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MODEL FOR OPTIMIZATION OF PHASE PROCESSES BY THE LINEAR PROGRAMMING METHOD

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Abstract: *The problem of optimization of a multiphase production process by the linear programming method is very frequent in manufacturing practice. Instead of a classical solution to the problem, i.e. creation of constraint equations per production phase, the paper proposes the methodology of forming of a unique (summary) table which can be used for creation of a complex mathematical model of the problem in a matrix form.*
The method is illustrated on the example of an optimization process of manufacturing and mounting of a hydraulic valve for regulation of pressure and flow, which is foreseen for installation on hydraulic bar feeders for CNC machines.

Key words: *Phase process, linear programming, optimization*

1. INTRODUCTION

Production processes can be single phase and multiphase processes. Single phase ones are those in which final products are directly made of raw material. Unlike single phase processes, a multiphase process can be divided into several phases in which the results of previous phases can have effects on later ones. To be more precise, a multiphase process can be divided into several single phase processes which are interconnected and conditioned by the given constraints.
Programming of a multiphase process is commonly reduced to the problem of process realization by individual phases for the purpose of achieving an optimum production result. The number of levels of division of a multiphase process is conditioned by objective circumstances, such as the technological process of manufacturing and mounting of the product, although division can sometimes be a consequence of subjective decisions, too [2].
The paper deals with the programming of multiphase processes which can be mathematically solved by linear programming methods. The procedure of creation of the mathematical model of multiphase process optimization is presented on the example of manufacturing and mounting of a hydraulic valve for regulation of pressure and flow, which is foreseen for installation of hydraulic bar feeders for CNC machines.
The look and mounting structure of the hydraulic valve with its accompanying parts is shown in Figure 1. The product is formed in four phases: in the first phase the parts (D1, D2, ... D8) are manufactured from semi-finished products, in the second phase the parts and standard parts (GR1, GR2, ... GR8) are used for making subassemblies (PS1, PS2 and PS3), in the third phase the main assembly (GS) is made, and the product (P) is formed in the fourth phase.
The task is as follows: It is necessary to program a multiphase production process so that the optimum quantity of regulating valves could be manufactured from the available quantities of semi-finished products and standard parts and thus acquire a maximum profit from the sale of those valves. There are no market constraints with respect to the quantity of products for sale.

Table 1. Specification of necessary semi-finished products for manufacturing valve parts

Designation	Name of part	Unit	Qty/product	Semi-finished product				Mass of finished component [kg]
				Material	Dimensions [mm]	Mass of work piece [kg]	Designation	
D_1	Control block	Pc.	1	S355JR	≠ 60×80×130	4.888	S_1	3.360
D_2	Block	Pc.	1	S355JR	≠ 60×80×92	3.459		2.046
D_3	Flow regulating spindle	Pc.	1	C45E	Ø32×100	0.646	S_2	0.243
D_4	Pressure regulating spindle	Pc.	1	C45E	Ø18h9×107	0.219	S_3	0.176
D_5	Screw 005	Pc.	1	C45E	Ø16×60	0.097	S_4	0.057
D_6	Piston	Pc.	1	C45E	Ø46×40	0.534	S_5	0.105
D_7	Ring	Pc.	1	C45E	Ø55×10	0.191	S_6	0.078
D_8	Bushing	Pc.	1	C45E	Ø30h9×35	0.199	S_7	0.073

