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HYBRID OPTIMIZATION ALGORITHM FOR DETERMINING SOUND ABSORPTION

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Abstract - *The great variety of biologically inspired algorithms and the possibility of application in different fields of engineering initiates the application in the search of the space of possible values of the coefficients of empirical models for acoustic impedance. The application of two biologically inspired algorithms: beluga optimization (BWO) and the marine predators algorithm, as well as their hybridization, was investigated in the paper. In order to obtain satisfactory coefficients of empirical models, the BWO algorithm was modified by adding a part of the marine predators algorithm that refers to the change of search agents in the BWO algorithm. The new model provides satisfactory predictions of the sound absorption coefficient of open-cell polyurethane foams, compared to experimental results obtained in an impedance tube. In relation to the known empirical models for impedance where the constants are determined by the linear regression method, the hybrid algorithm provides satisfactory predictions of the sound absorption coefficient of polyurethane foams.*

1. INTRODUCTION

Noise pollution is a serious environmental problem of the modern world. Due to the rapid development of the industry, which directly affects noise pollution, there are consequences for human health such as: stress, sleep disorders, cardiovascular problems and others [1]. Because of the negative impact on human health, it is very important to investigate the acoustic properties of materials that can be used in noise control.

An effective tools used in noise control are porous materials. Porous absorption materials are characterized by a high sound absorption coefficient. Polyurethane foams are widely used in various industries and noise protection systems due to their excellent mechanical properties and availability in a wide range of densities and thicknesses. [2]. Elastic porous foams PU with open cell are generally considered good sound absorbers [3].

With the development of new materials, for application in noise control, it is necessary to find empirical and/or theoretical models, which implementation would enable the determination of the acoustic properties of materials. Empirical models in combination with measurement results are used to determine the dependence of absorption coefficients as a function of frequency and thickness of the material. To determine the coefficients in the empirical model, Radičević [2], using the method of least squares, proposed a new empirical model for determining the acoustic properties of low-density polyurethane foams. The new model provides satisfactory predictions of the sound absorption coefficient of open-cell polyurethane foams, compared to experimental results obtained in an impedance tube.

This paper presents the application of newer biologically inspired algorithms, namely the beluga whale algorithm, BWO (2022, [4]), as well as the marine predator algorithm, MPA (2020, [5]), for determining the coefficients in the empirical model for determiningsound absorption coefficient. In order to obtain the best possible solutions, hybridization of these two algorithms was also carried out. The model for determining the characteristic acoustic impedance was derived based on the known dependencies defined by Delany and Bazley [6].

The accuracy of the results obtained by the proposed algorithms was checked by the results of measuring the absorption coefficients given in [2], and for the material polyurethane foam HR 3744.

2. EMPIRICAL MODELS FOR DETERMINING ACOUSTIC PROPERTIES OF MATERIALS

When selection a material, which is used in noise control, it is necessary to know sound absorption coefficient and the properties of the material under consideration. In the case of polymer foams, aforementioned provides the sound absorption coefficient at normal incidence α_n and the normal surface impedance Z_N. These quantities can be determined by experimental measurements, using the transfer method, described in the standard EN ISO 10534-2 [7]. However, these sizes can be predicted based on material properties: porosity, air flow resistance, pore geometry, etc. Empirical models using these material properties can be used to estimate acoustic properties.

The best-known and one of the first empirical models is the Delany-Bazley model [6], for determining the acoustic impedance and propagation coefficient of fibrous absorption materials. In this model, the only input parameter is the air flow resistance, which can be measured relatively easily. Dunn & Davern [8] retained the same form of equations developed by Delany & Bazley and calculated new values of regression constants for polyurethane foams.

2.1 Model for characteristic acoustic impedance

In order to demonstrate the justification of applying a biologically inspired algorithm to determine the characteristic acoustic impedance, in this work, the model defined by Delany-Bazley was applied[6].

The dependencies in this model are given in Eqs(1) – (4), [2].

$$Z_{CR} = \rho_0 c_0 \left[1 + C_1 \left(\frac{\sigma}{\rho_0 f} \right)^{C_2} \right] \tag{1}$$

$$Z_{CI} = -\rho_0 c_0 \left[C_3 \left(\frac{\sigma}{\rho_0 f} \right)^{C_4} \right] \tag{2}$$

$$\alpha = \left(\frac{2\pi f}{c_0} \right) \left[C_5 \left(\frac{\sigma}{\rho_0 f} \right)^{C_6} \right] \tag{3}$$

$$\beta = \left(\frac{2\pi f}{c_0} \right) \left[1 + C_7 \left(\frac{\sigma}{\rho_0 f} \right)^{C_8} \right] \tag{4}$$

where:

Z_{CR}, Z_{CI} – real and imaginary part of the characteristic acoustic impedance, Z_C;
α, β – real and imaginary parts of propagation constant, Γ,
σ – air flow resistance,
f – frequency,
ρ_0 – the density of air and
c_0 – the speed of sound in the air

