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SPATIAL ALIASING AND FALSE DETECTION OF STANDING WAVES IN EXPERIMENTAL DETERMINATION OF DISPERSION RELATIONSHIP BY CORRELATION METHOD

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Abstract - *The experimental determination of a dispersion relation by measurements of wave properties in different points in space meets various challenges due to the spatial aliasing, which is a consequence of description of a continuous wave by a discrete set of measurement points. The paper presents an analysis of the phenomenon of false detection of standing waves during experimental determination of dispersion relationship of flexural waves in beams by correlation method. The false detection of standing waves, manifested as zero group velocity of the waves that does not correspond to a real resonance, is a consequence of equidistant spacing of measurement points in experimental setup, and may be avoided by different positioning of measurement points in space. The results of the analysis are verified by comparison between the results of experimental setups with equidistant measurement points and measurement points with distances to excitation point proportional to prime numbers.*

1. INTRODUCTION

One of key elements of any definition of waves is that a wave is propagation of a disturbance through a medium. The disturbed part of the medium is the wavefield, and the border of a wavefield is a wavefront, which expands with propagation of a wave. Many complex theoretical and practical problems of wave propagation are solved by application of the Huygens principle, which describes the wave propagation by interference of spherical waves that are expanding with constant velocity c around all points of a wavefront. The velocity of propagation of the disturbance by spherical waves c is called “the phase velocity” of the wave, while the velocity of propagation of energy (or mass in the case of mechanical waves) by a wave (which does not have to be equal to phase velocity) is called “the group velocity” of the wave v. The knowledge of the phase velocity of a wave in a medium, therefore, is the key to the applications of the Huygens principle. An important property of wave motion is that the phase velocity c and the group velocity v of a harmonic wave, in general, depend on the frequency f of the harmonic wave. Since any wave may be represented as a sum of harmonic waves using the Fourier transformation, different frequency components of a wave propagate with different velocities, which leads to an effect of decomposition of waves that is called “dispersion”. For that reason, the all-important relationship between the phase velocity and the frequency of a wave, $c(f)$, is called “the dispersion relationship” (abbreviated as “DR” in the following text). Since the phase velocity c, frequency f, angular frequency $\omega = 2\pi/f$, wavelength λ, and the wavenumber $k = 2\pi/\lambda$ of a harmonic wave are related by the equations $\lambda = c/f$ and $\omega = c/k$, the DR is most often expressed in the analytical form $F(\omega,k) = 0$ or $\omega = \omega(k)$, because it enables simple calculation of both phase velocity (as $c = \omega/k$) and the group velocity (as $v = d\omega/dk$).

Theoretical calculation of the DR is possible only in a limited number of cases with high symmetry and simple structure of the medium, and the alternative for the other cases is the experimental determination of DR [1]. Even in the cases when it is possible to determine the DR by theory, experimental determination of DR is used for characterization of materials [2][3]. The main concept for experimental determination of the DR consists in measurement, in multiple points of a structure, of the response (usually acceleration) to an excitation (usually impulse or harmonic). The responses at these points at a particular excitation frequency (obtained by direct measurement or spectral decomposition) are then used to construct the “optimal” wavefield, with a known wavenumber, that has minimal deviation from the measured responses. Different approaches to construction of the wavefield lead to different methods for experimental determination of the DR [4]. The subject of this paper is the correlation method [5], which, in suitable structures, uses harmonic progressive waves to construct the test wavefields, and the simple sequential search of wavenumber space to find the wavenumber of optimal wavefield. It has been already shown that a straightforward application of the correlation method is limited to a low-frequency range due to the spatial aliasing [4], but the research presented in this article warns that even that frequency range is reduced in the case of equidistant measurement points due to the false detection of standing waves. Fortunately, the research shows that the false detection of standing waves may be avoided by non-equidistant positioning of measurement points.

