value of wavevector to be $k_0 \approx 9{,}3$ m$^{-1} \approx 0{,}44$ (π/d). Figure 4 shows that, as explained, not only values $\pm k_0$, but also other values that satisfy condition (11) represent strong local maxima of the correlation function, requiring limitation of the wavevectors to the IBZ for experimental detection of the DR.

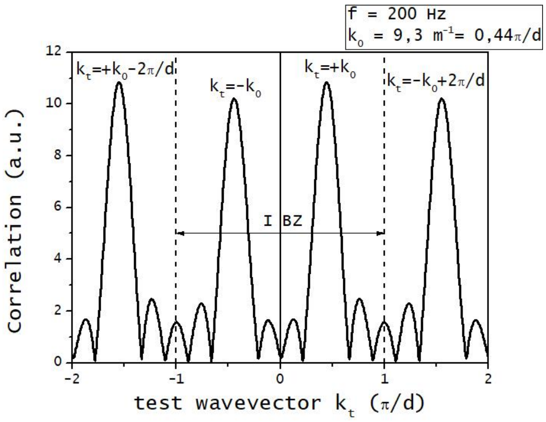

Figure 4: Correlation function for the case of equidistant measurement points

4.3. Dispersion relationships

In this section are presented results of the application of the correlation method for experimental calculation of the DR.

4.3.1. Average distance larger than d_{max}

The results of the application of the correlation method to the measurements taken in equidistant measurement points with distance $d \approx 15$ cm, with the correlation function calculated for test values of wavevectors in the range $(0,+5\pi/d)$, are shown in Figure 7. The pairs of corresponding wavevector–frequency values that represent the experimentally determined DR are presented with full circles. The line that passes through origin represent the DR theoretically predicted by (1). The other lines represent the aliases, i.e., the wavevector values that satisfy condition (11) with respect to the theoretically predicted DR. There are two kinds of aliases, the increasing aliases that have positive slope, and the decreasing aliases, which have negative slope. The positive slope (positive group velocity $d\omega/dk$) of increasing aliases indicates energy transfer in the direction of x-axis, which means that increasing aliases represent aliases of the waves propagating in the direction of incidence of the excited waves. Conversely, the decreasing aliases represent aliases of the waves propagating in the opposite direction. It should be noted that the decreasing aliases have positive phase velocities ($\omega/k > 0$) and negative group velocities ($d\omega/dk < 0$) because they represent aliases of the negative solutions of equation (1) for wavenumbers.

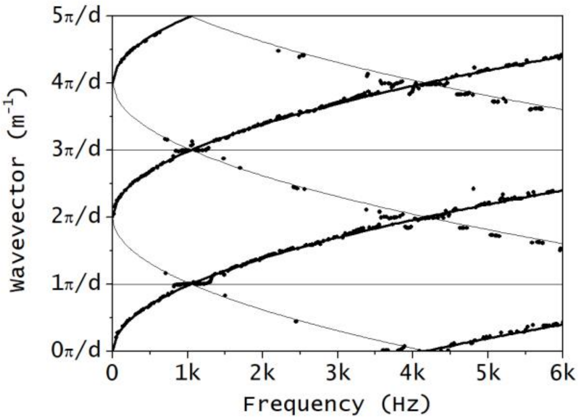

Figure 7: Wavevectors detected by the straightforward implementation of correlation method to equidistant measurements with $d \approx$ 15 cm ($\pi/d \approx 21$ m^{-1})

The Figure 7 clearly shows that the detected values of wavevectors corresponding to frequencies used in the experiment belong to different aliases of wavevector values with respect to the values predicted by (1), and that the correlation method is not capable of distinguishing them. Majority of the wavevectors calculated by correlation method describe waves propagating in the direction of x-axis, but some of them describe waves propagating in the opposite direction, so that correlation method is not capable of detecting even the direction of propagation of the wave, as explained by Brillouin [14].

However, if the range of test wavevectors is reduced to the IBZ according to the equation (12), the obtained results are shown in the Figure 8. The circles and lines have the same meanings as in the Figure 7. The first feature of the obtained results is that, with reduction to $|k| < \pi/d$, the correlation method, as predicted, leads to correct determination of DR within the frequency range $f < f_{BZ}$. The lines in that frequency range represent theoretical predictions given by (1). Furthermore, almost all detected wavenumbers represent the waves propagating in the direction of x-axis, with the only exception occurring for a frequency where doublet resonances are observed. The mere existence of the doublets, otherwise not predicted by the theory, indicates that, under resonant conditions, the experimental setup does not satisfy the assumptions requested by the theoretical model that leads to (1). The existence of the doublets is still not properly explained and is worthy of further research.