The sound absorption coefficient at normal incidence, α_n, for a firmly supported layer of material of thickness *d*, can be obtained using expressions (5) and (6), with knowledge of characteristic acoustic impedance and propagation constant Γ.

$$Z_S = Z_C \coth \Gamma o \tag{5}$$

$$\alpha_n = 1 - \left| \frac{Z_S - \rho_0 c_0}{Z_S + \rho_0 c_0} \right|^2 \tag{6}$$

The values of the sound absorption coefficient, which will be used to verify the proposed algorithm, are given in Table 1, [2]. These values were obtained by measuring in an impedance tube using the transfer function method between two microphones, described in the standard SRPS EN ISO 10534-2 [7].

3. OPTIMIZATION ALGORITHMS

In the last decades, methods have been developed that are increasingly better at solving complicated real-world optimization problems. The main characteristic of these methods is that they were created as inspiration from nature and for this reason they are called biologically inspired methods. Biologically inspired optimization algorithms belong to the group of metaheuristic optimization methods that simulate natural processes and systems when performing search activities in the space of potential solutions, [9]. Among the best known, most popular, methods are: genetic algorithms (Genetic Algorithm - GA, John Holland, 1962), differential evolution (Differential Evolution - DE, R. Storn and K. Price 1996), particle swarm optimization (Particle Swarm Optmization - PSO, J. Kennedy and R. Eberhart in 1995), ant colony optimization (ACO M. Dorigo in the late 1990s), cuckoo search (CS – Xin-She Yang and Suash Deb, 2007), firefly algorithm (FA - Xin-She Yang, 2008), bat algorithm (Bat Algorithm – BA - Xin-She Yang, 2010), krill herd hlgorithm – (KHA – Amir H Gandomi and Amir H Alavi, 2012), gray wolf algorithm (Gray Wolf Optimizer, Seyedali Mirjalili, Seyed Mohammad Mirjalili, Andrew Lewis, 2014.

Table 1 *Absorption coefficient values for foam HR 3744, [2]*

	Material thickness (cm)									
f_C (Hz)	1	2	3	4	5	6	7	8	9	10
125	0.078945	0.052273	0.078038	0.091768	0.095615	0.099378	0.14166	0.14739	0.1988	0.23425
160	0.07159	0.064579	0.088787	0.095501	0.1163	0.13339	0.17142	0.1915	0.23306	0.28444
200	0.067147	0.071539	0.095544	0.1082	0.13866	0.16303	0.21552	0.26369	0.29383	0.36679
250	0.062784	0.081199	0.10469	0.12844	0.16959	0.20025	0.26771	0.3203	0.37254	0.47083
315	0.060061	0.082818	0.11674	0.15397	0.20809	0.24824	0.34862	0.41372	0.44536	0.56428
400	0.061533	0.088451	0.13883	0.19781	0.26972	0.31781	0.44411	0.49926	0.59604	0.73095
500	0.062915	0.10462	0.17202	0.25435	0.324	0.38727	0.55493	0.66397	0.75279	0.88024
630	0.07125	0.12535	0.22696	0.3171	0.44179	0.52143	0.74253	0.82768	0.8864	0.96609
800	0.086209	0.15796	0.29957	0.44704	0.59085	0.67178	0.88578	0.93204	0.94498	0.9434
1000	0.089769	0.19083	0.40787	0.5967	0.74673	0.80233	0.92326	0.92662	0.90819	0.8601
1250	0.1146	0.24294	0.56195	0.75699	0.84916	0.8587	0.86797	0.85938	0.83806	0.82517
1600	0.073466	0.30496	0.74481	0.86448	0.85371	0.84389	0.77397	0.80167	0.80959	0.90507
α_w	0.05	0.15	0.25	0.3(M)	0.35(M)	0.4(M)	0.55(M)	0.6(M)	0.65	0.75
f_C (Hz)	α_p									
250	0.063331	0.078519	0.105658	0.130203	0.172113	0.20384	0.277283	0.33257	0.370577	0.4673
500	0.065233	0.10614	0.17927	0.25642	0.34517	0.408837	0.580523	0.663637	0.745077	0.859093
1000	0.096859	0.197243	0.42313	0.600243	0.728913	0.777603	0.892337	0.906013	0.897077	0.876223

The advantage of these algorithms is that they are flexible, can be applied to a large number of optimization problems, as well as their adaptability to the optimization problem. It is important to note here that the function optimized by these methods does not have to be differentiable and continuous, and that there is no limit to the number of variables to be optimized. However, perhaps the most important advantage of these methods is that they are all algorithmically conceived and, as such, can be improved with simple modifications, thus achieving greater efficiency in finding the optimal solution.

In the continuation of the paper, two more recent algorithms are proposed: the beluga whale algorithm, BWO (2022, [4]), the marine predator algorithm, MPA (2020, [5]), as well as their hybridization.