2.THEORY

For the sake of simplicity, the present paper will consider propagation of flexural waves over a thin homogenous beam, which is a well-studied case in theory [6]. The flexural waves are suitable for experimental studies because they are simple for excitation and detection. The DR of flexural waves that propagate over a thin homogenous beam is given by the expression:

$$\omega = \sqrt{\frac{EI}{\rho A}} k^2 \qquad (1)$$

with ρ standing for density, E for Young's modulus of the material, while I represents the area moment of inertia, and A the area, of the cross-section of the beam.

Let a harmonic progressive flexural wave, with the angular frequency ω and the wavenumber k related by the DR (1), be excited in a homogenous thin beam by the action of harmonic transversal force $\underline{F}(t) = \underline{F}_\omega \cdot \exp(-i\omega t)$. If the x-axis is oriented along the longitudinal axis of the beam, and if one end of the beam is taken as the origin of the axis, then the transversal accelerations of the beam points will have values $\underline{a}(x,t) = \underline{a}_\omega \cdot [i \cdot \exp(kx-\omega t)]$, and the response to the excitation of the beam to the excitation, the accelerances $\underline{w}(x,t) = \underline{a}_t(x,t)/F(t)$ will have values $\underline{w}(x,\omega) = \underline{w}_\omega \cdot \exp(ikx)$, independent of t. Due to the orthogonality of harmonic functions, the correlation between the accelerance $w_t(x,\omega)$ of any harmonic wavefield, with the angular frequency ω and the wavenumber k_t, and the accelerance of the wavefield of the excited harmonic wave $\underline{w}(x,\omega)$, defined as

$$\underline{W}_\omega(k_t) = \int_{-\infty}^{+\infty} \underline{w}(x,\omega)\, \underline{w}_t^*(x,\omega)\, dx \qquad (2)$$

will be equal to zero unless the wavenumber of the harmonic wavefield is equal to the wavenumber of the excited wave, $k_t = k$, when the correlation tends to infinity. This property of the correlation is the basis for correlation method for experimental determination of DR. The method uses measurements of accelerations in a finite number of points L, with the coordinates x_l ($l = 1, 2,\ldots, L$), which are, in the case of thin homogenous beam placed along the beam. The measured accelerations and excitation force are used to obtain the accelerances in the measurement points $w_l(\omega) = w(x=x_l, \omega)$, which are then used for calculations of the sums that are estimations $\underline{\hat{W}}_\omega(k_t)$ of correlation between the measured wavefield at angular frequency ω and different harmonic progressive waves with wavenumbers k_t and accelerances $\underline{w}_t(x,\omega) = \underline{w}_\omega \cdot \exp(ik_t x)$.

$$\underline{\hat{W}}_\omega(k_t) = \underline{w}_{t\omega}^* \sum_{l=1}^{L} \underline{w}(x_l,\omega) e^{-ik_t x_l} \Delta x_l = \underline{w}_{t\omega}^* \sum_{l=1}^{L} \underline{w}_{l\omega} e^{-ik_t x_l} \Delta x_l \qquad (3)$$

where Δx_l represents the space interval around the point x_l. Since $\underline{\hat{W}}_\omega(k_t)$ is only an estimation of the correlation $\underline{W}_\omega(k_t)$, it is not equal to zero for $k \neq k_t$ and not infinite for $k=k_t$, but it should, anyway, have maximal value in the latter case. Using that property of $\underline{\hat{W}}_\omega(k_t)$, correlation method selects value for wavenumber of the measured wavefield to be equal to the test value k_t which has maximal value of estimated correlation (3), and that is the value that maximizes the correlation function $Y_\omega(k_t)$, given by the expression:

$$Y_\omega(k_t) = \left| \sum_{l=1}^{L} \underline{w}_{l\omega} \exp(-ik_t x_l) \Delta x_l \right| \qquad (4)$$

