Application of Sub Matrixes for Phase Process Optimization of Linear Programming

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The problem of optimization of a multiphase production process by the linear programming method is very frequent in manufacturing practice. Instead of a classical solution to the problem, by creating constraint equations per production phase, the paper proposes the methodology of application of sub matrixes for solving the complex mathematical model of the problem in a matrix form.

The method is illustrated in the example of an optimization process of manufacturing and assembly of a hydraulic valve for regulation of pressure and flow, which is intended for installation on hydraulic bar feeders for CNC machines. Problem is solved by MatLab software package.

Keywords: Phase process, linear programming, optimization, sub matrixes

1. INTRODUCTION

In contrast to the single-phase production process in which the final products are directly produced from semi-finished products, multiphase process can be divided into several single-phase processes, interrelated and conditioned by the given constraints.

The number of levels of multiphase process breakdown is determined by objective conditions such as technology production process and assembly of the product, although breakdown can sometimes be also due to subjective decisions [1] [2].

The paper [2] discusses programming of multiphase processes that can be mathematically solved by linear programming method. The procedure of forming a mathematical model to optimize multiphase process is present in the example of manufacturing and assembly of hydraulic valves for regulating pressure and flow, which is designed for installation on hydraulic feeders from rod material for CNC machines.

The task consists of the following procedure [2]: it is necessary to program the multi-phase production process in order to produce the optimal amount of control valves from available quantity of semi-finished product and standard parts while at the same time profit from the sale of the valve is maximum. Market limitations in the amount of product placement do not exist.

2. MATHEMATICAL MODEL

The mathematical model in its matrix form reads:
It is necessary to maximize the objective function:.

$$\mathbf{max\ F(X) = d\ X} \tag{1}$$

with satisfying the constraints with respect to available quantities:

$$\mathbf{M\ X \leq B} \tag{2}$$

and the non-negativity condition:

$$\mathbf{X \geq 0} \tag{3}$$

For the given example [2], the matrices **M** , **X** and **B** read:

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} \qquad B = \begin{bmatrix} 10026,24 \\ 38,733 \\ 12,255 \\ 6,456 \\ 53,36 \\ 38,142 \\ 22,694 \\ 500 \\ 600 \\ 500 \\ 700 \\ 500 \\ 600 \\ 1000 \\ 700 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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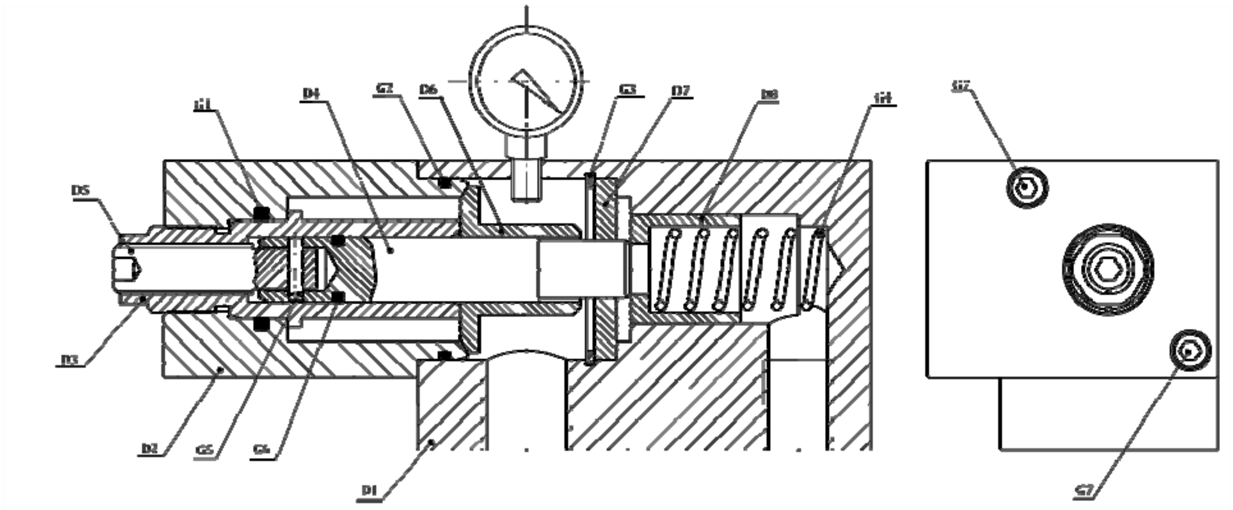

Fig. 1. Assembly structure of the product

$$
\mathbf{M} = \left[\begin{array}{cccccccc:ccc:c:c}
4{,}888 & 3{,}459 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0{,}646 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0{,}219 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0{,}097 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0{,}534 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0{,}191 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}199 & 0 & 0 & 0 & 0 & 0 \\
\hdashline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\hdashline
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\
\hdashline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\
\hdashline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1
\end{array}\right]
$$

It can be noted that the matrix of coefficients **M**, the matrix of variables **X** and the matrix of constraints **V** consist of several sub matrixes. In the given example, the matrix **M** is of dimension p x q and is composed of p x q = 5 x 4 = 20 sub matrixes. If the following marks are introduced:

- i – the number of different semi-finished products
- j – the number of standard parts
- k – the number of parts fabricated in first phase
- l – the number of sub-assemblies fabricated in second phase
- m – the number of main assemblies fabricated in third phase
- n – the number of products
- s – the number of phases

Main matrix **M** can be present in the following shape:

$$
\mathbf{M} = \left[\begin{array}{c:c:c:c}
M_{11} & M_{12} & M_{13} & M_{14} \\ \hdashline
M_{21} & M_{22} & M_{23} & M_{24} \\ \hdashline
M_{31} & M_{32} & M_{33} & M_{34} \\ \hdashline
M_{41} & M_{42} & M_{43} & M_{44} \\ \hdashline
M_{51} & M_{52} & M_{53} & M_{54}
\end{array}\right]
\begin{array}{c} q \\ \\ \\ \\ p \end{array}
$$

wherein matrix dimension are

$$p = s + 1$$

$$q = s$$

The generating of sub matrixes of matrix **M** can be performed in the following manner:

a) The formation of the first column of matrix **M** from its sub matrixes is as follows

Sub matrix **M11** is of dimension i x k , sub matrix **M21** is of dimension j x k, sub matrix **M31** is of dimension k x k, sub matrix **M41** is of dimension l x k, sub matrix **M51** is of dimension m x k. For presented example [2] these matrixes are:

$$M_{11} = \begin{bmatrix} 4{,}888 & 3{,}459 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0{,}646 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0{,}219 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0{,}097 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0{,}534 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0{,}191 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}199 \end{bmatrix}_i^k$$

$$M_{21} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_j^k$$

$$M_{31} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}_k^k$$

$$M_{41} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_l^k$$

$$M_{51} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_m^k$$

b) The formation of the second column of matrix **M** from its sub matrixes for given example is as follows

$$M_{12} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_i^l \quad M_{22} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_j^l$$

$$M_{32} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}_k^l \quad M_{42} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}_l^l;$$

$$M_{52} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}_m^l;$$

c) The formation of the third column of matrix **M** from its sub matrixes for given example is as follows

$$M_{13} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_i^m; \quad M_{23} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_j^m; \quad M_{33} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_k^m;$$

$$M_{43} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}_l^m; \quad M_{53} = \begin{bmatrix} -1 \end{bmatrix}_m^m;$$

d) The formation of the fourth column of matrix **M** from its sub matrixes for given example is as follows

$$M_{14} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_i^n; \quad M_{24} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}_j^n; \quad M_{34} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_k^n;$$

$$M_{44} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_l^n; \quad M_{54} = \begin{bmatrix} 1 \end{bmatrix}_m^n;$$

One can note the following:

- sub matrixes $\mathbf{M_{12}}$, $\mathbf{M_{13}}$ и $\mathbf{M_{14}}$ are zero matrixes because in phase processes achievement of sub-assemblies, main assemblies and products is not possible just from semi-finished products;
- sub-matrixes $\mathbf{M_{41}}$, $\mathbf{M_{51}}$ и $\mathbf{M_{52}}$ are also zero matrices because it is not possible to achieve part from sub-assemblies and assemblies, nor is it possible to achieve sub-assembly from main assembly;
- sub-matrixes $\mathbf{M_{31}}$, $\mathbf{M_{42}}$ and $\mathbf{M_{53}}$ are diagonal matrices wherein the elements on the main diagonal have a value of -1, so they represent a scalar matrices, ie.

$$M_{31} = [-1a]_k^k$$
$$M_{42} = [-1a]_l^l$$
$$M_{53} = [-1a]_m^m$$

which are negative of

$$M_{31} = -E = [-a]_k^k$$
$$M_{42} = -E = [-a]_l^l$$
$$M_{53} = -E = [-a]_m^m$$

The mentioned matrixes can be defined solely on *i, j, k, l, m, n* and *s* values, while the other sub matrixes of main matrix **M** can be formed on the basis of data for each phase of development.

The matrix **X** is a matrix of columns and will have as many sub matrixes as phases in the process that is. *r = s*, so there are 4 sub matrixes in the given example:

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}_r^1,$$

The number of elements of sub matrixes are:

- for matrix $\mathbf{X_1}$ the number of elements is equal to the number of parts k,
- for matrix $\mathbf{X_2}$ the number of elements is equal to the number of sub-assemblies l,
- for matrix $\mathbf{X_3}$ the number of elements is equal to the number of main assemblies m, and
- for matrix $\mathbf{X_4}$ the number of elements is equal to the number of products n.

Sub matrixes of matrix **X** for given example are:

$$X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix}_k^1; \quad X_2 = \begin{bmatrix} x_9 \\ x_{10} \\ x_{11} \end{bmatrix}_l^1;$$

$$X_3 = [x_{12}]_m^1; \quad X_4 = [x_{13}]_n^1.$$

The matrix **B** is also column matrix and consists of *c=s+1* column matrixes:

$$B = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix}_c^1$$

The number of elements of sub matrixes is:

- the number of elements of the matrix $\mathbf{B_1}$ is equal to the number of semi-finished products,
- the number of elements of the matrix $\mathbf{B_2}$ is equal to the number of standard parts,
- the number of elements of the matrix $\mathbf{B_3}$ is equal to the number of parts,
- the number of elements of the matrix $\mathbf{B_4}$ is equal to the number of sub-assemblies,
- the number of elements of the matrix $\mathbf{B_5}$ is equal to the number of main assemblies.