3.1 Beluga whale alghorithm

Inspired by the behavior of the beluga whale while swimming, hunting and "falling", Figure 1, Changting Zhong, Gang Li and Zeng Meng proposed the Beluga Whale Algorithm (Beluga whale optimization – BWO) [4]. Belugas are social animals, living in groups that vary from 2 to 25 members. It is observed that pair of belugas have been swimming in sync or in a "mirror" mode. Beluga whales generally feed in a group and attack prey by directing fish into shallow water, sharing information among themselves taking into account the position of the best candidate. Hence, they share information about the positions considering the best candidate. During migrations and hunting, belugas are threatened by humans, killer whales and polar bears, so a small number of belugas do not survive and end up on the seabed, this phenomenon is called "whale fall".

These three dominant beluga whale behavior are mathematically formulated to form an optimization algorithm, [4].

For behavior modeling, beluga whales are considered as search agents that can move in the search space, changing their position vectors. Therefore, each beluga is a candidate for the best solutions, which is updated during optimization. The matrix to positions of search agents, [4]:

$$X = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,d} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,d} \\ \cdots & \cdots & \cdots & \cdots \\ x_{n,1} & x_{n,2} & \cdots & x_{n,d} \end{bmatrix}$$

where:

n – the population size of beluga whales,

d – the number of variables being optimized.

For each beluga whale (search agent), the obtained fitness values are stored as follows, [4]:

$$F_X = \begin{bmatrix} f\left(x_{1,1}, x_{1,2}, \ldots, x_{1,d}\right) \\ f\left(x_{2,1}, x_{2,2}, \ldots, x_{2,d}\right) \\ \cdots \\ f\left(x_{n,1}, x_{n,2}, \ldots, x_{n,d}\right) \end{bmatrix}$$

The BWO algorithm can transition from the exploration phase to exploitation depending on the balance factor, which is modeled as given in Eq. (7), [4].

$$B_f = B_0\left(1 - \frac{T}{2T_{\max}}\right) \quad (7)$$

where:

T - the current iteration,

T_{max} – maximum iterative number,

B_0 – randomly changes between (0, 1) at each iteration.

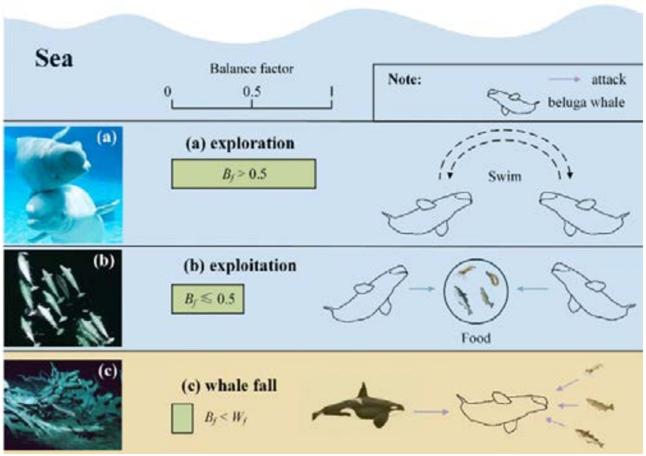

Fig. 1 *Behaviors of beluga whales, (a) swim - exploration phase; (b) foraging - exploitation phase, (c) whale fall- whale fall phase.,[4]*

As previously stated, belugas swim in pairs in a synchronized or mirrored mode. The mathematical model of this behavior is given by the equation (8), [4]

$$\begin{cases} X_{i,j}^{T+1} = X_{i,p_j}^{T} + \left(X_{r,p_1}^{T} - X_{i,p_j}^{T}\right)(1+r)\sin(2\pi r_2),\ j == even \\ X_{i,j}^{T+1} = X_{i,p_j}^{T} + \left(X_{r,p_1}^{T} - X_{i,p_j}^{T}\right)(1+r)\cos(2\pi r_2),\ j == odd \end{cases} \quad (8)$$

where:

T – - the current iteration,

$X_{i,j}^{T+1}$ – the new position for the ith beluga whale on the jth dimension,

p_j, $j = 1,2,\ldots,d$ – is a random number selected from d,

X_{i,p_j}^{T} – position of the ith beluga whale on p_j variable,

X_{i,p_j}^{T} и X_{i,p_j}^{T} – the current positions for ith and rth beluga whale (r - randomly selected beluga whale)

r_1, r_2 – random number between (0,1).

Due to the improvement of convergence, the Levy flight strategy is introduced in the exploitative phase of the BWO. The mathematical model of beluga behavior during feeding is given by the equation (9), [4].

$$X_i^{T+1} = r_3 X_{best}^{T} - r_4 X_i^{T} + C_1 \cdot L_F \cdot \left(X_r^{T} - X_i^{T}\right) \quad (9)$$

where:

T – current iteration,

X_i^T и X_r^T – current position for the ith beluga whale and a random beluga whale,

X_i^{T+1}– the position of new position of the ith beluga whale,

X_{best}^{T} – the best position among beluga whales,,

r_3, r_4– random number between (0,1)

C_1=2r$_4$ (1 – T/T_{max}) – random jump strength that measuring the intensity of Levy flight.