As explained, the correlation method uses progressive waves to construct the test wavefields, which implies that the method is applicable to the wavefields that consist of progressive waves. However, in all real structures, due to their finite dimensions and reflections from the boundaries, exist standing waves with the corresponding natural frequencies of the structures. While the standing waves may be considered as a superposition of progressive waves moving in opposite directions (therefore having wave vectors with equal intensity, but opposite signs), they do not transfer energy and their group velocities are zero. Since the group velocity is derivative of DR, the DR obtained by correlation method has inflexion point at resonant frequencies of the structure, which is different from DR of progressive waves that the correlation method intends to reveal. Therefore, it may be concluded that the existence of standing waves prevents the application correlation method. This paper focuses on the phenomenon of false detection of standing waves that arises when experimental determination of DR is performed by correlation method with data obtained using equidistant measurement points (abbreviated as DR-CM-ED). This phenomenon, which will be shown to be avoidable, further reduces applicability of correlation method for determination of DR.

The measurement points are most frequently taken to be equidistant, $x_l = l \cdot d$, where d is the distance between adjacent measurement points. There are several reasons for such approach, and one of dominant is application of the most popular signal processing technique, FFT (Fast Fourier Transformation), which requires measurement from equidistant input points. In such a case, the correlation function has the form:

$$Y_\omega = \left| \sum_{l=1}^{L} \underline{w}_{l\omega} \exp(-i \cdot l \cdot d \cdot k_t) \right|. \qquad (5)$$

It has been shown [4] that the correlation function in the case of equidistant measurement point is periodic in wavenumber space with the period $2\pi/d$, $Y_\omega(k+2\pi/d) = Y_\omega(k)$, as illustrated in Fig. 1. Since it means that the maxima of the correlation function are also periodic, the periodicity of the correlation function has a consequence that the DR obtained DR-CM-ED is also periodic with the same period, $2\pi/d$, $\omega(k+2\pi/d) = \omega(k)$, as Fig. 2 illustrates. Such a periodicity is a well-known consequence of the equidistant selection of measurement points, and it occurs in general in all cases of propagation of waves through periodic structures [7]. The basic period in the wavenumber space, $-\pi/d < k < \pi/d$, is called "the Brillouin zone" (BZ) or "the first Brillouin zone". If the limits of the first Brillouin are denoted as $\pm\pi/d = \pm k_{BZ}$, and the corresponding values of angular frequency and frequency as ω_{BZ} and f_{BZ}, then the period of a DR-CM-ED may be written as $2k_{BZ}$.

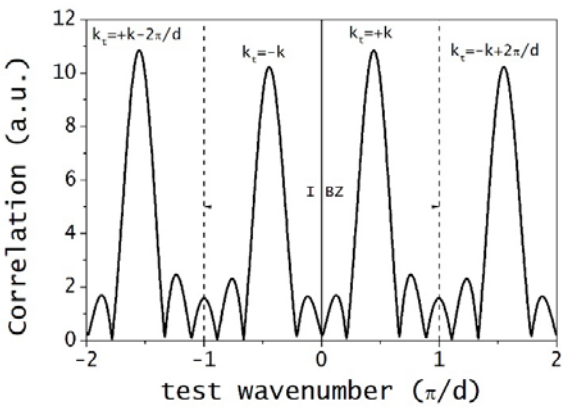

Fig. 1 *Correlation function of a DR-CM-ED*

For physical reasons, any DR is even function, $\omega(-k) = \omega(k)$, because the reversed sign of k means only change of direction of propagation, which does not affect phase velocity due to the

symmetry properties of space and matter. Due to the periodicity and even parity of a DR-CM-ED, its derivative has to be periodic with the same period,

$$\left.\frac{d\omega}{dk}\right|_{k+\frac{2\pi}{d}} = \left.\frac{d\omega}{dk}\right|_{k} \tag{6}$$

and has to have odd parity

$$\left.\frac{d\omega}{dk}\right|_{-k} = -\left.\frac{d\omega}{dk}\right|_{k}. \tag{7}$$