The application of the correlation method for frequencies above f_{BZ} leads to aliasing, as the waves with wavevectors $|k| > \pi/d$ have the same accelerations in measurement points as the waves with corresponding wavevector values in the IBZ. The lines in the frequency range $f > f_{BZ}$ represent the aliases of the theoretical predictions given by (1). It may be noted that the majority of the waves experimentally detected by the correlation method have positive group velocities, and that the sign of experimentally determined phase velocity depends on the wavenumber so that phase velocity is positive if the wavenumber is between even and odd multiples of π/d (hence between $(2z)\pi/d$ and $(2z+1)\pi/d$), while phase velocity is negative if the wavenumber is between odd and even multiples of π/d (hence between $(2z+1)\pi/d$ and $(2z)\pi/d$). Therefore, the group velocity may be correctly determined by correlation method even for frequencies higher than above f_{BZ}, but the phase velocity cannot.

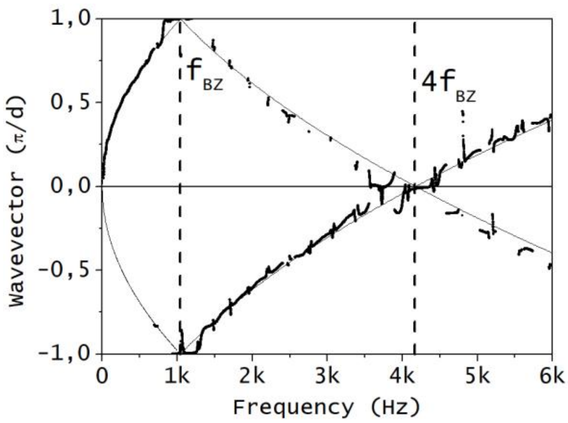

Figure 8: Dispersion relationship from Fig.7 reduced to the first Brillouin zone ($\pi/d \approx 21$ m^{-1})

A notable deviation from the theoretical predictions is observed in vicinity of borders of the Brillouin zones, where $k = z{\cdot}\pi/d = z{\cdot}k_{BZ}$ and, according to the equation (1), $f = z^2{\cdot}f_{BZ}$. The borders of the Brillouin zones are marked by the thick vertical dotted lines in the Figure 8, and experimentally detected wavevectors are almost independent of the frequency in the frequency ranges around the borders. Since the Brillouin zones are only the consequence of equidistant spatial distribution of measurement points, the observed departure from monotonicity has also to be an artefact introduced by the experimental setup. The explanation of the observed behaviour represents a different phenomenon related to the experimental technique of measuring DRs, and it will be discussed in a separate paper [15].

4.4. Average distance smaller than d_{max}

Figure 10 presents the results of application of the correlation method to the measurements taken in the second experiment, with non-equidistant measurement points, and with average distance d = 5 cm being smaller than the maximal distance d_{max} corresponding to the frequency range $f < 6$ kHz.

The figure shows that the experimentally obtained DR agrees with the theoretical predictions in the whole frequency range used in the experiment. As it was the case with the measurements with $d > d_{max}$, almost all detected wavevectors describe the waves propagating in the direction of x-axis, with the exceptions for frequencies that correspond to the observed doublet resonances.

5. CONCLUSIONS

The paper presented an analysis of the concept for determination of dispersion relationship of flexural waves in beams using correlation method. It was shown that the spatial aliasing restricts applicability of correlation method in experiments with equidistant measurements to the first Brillouin zone of wavevector space, which reflects in reduction of the frequency range for application of the correlation method. The derived analytical expression (14) determines the maximal average distance between the measurement points d_{max} that enables application of the correlation method for experimental determination of DR in thin beams in the selected frequency range $f < f_{max}$.

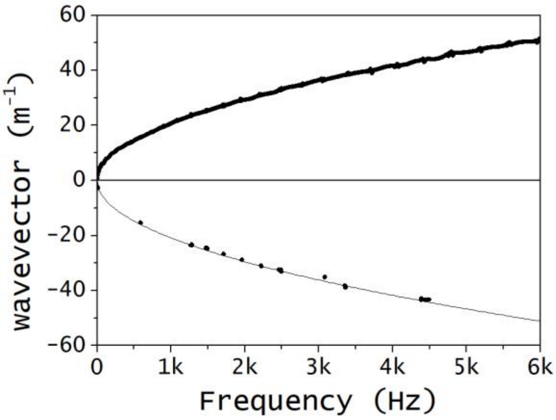

Fig. 9: Wavevectors detected by the correlation method with the average distance between the measurements $d \approx 5$ cm $< d_{max}$

Two experiments that were carried out confirmed the validity of application of the expression (14). The results of the experiment with $d < d_{max}$ were in perfect agreement with the theoretical predictions (1), while the results of the experiment with $d > d_{max}$ agreed with the theoretical predictions only in the limited frequency range where $f < f_{BZ}$. Therefore, the obtained results may be useful for planning the experiments for determination of the DR of flexural waves in thin beams, but may also be used as a starting point for experimental determination of other waves in other mechanical structures.

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