L_F - Levy flight function, [4].

To ensure a constant number of population sizes, beluga positions and step sizes of whale falls are used to determine the updated position. The mathematical model of the whale fall behavior is represented by the equation (10), [4].

$$X_i^{T+1} = r_5 X_i^{T} - r_6 X_r^{T} + r_7 X_{step} \quad (10)$$

где je:

r_5, r_6 и r_7 – random numbers between (0,1)

X_{step} – the step size of whale fall: X_{step}=f(u_b, l_b, C_2)

u_b, l_b – upper and lower boundary of variables

C_2 – step factor which is related to the probability of whale fall and population size, [4].

Based on all the given mathematical models and the mentioned approximations, in Figure 2, the proposed BWO algorithm is given in the form of a pseudo code, [4].

3.2 Marine predator algorithm

The Marine Predator Algorithm (MPA) is a metaheuristic algorithm that is biologically inspired by the feeding behavior of marine predators. This algorithm was proposed by Afshin Faramarzi, Mohammad Heidarinejad, Seyedali Mirjalili, Amir H. Gandomi [5]. When attacking prey, predators follow two strategies: Lévy, when prey concentration is low, and Brownian movement when prey is abundant. In order to make a trade-off between the two strategies, predators measure the ratio of velocity from the prey to them.

```
Initialization of search agents (beluga whale) X_k, k = 1,..,n%% n –the number of agents
Generating the best solution, F_best
Broj_iter=1000; %% maximum number of iterations
While i = 1,...,Broj_iter
        Calculating the probability of failure,W_f
        Calculation of the balance factor, B_f
        While k=1,...,n
            If B_f(k)> 0.5 %% search phase
                Random generation p_j (j = 1,...d); %% d  - number of variables
                A random beluga selection X_r;
                Updating the new position of k-th beluga
            Else if B_f(k) ≤ 0.5 %% expoatation phase
                Update random jump strength C_1;
                Calculation of the value of Levy flight
                Updating the new position of k-th beluga;
            End If
            New position limit check and calculation F_c; %% F_c - objective function
        End While
        While k=1,...,n
            If B_f(k) ≤ W_f %% beluga decay phase
                Factor update C_2;
                Step size update, X_step;
                New position update k-th belugas;
                New position limit check and calculation F_c;
            End If
        End While
        Searching for the currently best solution F_c^Best;
End While
```

Fig. 2 *Pseudo code of BWO algorithm,[4]*

The mathematical model of the MPA algorithm is presented below. Like most metaheuristics, MPA starts by initializing a set of solutions in the search space using Eq (11), [10]:

$$X_0 = X_{min} + rand\left(X_{max} - X_{min}\right) \tag{11}$$

where:

X_{min}, X_{max} – lower and upper bound for variables,
rand – uniform random vector in the range (0,1)

Bassed on the survival of the fittest, the top predator is the best one at foraging. Based on this, the matrix of the best solution (top predator), the so-called *Elite* matrix, is formed (12). This matrix array is used to find prey based on information about the position of the prey.

$$Elite = \begin{bmatrix} X_{1,1}^I & X_{1,2}^I & \dots & X_{1,d}^I \\ X_{2,1}^I & X_{2,2}^I & \dots & X_{2,d}^I \\ \dots & \dots & \dots & \dots \\ X_{n,1}^I & X_{n,d}^I & \dots & X_{n,d}^I \end{bmatrix}_{nxd} \tag{12}$$

where:

$\overrightarrow{X^I}$ - represents the top predator vector,
n – number of search agents,
d – search space.

In this algorithm, there is also a prey matrix, which serves to update the positions of the predators, 14.

$$Prey = \begin{bmatrix} X_{1,1} & X_{1,2} & \dots & X_{1,d} \\ X_{2,1} & X_{2,2} & \dots & X_{2,d} \\ \dots & \dots & \dots & \dots \\ X_{n,1} & X_{n,d} & \dots & X_{n,d} \end{bmatrix}_{nxd} \tag{13}$$

The MRA optimization process is divided into three main optimization phases taking into account the different velocity ratio between predator and prey. The first stage can be considered when the velocity ratio between predator and prey is high. In contrast, unit ratio, predator and prey move at almost the same pace, and low speed ratios represent the second and third stages. Details of each step are addresed below.