Furthermore, since the derivative of a DR, $d\omega/dk$, is the group velocity of a wave, it has to be a continuous function of k, because any physical quantity in classical mechanics has a single value. The continuity of the first derivative for any wavenumber $k = k_0$ may be expressed as

$$\lim_{k\to k_o}\left(\frac{d\omega}{dk}\right) = \left.\frac{d\omega}{dk}\right|_{k_0}, \tag{8}$$

which means that the left and the right limit of the derivative of DR have to be equal for any wavenumber $k = k_0$

$$\left.\frac{d\omega}{dk}\right|_{k_o^-} = \left.\frac{d\omega}{dk}\right|_{k_o^+}. \tag{9}$$

Since the equations (7) and (9) hold for all wavenumbers, they have to hold also for $k = 0$. From (7) follows that the first derivatives of the DR on the left side and right side of zero have opposite signs

$$\left.\frac{d\omega}{dk}\right|_{0^-} = -\left.\frac{d\omega}{dk}\right|_{0^+}, \tag{10}$$

and from (9) it follows that they have to have the same sign

$$\left.\frac{d\omega}{dk}\right|_{0^-} = \left.\frac{d\omega}{dk}\right|_{0^+}. \tag{11}$$

Both (10) and (11) may hold only if the derivative of the DR for $k = 0$ is zero. Due to the periodicity, the first derivative of a DR-CM-ED also has to be equal to zero for all points $k = \pm z\cdot 2k_{BZ} = \pm m\cdot k_{BZ}$, where z is an integer, and $m = 2z$ is an even number.

On the other hand, the equations (6) , (7) and (9) have also to hold for all wavenumbers, and therefore also for the border of the Brillouin zone, $k = +k_{BZ}$. However, due to the continuity (9) the first derivatives of the DR-CM-ED on the left side and right side of $+k_{BZ}$ have to have the same signs

$$\left.\frac{d\omega}{dk}\right|_{k_{BZ}^-} = \left.\frac{d\omega}{dk}\right|_{k_{BZ}^+}, \tag{12}$$

and, at the same time, due to the combination of the requirements for periodicity (6) and odd parity (7), they have to have opposite signs

$$\left.\frac{d\omega}{dk}\right|_{k_{BZ}^-} \overset{(7)}{=} -\left.\frac{d\omega}{dk}\right|_{-k_{BZ}^-} \overset{(6)}{=} -\left.\frac{d\omega}{dk}\right|_{-k_{BZ}^-+2k_{BZ}} = -\left.\frac{d\omega}{dk}\right|_{k_{BZ}^+} \tag{13}$$

which is, again, only possible if the derivative of the DR for $k = \pm k_{BZ}$ is zero. Due to the periodicity, the first derivative of a DR-CM-ED also has to be equal to zero for all points $k = k_{BZ} \pm z\cdot 2k_{BZ} = \pm m\cdot k_{BZ}$, where z is an integer, and $m = (2z+1)$ is an odd number.

In conclusion, the physical requirements for even parity and continuity of DR and the mathematical requirement for periodicity of DR-CM-ED mean that the first derivative of the DR-CM-ED has to be equal for all wavenumbers satisfying conditions $k = \pm m\cdot k_{BZ}$ with m being an integer.

$$\left.\frac{d\omega}{dk}\right|_{k=m\cdot k_{BZ}} = 0, \tag{14}$$

On the other hand, the equation (1) shows that the first derivative of the DR of flexural waves a thin homogenous beam is equal to zero only for k=0, which means that the DR-CM-ED cannot be a good description of the DR for wavenumbers satisfying conditions $k = \pm m\cdot k_{BZ}$ (in other word, in centres and borders of Brillouin zones).