Sub matrixes $\mathbf{B_3}$, $\mathbf{B_4}$, $\mathbf{B_5}$, are zero matrixes, while sub matrix $\mathbf{B_1}$ represents available quantity of semi-finished products and sub matrix $\mathbf{B_2}$ represent available amount of standard parts. For given example these sub matrixes are:

$$B_1 = \begin{bmatrix} 10026,24 \\ 38,733 \\ 12,255 \\ 6,456 \\ 53,36 \\ 38,142 \\ 22,696 \end{bmatrix}_i^1; \quad B_2 = \begin{bmatrix} 500 \\ 600 \\ 500 \\ 700 \\ 500 \\ 600 \\ 1000 \\ 700 \end{bmatrix}_j^1; \quad B_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_k^1;$$

$$B_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_l^1; \quad B_5 = [0]_m^1$$

3. PROBLEM SOLVING

For solving the multiphase problem of linear programming with sub matrixes the software package MATLAB is used. For data from the mentioned example program code is written in the following form:

```
i=7;
j=8;
k=8;
l=3;
m=1;
n=1;
s=4;

M11=[4.888 3.459 0 0 0 0 0 0; ...
    0 0 0.646 0 0 0 0 0;...
    0 0 0 0.219 0 0 0 0;...
    0 0 0 0 0.097 0 0 0;...
    0 0 0 0 0 0.534 0 0;...
```

```
    0 0 0 0 0 0 0.191 0;...
    0 0 0 0 0 0 0 0.199;]

M21=zeros(j,k)
M31=-eye(k)
M41=zeros(l,k)
M51=zeros(m,k)

M12=zeros(i,l)
M22=[0 1 0;...
     0 1 0;...
     0 0 1;...
     0 0 1;...
     1 0 0;...
     1 0 0;...
     0 0 0;...
     0 0 0;]
M32=[0 0 1;...
     0 1 0;...
     1 0 0;...
     1 0 0;...
     1 0 0;...
     0 0 0;...
     0 0 1;...
     0 0 1;]
M42=-eye(l)
M52=zeros(m,l)

M13=zeros(i,m)
M23=[0;0;0;0;0;0;0;0]
M33=[0;0;0;0;0;1;0;0]
M43=[1;1;0]
M53=-eye(m)

M14=zeros(i,n)
M24=[0;0;0;0;0;0;2;1]
M34=[0;0;0;0;0;0;0;0]
M44=[0;0;1]
M54=[1]

M=[M11 M12 M13 M14;
   M21 M22 M23 M24;
   M31 M32 M33 M34;
   M41 M42 M43 M44;
   M51 M52 M53 M54;]

B1=[10026.24;38.733;12.255;6.456;53.36;38.1
42;22.696]
B2=[500;600;500;700;500;600;1000;700]
B3=zeros(k,1)
B4=zeros(l,1)
B5=zeros(m,1)
B=[B1;B2;B3;B4;B5]

F=[0 0 0 0 0 0 0 0 0 0 0 0 0 100]'

[x,fval,exitflag,output]=linprog(-
F,M,B,[],[],[],[],[])
```

After the sixth iteration the following solution is obtained:

```
x =
  1.0e+003 *

    1.1322
    1.0206
    0.0577
    0.0560
    0.0632
    0.0813
    0.1648
    0.1026
    0.0560
    0.2317
    0.0762
    0.0560
    0.0560

fval =
 -5.5959e+003

exitflag =
     1
output =
      iterations: 6
       algorithm:  'large-scale:  interior
point'
    cgiterations: 0
```

Therefore, based on the available quantity of semi-finished products and standard parts it is possible to produce 56 units of a product, where the profit of 5.600,00 of monetary units is achieved.

4. CONCLUSION

The paper [2] presents the methodology of a mathematical modeling of the phase process of linear programming in matrix form. Unlike the traditional way of mathematical modeling where for each phase equations of restrictions for certain category of recourses are defined, by applying this methodology it is possible to define the matrixes **X**, **M** and **B** from the summary table, which defines the forming of the product in phases, which significantly reduces errors occurrence in the mathematical modeling process.

For multiphase problems in linear programming that have many constraints and variables, matrixes **X**, **M** and **B** can be expressed by sub matrixes which simplifies the process of mathematical modelling because the majority of sub matrixes can be defined automatically.

Due to limited space, in this paper, four phase production problem is analyzed on practical example only with constraints of the available quantities of semi-finished products and standard parts. For solving the problem MatLab software package is used.

Real practical problems are more complex because products are more complex, requiring a greater number production phases, and mathematical models have more resource constraints such as: labor restriction, the available capacity of machines, market constraints, the available monetary fund and etc.

Further authors efforts will be focused on the software solving of the problem with unlimited number of phases and large number of different types of constraints.

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