Phase 1. High velocity ratio ($v \geq 10$),

Space search (prey search) is represented by the equation 14.,[5].

$$\begin{gathered} While\ Iter < \frac{1}{3} Max\_iter \\ \overrightarrow{stepsize_i} = \overrightarrow{R_B} \otimes \left(\overrightarrow{Elite_i} - \overrightarrow{R_B} \otimes \overrightarrow{\Pr ey_i}\right),\ i = 1,..,n \\ \overrightarrow{\Pr ey_i} = \overrightarrow{\Pr ey_i} + P \cdot \vec{R} \otimes \overrightarrow{stepsize_i} \end{gathered} \tag{14}$$

where:

$\overrightarrow{R_B}$ – vector containing random numbers based on Normal distribution representing the Brownian motion,
P = 0,5 – constant number,
$\vec{R}$ – vector of uniform random numbers in [0,1],
Iter – the current iteration,
Max_Imer – maximum number of iterations.

Phase 2. Unit velocity ratio ($v \approx 1$),

This phase occurs in the intermediate phase of optimization process, where exploration gradually changes to exploitation. In this phase, both exploration and exploitation are important. Therefore, half of the individuals are designated for research, and the other half for exploitation. In this phase, the prey is responsible for the hunt and the predator is responsible for the search. The mathematical model of this phase is represented by the following equations 15., for the first half of the population [5].

$$\begin{gathered} While\ \frac{1}{3} Max\_iter < Iter < \frac{2}{3} Max\_iter \\ \overrightarrow{stepsize_i} = \overrightarrow{R_L} \otimes \left(\overrightarrow{Elite_i} - \overrightarrow{R_L} \otimes \overrightarrow{\Pr ey_i}\right),\ i = 1,..,n/2 \\ \overrightarrow{\Pr ey_i} = \overrightarrow{\Pr ey_i} + P \cdot \vec{R} \otimes \overrightarrow{stepsize_i} \end{gathered} \tag{15}$$

For the other half of the population, the model is represented by Eq. 16., [5].

$$\begin{gathered} \overrightarrow{stepsize_i} = \overrightarrow{R_B} \otimes \left(\overrightarrow{R_B} \otimes \overrightarrow{Elite_i} - \overrightarrow{\Pr ey_i}\right),\ i = n/2,..,n \\ \overrightarrow{\Pr ey_i} = \overrightarrow{Elite_i} + P \cdot CF \otimes \overrightarrow{stepsize_i} \end{gathered} \tag{16}$$

where:

$\overrightarrow{R_L}$ – vector of random numbers based on Lévy distribution representing Lévy movement. In this phase, the first half of the prey moves with the Levy strategy, while the other half moves with the Brownian movement strategy.

CF – adaptive parameter to control the step size for predator movement and is generated using Eq. 17., [5].

$$CF = \left(1 - \frac{Iter}{Max\_Iter}\right)^{\left(2 - \frac{Iter}{Max\_Iter}\right)} \tag{17}$$

Phase 3. Low velocity ratio ($v \approx 1$),

This is the exploitation phase and is represented by Eq 18., [5].

$$\begin{gathered} While\ Iter > \frac{2}{3} Max\_iter \\ \overrightarrow{stepsize_i} = \overrightarrow{R_L} \otimes \left(\overrightarrow{R_L} \otimes \overrightarrow{Elite_i} - \overrightarrow{\Pr ey_i}\right),\ i = 1,..,n \\ \overrightarrow{\Pr ey_i} = \overrightarrow{Elite_i} + P \cdot CF \otimes \overrightarrow{stepsize_i} \end{gathered} \tag{18}$$

The behavior of marine predators can be significantly affected by environmental issues, such as eddy formation or the Fish aggregating Devices (FAD) effect. As a result, the predators spend 80% of their time searching for prey in the vicinity, while the rest of the time they search for prey in other environments. The FADs are considered as local optima and their effect as trapping in these points in search space. Consideration of these longer jumps during simulation avoids stagnation in local optima. The mathematical model of this behavior can be represented by Eq. 19.,[5].

$$\overrightarrow{Prey_i} = \begin{cases} \overrightarrow{Prey_i} + CF\left[\vec{X}_{\min} + \vec{R} \otimes \left(\vec{X}_{\max} - \vec{X}_{\min}\right)\right] \otimes \vec{U}, \; if\ r \le FAD_s \\ \overrightarrow{Prey_i} + \left[FAD_s(1-r) + r\right]\left(\overrightarrow{Prey_{r1}} - \overrightarrow{Prey_{r2}}\right), \; if\ r > FAD_s \end{cases}, \tag{19}$$

where r is a random number in the range [0,1], a $\vec{U}$ is the binary vector with arrays including zero and one. FADs=0.2 indicates the influence of FADs on the updating process.