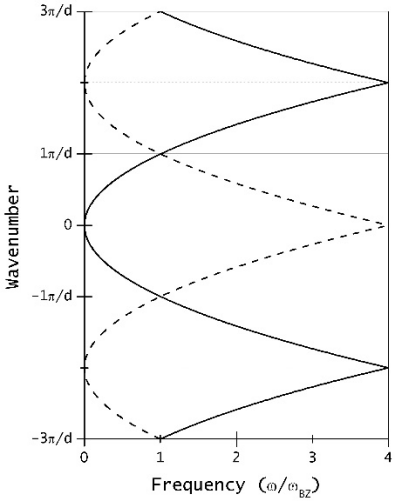

Fig. 2 *Theoretical prediction of DR-CM-ED (solid line – centres in even BZ, dashed line – centres in even BZ)*

The reason for the observed failure of DR-CM-ED is periodicity of the array of measurement points, which is caused by their equidistant positions. It is well-known fact from studies of waves in periodic structures [7] that waves with $k = \pm m\cdot k_{BZ}$ in discrete periodic structures (e.g. crystals) degenerate into oscillations for even values of m, while for odd values of m they represent standing waves. It is easy to demonstrate because in all points of a periodic discrete structure with period d, $x_l = l\cdot d$, a wave with $k = \pm m\cdot k_{BZ}$ for even m=2z has phase factors $\exp(ikx_l) = \exp(i\cdot z\cdot l\cdot 2\pi) = 1$, which means that phases of the wave are equal in the same points, implicating that the wave degenerates into whole-body (or rigid-body) oscillations. On the other hand, if m=2z+1 is odd, then the phase factors have values $\exp(ikx_l) = \exp(i\cdot(2z+1)\cdot l\cdot\pi) = (-1)^l$, which means that the adjacent points have opposite phases, implicating that the wave is a standing wave. The same relationships hold for the accelerations of a progressive wave with $k = \pm m\cdot k_{BZ}$ taken in an array of equidistant points, which means that such a progressive wave will be falsely detected as standing wave or whole-body oscillation.

At first glance, the effect of false detection of standing waves and whole-body oscillations may be easily avoided by selecting non-periodic positioning of measurement points. However, the Fourier theorem shows that any finite non-periodic array may be decomposed in an infinite sum of periodic arrays with the period equal to the least common denominator of differences between the members of array (the distances between the measurement points). Since, by definition, irrational distances cannot be measured, the longest practical

period of a set of distances between the measurement points is obtained if the positions of the measurement points x_l are proportional to prime numbers.

3.EXPERIMENT

The experiment was carried with the aim to apply correlation method for determination of DR of flexural waves on a free homogenous thin beam using a set of equidistant measurement points and a set of measurement points with distances proportional to prime numbers. The complete experiment is described in the reference [4], and here will be repeated just the part about the test object and positioning of the measurement points, which are the most relevant for further discussion of the obtained results.

The beam was a steel rod with length $D \approx 1{,}65$ m, and roughly square cross-section with side $b \approx 1$ cm. The end parts of the rod, with the length of around 25 cm, were resting on soft sponges, with the aim to emulate a beam with free ends. The measurements were organized in three series, with different positions of measurement points.

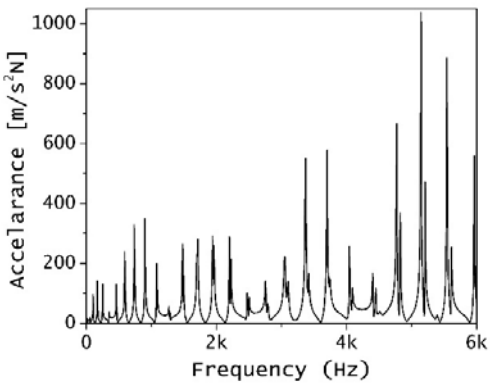

Fig. 3 *Accelerance of the beam measured at the point x=90 cm from the beginning of the rod*

In the first series, the measurement points were uniformly distributed at 10 equidistant positions with distances $d \approx 15$ cm, with the first measurement point being at the distance d from the beginning of the rod and the last being at the distance d from the end of the rod. Therefore, the arrangement of the measurement points was symmetric with respect to the centre of the rod.

In the second series, 10 measurement points were selected to have the distances to the beginning of the rod (the excited end) being proportional to prime numbers sequence 5, 7, 11, 13, 17, 19, 23, 29, 31 and 37. The selected set of measurement points covered the part of the rod between 20 cm and 148 cm.

In the third series, 31 measurement points were selected to have the distances to the beginning of the rod (the excited end) being proportional to prime numbers sequence 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97,101, 103, 107, 109, 113 and 127. The selected set of measurement points covered the part of the rod between 7.5 cm and 157.5 cm.

4. RESULTS AND DISCUSSION

The accelerance amplitude spectra for all cases revealed the expected resonant behaviour, as Fig. 3 illustrates. The resonant peaks at low frequencies (14 Hz – 590 Hz) are singlets, while at higher frequencies the resonant peaks appear to be doublets, with the frequency split of the doublets increasing with resonant frequency.

The frequencies are detected at frequencies close to the natural frequencies predicted by theory for flexural vibrations of a thin homogenous beam with free ends:

$$f_m = (2m)^2 \left(\frac{1}{2\pi} \sqrt{\frac{EI}{A\rho}} \left(\frac{\pi}{2D} \right)^2 \right), m = 0,1,2... \qquad (15)$$

and these are the frequencies of the standing waves that form at the beam.

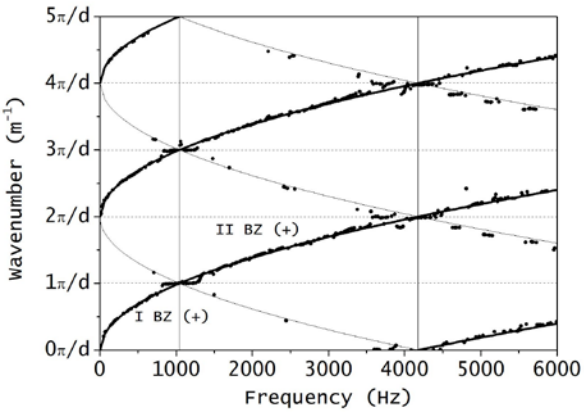

Fig. 4 *DR-CM-ED obtained in the experiment (dots-experimental data, lines-theory)*

As explained in the theoretical analysis, in the case of the equidistant measurement points the obtained DR (it is the DR-CM-ED), presented by points in the Fig. 4, is periodic function of wavenumber k, with the period $2k_{BZ} = (2\pi/d) \approx 42\text{m}^{-1}$, so that the frequency $f_{BZ} \approx 1044$ Hz corresponds to the end of the first Brillouin zone ($k_{BZ} = \pi/d$), and the frequency $4 \cdot f_{BZ} \approx 4176$ Hz corresponds to the center of the next Brillouin zone ($2 \cdot k_{BZ}$). Even in such a large-scale view (0-6000 Hz) it is clear that at frequencies corresponding to wavenumber values $k = m \cdot k_{BZ}$ the obtained DR departs both from the monotonously increasing trend within Brillouin zones and from the predictions of theory, presented by solid lines in Fig. 4.