MPA saves memory by saving the old prey position. After updating the current solution, the fitness values of the current solution and the previous solution are compared and declared the best, if the value of the previous solution is better than the current solution. The pseudo code of the MPO algorithm is given in Figure 3.

```
Initializing Search Agents - Prey; k = 1,...,n%% n –  number of agents
Broj_iter=1000; %% maximum number of iterations
While i =1,...,Broj_iter
        Calculating the objective function (F_c);
        formation of the Elite matrix
        If i < Broj_iter/3
                Prey update, Eq. (16);
        Else if Broj_iter /3 < i < 2
                Broj_iter/3
                While k=1,...,n
                        If j<n/2
                                Prey update, Eq. (17)
                        Else
                                Prey update Eq. (18)
                End While
        Else if i > 2· Broj_iter /3
                Prey update Eq. (19)
        End If
        Memorizing and updating the Elite matrix
        Applying the FADs effect and updating, Eq. (20)
        Find F_c^Best; %% the best predator
End While
```

Fig. 3 *Pseudo code of the MPO algorithm*,[5]

3.3 Hybrid algorithm BW-MPA

Hybridization of optimization algorithms is performed in order to obtain better solutions in a shorter time frame. Usually, similar algorithms are used for the combination of optimization algorithms, that is, algorithms that have the same or similar basis in nature. For this reason, the algorithms presented above were chosen. Namely, beluga whales are members of marine mammals, which can also be said to be predators, except that they hunt in groups.

The hybridization of these two algorithms is reflected in the fact that each search agent, within the BWO algorithm, is positioned as the best predator from the MPA algorithm. Then, the search of the space of possible solutions is performed using the BWO algorithm, where after finding the best solution, an even better position of the agent from the BWO algorithm is checked and searched using the part of the MPA algorithm, Figure 4.

4. RESULTS AND DISCUSSION

The specificity of determining the coefficients of the empirical model of sound absorption is that the exact boundaries of the search space are not known in advance. In other words, the search space is infinite. With the selected empirical model, it is necessary to find eight coefficients, C_i, $i = 1,\ldots,8$. The limits in which they are searched are in the range of $-\infty$ to $+\infty$. These limits are defined by the vector values l_i for the lower limit, and u_i for the upper limit, Eq. 20.

$$\begin{aligned} l &= \left[-\infty-\infty-\infty-\infty-\infty-\infty-\infty-\infty\right] \\ u &= \left[+\infty+\infty+\infty+\infty+\infty+\infty+\infty+\infty\right] \end{aligned} \tag{20}$$

```
1   Initialization of search agents (beluga whale) X_k, k = 1,..,n%% n –the number of agents
2   Calculating the objective function (F_C);
3   Formation of the Elite matrix
4   If i < Broj_iter/3
5           Ažuriranje plena Eq. (16)
6   Else if Broj_iter /3 < i < 2
7           Broj_iter/3
9   While k=1,...,n
10          If j<n/2
11              Ažuriranje plena Eq. (17)
12          Else
13              Ažuriranje plena Eq. (18)
14          Else if i > 2· Broj_iter /3
16              Ažuriranje plena Eq. (19)
17          End If
18  End While
19  Memorizing and updating the Elite matrix
20  Applying the FADs effect and updating, Eq. (20)
21  Find F_c^Best; %% the best predator
22  F_C^Best = Predator_c^Best;
23  l = −rand(1,d) · Predator_c^Best; u = rand(1,d) · Predator_c^Best;
24  %% Modified Beluga algorithm:
25  %% after finding the best solution, F_c^Best, line 29 -  BWO pseudocode
26  %% beluga is seen as a predator and within a cycle While i = 1,...,n
27  %% the part of the MPA algorithm that finds the best predator is inserted
28  While i = 1,...,Broj_iter
29          ...
30          Searching for curent the best solution F_c^Best;
31          Pozicija_plena = Pozicija_beluge_best;
32          Formation of the Elite matrix
33          If i < Broj_iter/3
34                  Ažuriranje plena Eq. (16)
35          Else if Broj_iter /3 < i < 2
36                  Broj_iter/3
37                  While k=1,...,n
38                          If j<n/2
39                                  Ažuriranje plena Eq. (17)
40                          Else
41                                  Ažuriranje plena Eq. (18)
42                  End While
43          Else if i > 2· Broj_iter /3
44                  Ažuriranje plena Eq. (19)
45          End If
46          Memorizing and updating the Elite matrix
47          Applying the FADs effect and updating, Eq. (20)
48          Finding F_c^Best;
49  End While
```

Fig. 4 *Pseudo code of hybrid BW-MPA algorithm*

For all three algorithms, BWO, MPA and BW-MPA, the algorithm parameters used are:

The number of search agents is 20

The maximum number of iterations is 1000.

Algorithms were applied as suggested in the literature [4] and [5], that is, no modifications were performed. Conventionally speaking, the only modification that has been made is related to the hybridization of these algorithms. The first modification refers to the reduction of the search space, so that, after setting the limits of the variables, equation 20, the positioning of the best search agent was performed, using the MPA algorithm and the fitness values, obtained in this step, are used as a basis for defining the limits of the search space, equation 21, line 23, BW-MPA.

$$\begin{aligned} l &= -rand(1,d) \cdot Predator_c^{Best} \\ u &= rand(1,d) \cdot Predator_c^{Best} \end{aligned} \tag{21}$$

where

d – number of variables,

$Predator_c^{Best}$ – the best solution of the objective function according to the MPA algorithm.