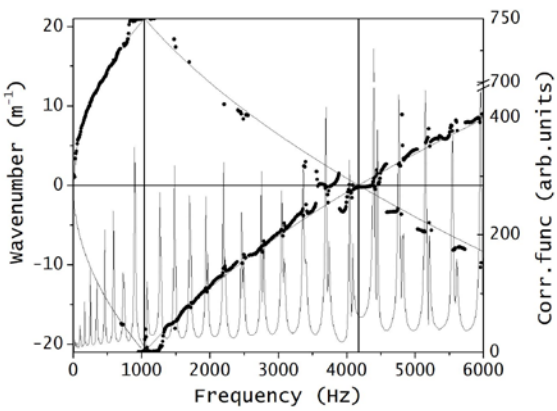

Fig. 5 *Restricted DR-CM-ED and correlation function obtained in the experiment (dots-experimental data, thick lines-theoretical DR, thin line-correlation function)*

In order to increase visibility and use the periodicity of the obtained results, the DR-CM-ED is frequently presented confined to the first Brillouin zone, as shown in the Fig. 5. The DR-CM-ED is presented in the figure by the points, while the thin line presents the correlation function, which indicates resonances. The figure clearly presents the influence of the

standing waves to the experimental determination of DR in general: close to the natural frequencies of the beam, correlation method is unable to distinguish the incident (the monotonously increasing branches) and reflected (the monotonously increasing branches) progressive waves. Furthermore, the figure also shows that the DR-CM-ED contains ranges of false standing waves at the ends of Brillouin zones (flat DR-CM-ED near $f_{BZ} \approx 1044$ Hz) and false oscillations (flat DR-CM-ED near $f_{BZ} \approx 4176$ Hz). Finally, Fig. 5 also indicates that there is interaction between real and false standing waves detection, observable in the range 3500-4500 Hz.

With the aim to clearly show the effect of false standing wave detection, Fig. 6 shows the part of DR-CM-ED containing the first Brillouin zone and the range close to its boundary $k = k_{BZ}$. The figure also shows the theoretical predictions for DR of incident (solid line) and reflected (dashed line) progressive waves. It is clear that the DR-CM-ED is flat, indicating detection of standing waves, in the area range where the values of wavenumbers of incident and reflected waves coincide due to periodicity artificially induced by positioning of measurement points.

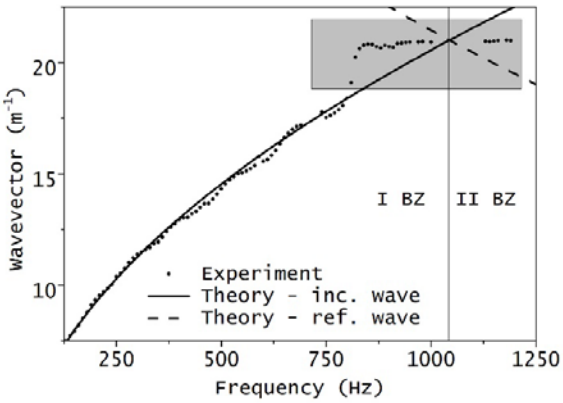

Fig. 6 *DR-CM-ED within the I BZ and close to the boundary between the I BZ and II BZ*

In order to test the effects of different positions of measurement points, three sets of measurement points were formed for application of correlation method for determination of DR:

- "equidistant" set of data obtained in 10 equidistant measurement points during the first series of measurements,
- "mixed" set, containing data obtained in 10 equidistant measurement points during the first series of measurements and the data obtained in 10 non-equidistant measurement points during the second series of measurements,
- "primed" set, containing data obtained in 31 non-equidistant measurement points during the third series of measurements.

Fig. 7 shows that the periodicity of the measurement points is indeed the sole reason for the false detection of standing waves near the boundary of Brillouin zone. The false detection occurs not only when the measurement points are equidistant, but also when there is a subset of equidistant points within the measurement points set. When all the positions of the measurement points are proportional to prime numbers, the detection of false standing waves disappears, and the DR in the region follows theoretical predictions, shown by the solid line.