The second modification refers to the incorporation of a part of the MPA algorithm, which attempts to check whether the best solution obtained by the BWO algorithm is also the best predator, lines 30 – 38, BW-MPA pseudocode, Figure 4.

Figure 5 shows a comparative view of the results obtained by the proposed algorithms.

Parameter	BWO	MPA	BW-MPA
Duration of the algorithm [sec]	***407.4369253343***	***1510.8727405994***	***571.7954629894***
C_1	0.65264	0.42371	0.43462
C_2	0.07088	0.30741	0.29065
C_3	-6.38976	-5.00000	-11.71252
C_4	-3.27006	-3.03414	-3.81486
C_5	0.36615	0.30101	0.30681
C_6	0.03847	0.13982	0.13069
C_7	0.10352	0.09744	0.09994
C_8	0.94643	0.94116	0.93413
The objective function Δ	***0.2572380020***	***0.2486829801***	***0.2474991507***
The worst solution	12.4443331703	10.4778114079	26.9913366998
Middle value	0.3145679399	0.3395845575	0.4516901084
Standard deviation	0.4399196086	0.5602260743	1.1194907950

Fig. 5 *Results obtained by the proposed algorithms*

Figures 6, 7 and 8 show convergence curves in the process of searching for the best solution.

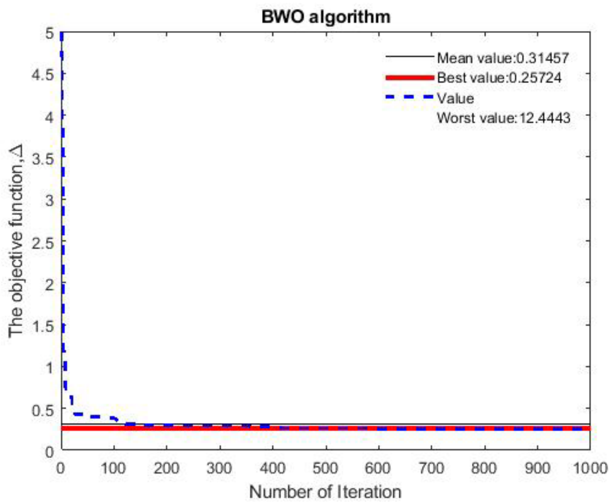

Fig. 6 *Iterative process in determining the coefficients, Ci,i=1,…,8; when implementing the BWO algorithm*

Analyzing the results given in figure 5, as well as the curves given in figures 6, to 8, it is observed that all three diagrams have an excellent convergence. Although, in practice, the search space is unlimited, Eq. 20, all three algorithms already enter the zone of the best possible solution after ten iterations..

By applying these modifications, it was expected that the hybrid algorithm works slower than the individual algorithms. From Figure 5, it can be seen that the search time is slightly longer than that of the BWO algorithm, and significantly less than that of the MPA algorithm. Here we should be honest and say that the duration of the search with the MPA algorithm is determined by the very nature of the algorithm: the number of iterations, at least three cycles of adjustment within one iteration and the like, which in fact leads to an increase in the duration of the search compared to the BWO algorithm.

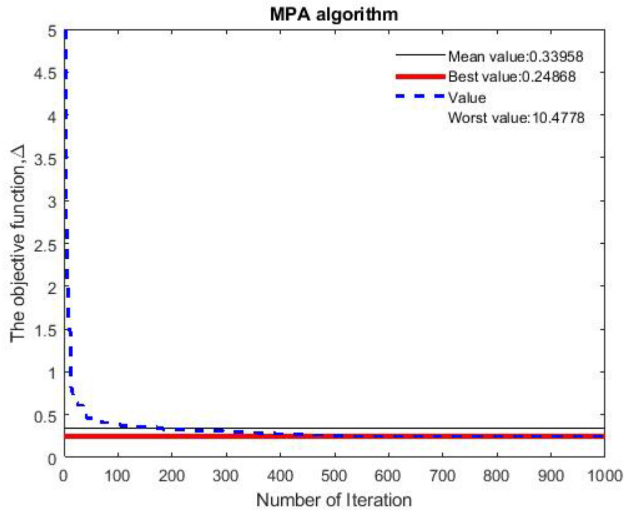

Fig. 7 *Iterative process in determining the coefficients, Ci,i=1,…,8; when implementing the MPA algorithm*