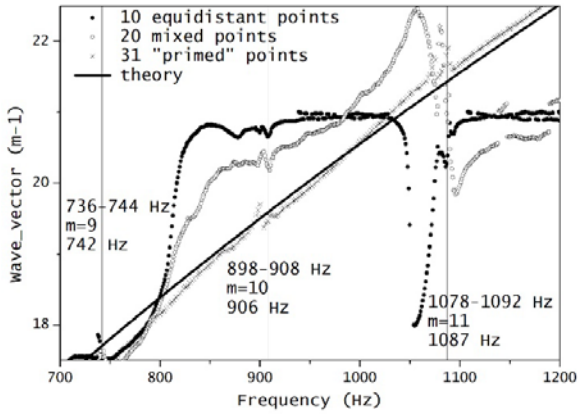

Fig. 7 *DR-CM-ED obtained using three different sets of measurements points*

Fig. 7 also shows that the detection of real standing waves, that arise at natural frequencies of the structure, as expected, does not depend on positions of measurement points. The natural frequencies of the beam in the observed range correspond to resonant frequencies (15) for values $m = 9$, $m = 10$ and $m = 11$.

5. CONCLUSIONS

This article presents research of false detection of standing waves during experimental determination of dispersion relationship using by correlation method and equidistant measurement points.

The theoretical analysis revealed that a progressive wave with wavenumber k, discretized in an equidistant set of measurement points with distance d between them, cannot be distinguished from a standing wave when $k = (2z+1)\pi/d$ (ends of Brillouin zones) and from whole-body oscillation when $k = (2z)\pi/d$ (centers of Brillouin zones). Since standing waves and whole-body oscillations do not transfer energy, the false detection causes false inflection points ("flat ranges") in the dispersion relationships detected by correlation method. This theoretical analysis shows that the false detection of standing waves is an artifact of equidistant measurement points and the simplest way to avoid it is by positioning measurement points so that their distances to excitation point are proportional to prime numbers.

The experiment that was designed to verify the theoretical considerations confirmed the presented conclusions. Furthermore, the experiment revealed that even the application of correlation method to experimental data obtained using non-equidistant set of points are sensitive to false standing waves detection if the set contains a subset with equidistant points.

Due to their finite dimensions, in all real structures exist real standing waves, which are also detected by correlation method and cause inflection points in DR. Since they are real, their detection, and the corresponding deviation of experimental DR from the true values, cannot be avoided by positioning of measurement points. Therefore, the correlation method is not applicable in vicinity of resonant frequencies of structures.

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REFERENCES

[1] E.B. Groth., T. G. R.Clarke, G. Schumacher da Silva, I. Iturrioz and G. Lacidogna, "The elastic wave propagation in rectangular waveguide structure: Determination of dispersion curves and their application in nondestructive techniques", *Applied Sciences*, 10(12), p.4401, 2020

[2] J.G. McDaniel, P. Dupont, L. Salvino, "A wave approach to estimating frequency-dependent damping under transient loading", *Journal of Sound and Vibration*, Vol. 231(2), 433-449, 2000

[3] J.G. McDaniel and W.S. Shepard Jr, "Estimation of structural wave numbers from spatially sparse response measurements", *The Journal of the Acoustical Society of America*, Vol. 108(4), 1674-1682, 2000

[4] V. Sindjelic, S. Ciric-Kostic, A Nikolic, Z. Soskic, "Extension of the frequency range for experimental determination of dispersion relationship of flexural waves in beams by correlation method", *IMK-14 – Research & Development in Heavy Machinery*, Vol. 26(4), pp. 95-107, 2020

[5] N.S. Ferguson, C.R. Halkyard, B.R. Mace, and K.H. Heron, "The estimation of wavenumbers in two-dimensional structures", *Proceedings of ISMA2002: International Conference on Noise and Vibration Engineering, Leuven (Belgium) 16-18 Sep 2002*. pp.799-806, 2002

[6] C.F. Beard, *Structural Vibrations – Analysis and Damping*, Elsevier, Oxford, (UK), 1996

[7] L. Brillouin, *Wave propagation in periodic structures: electric filters and crystal lattices*, Dover publications; 1953.