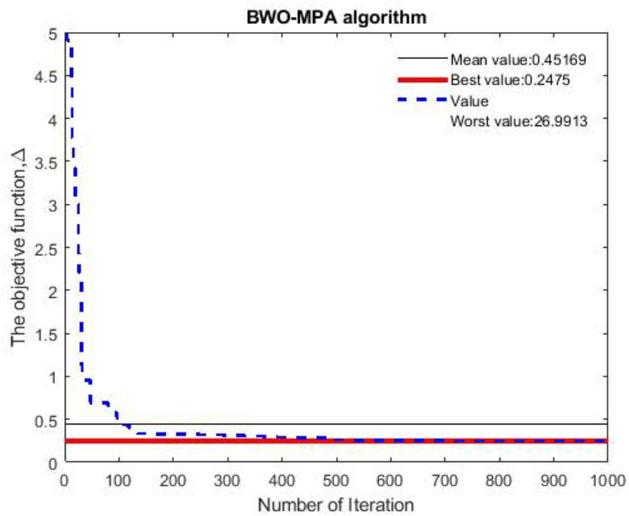

Fig. 8 *Iterative process in determining the coefficients, Ci,i=1,…,8; when applying the hybrid BW-MPA algorithm*

Furthermore, the results obtained by applying these three algorithms to the determination of the coefficients in the empirical model for determining the sound absorption coefficient show that practically all three algorithms give approximate results, with slightly better results obtained by applying the BWO-MPA algorithm Δ = **0.2474991507**. This can be explained by the similar nature of the mentioned algorithms.

The second conclusion can serve as a guideline for further research, and it would refer to the rapid convergence towards the best solution. This actually means that the algorithms can enter the space of possible local minima. In order to avoid this, and especially in the case of an infinite search space, it is necessary to modify these algorithms to avoid the possibility of entering the space of some of the local minima.

Figure 9 shows the dependence of the sound absorption coefficient α_n depending on the thickness of the material, d [*m*] and the frequency *f* [Hz], based on the obtained results of the coefficients, C_i, i = 1,..,8., from the Table in the Figure 5.

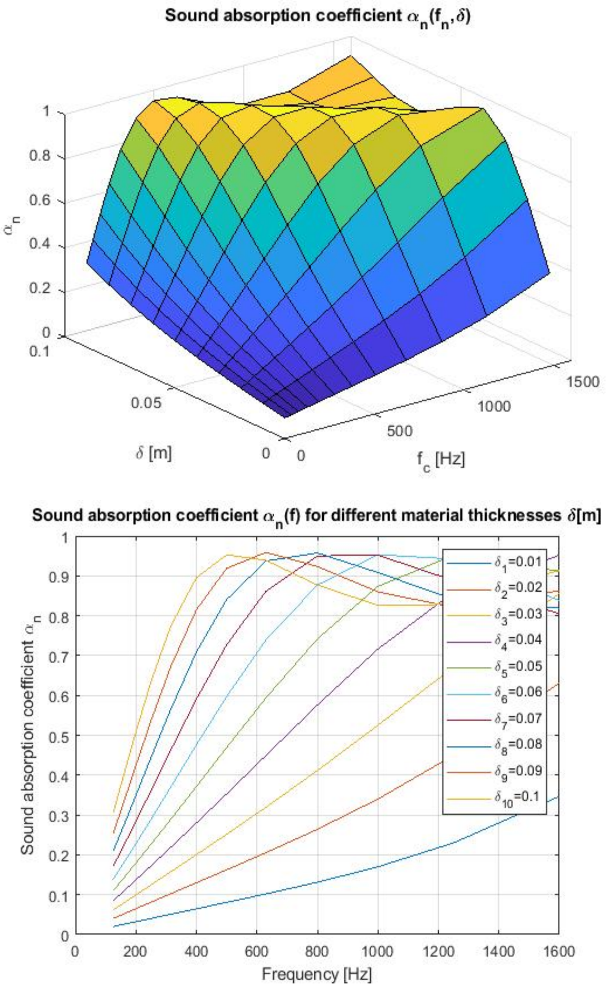

Fig. 9 *Graphical representation of the sound absorption coefficient $\alpha_n=F(f,\delta)$*

5. CONCLUSION

In this paper, the application of the hybridized BWO-MPA algorithm is proposed, by combining a part of the MPA algorithm in the BWO algorithm. The proposed modification was reflected in the fact that the MPA algorithm was used to narrow the infinite search space. This was done by positioning the best search agent, applying the MPA algorithm, and the fitness values, obtained in this step, are used as a basis for defining the boundaries of the search space, equation 21, line 23, pseudocode from Figure 4. A further modification included part of the MPA algorithm in the search cycle of the BWO algorithm, which checks whether the best solution obtained by the BWO algorithm is also the best predator, lines 30 – 38, BW-MPA pseudocode, Figure 4.

Fast convergence, when searching the infinite space of possible solutions, is good, however, it hides the danger of entering the space of a local minimum. Additional modifications of the proposed algorithm, in the continuation of the research, should remove or at least mitigate the danger of entering the local minimum space.

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