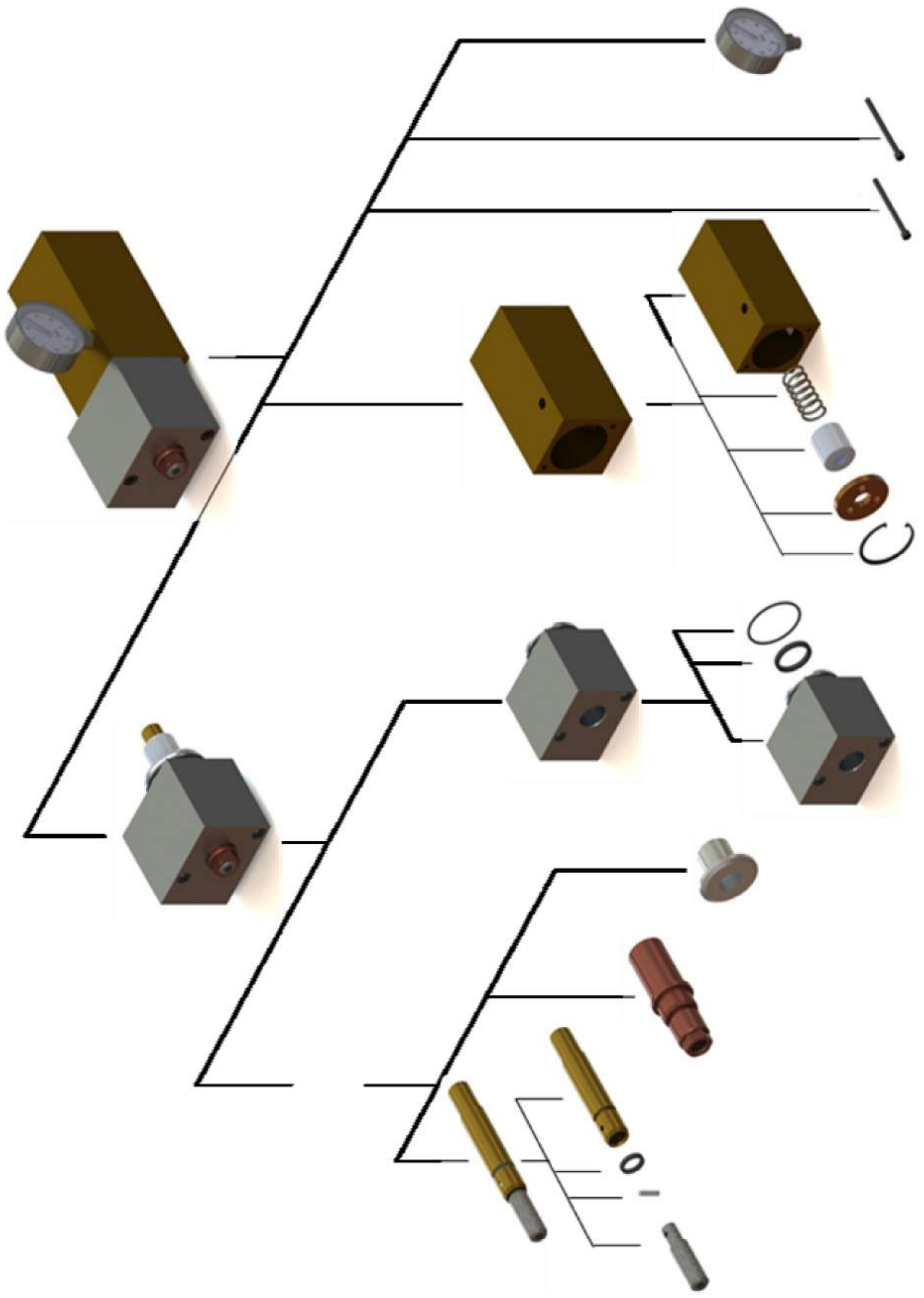

Fig. 1. Assembly structure of the product

Parts D1 through D8 are manufactured from semi-finished products by cutting operation. The necessary dimensions and quantities of semi-finished products for parts manufacturing are shown in Table 1.

The available quantities of semi-finished products in the company are shown in Table 2.

Table 2. Available quantities of semi-finished products used for manufacturing valve parts

Semi-finished products						
Designation	**Name**	**Unit**	**Avail. Qty**	**Material**	**Dimensions [mm]**	**Mass*[kg]**
S_1	Steel plate	Pc.	4	S355JR	2000×2000×80	2506.560
S_2	Cold drawn steel bars	Pc.	3	C45E	Ø32×2000	12.911
S_3	Cold drawn steel bars	Pc.	3	C45E	Ø18×2000	4.085
S_4	Cold drawn steel bars	Pc.	2	C45E	Ø16×2000	3.228
S_5	Cold drawn steel bars	Pc.	2	C45E	Ø46×2000	26.680
S_6	Cold drawn steel bars	Pc.	1	C45E	Ø55×2000	38.142
S_7	Cold drawn steel bars	Pc.	2	C45E	Ø30×2000	11.348
* The mass of semi-finished products refers to the unit of measurement (one steel plate, bar, etc.......)						

Table 3. Specification of standard parts and available quantity

Designation	**Name**	**Standard/ Manufacturer**	**Unit**	**Avail. Qty**
GR_1	Sealing	RubeliGuiquoz	Pc.	500
GR_2	O-ring	RubeliGuiquoz	Pc.	600
GR_3	Seeger ring	DIN472	Pc.	500
GR_4	Spring	DIN 2098	Pc.	700
GR_5	Elastic pin	DIN 1481	Pc.	500
GR_6	Sealing	RubeliGuiquoz	Pc.	600
GR_7	Screw M6x80	DIN 912	Pc.	1000
GR_8	Manometer	WIKA 0-10 bar	Pc.	700

The available quantities of standard parts which are purchased on the market and which are necessary for completion of the valves are shown in Table 3.

2. PRODUCTION PHASES AND CONSTRAINT EQUATIONS

The process of manufacturing and mounting parts, subassemblies and assemblies takes place in phases presented in the following tables. In the first phase semi-finished products S1, S2, . . . S7 are cut for the purpose of making parts D1, D2, . . . D8 in quantities x1, x2, . . . , x8. The necessary and available quantities of semi-finished products are shown in Table 4.

Table 4. Phase 1

			Products of Phase 1								**Available quantities**
			D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	
Phase 1	**Semi-fin. products**	S_1	4.888	3.459							10026.24
		S_2			0.646						38.733
		S_3				0.219					12.255
		S_4					0.097				6.456
		S_5						0.534			53.36
		S_6							0.191		38.142
		S_7								0.199	22.696
	Quantity		x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	

For the first phase of production it is necessary to determine the variables $x_1, x_2, \ldots, x_8$ which satisfy the non-negativity conditions

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \geq 0 \quad (1)$$

and constraints with respect to the available quantities:

$$4.888 \cdot x_1 + 3.459 \cdot x_2 \leq 10026.24 \quad (2)$$

$$0.646 \cdot x_3 \leq 38.733 \quad (3)$$

$$0.219 \cdot x_4 \leq 12.255 \quad (4)$$

$$0.097 \cdot x_5 \leq 6.456 \quad (5)$$

$$0.534 \cdot x_6 \leq 53.36 \quad (6)$$

$$0.191 \cdot x_7 \leq 38.142 \quad (7)$$

$$0.199 \cdot x_8 \leq 22.696 \quad (8)$$

After the completion of Phase 1, the remaining quantities of semi-finished products are

$a = 10026.24 – 4.888 \cdot x_1 – 3.459 \cdot x_2$ semi-fin. product S_1

$b = 38.733 – 0.646 \cdot x_3$ semi-fin. product S_2

$c = 12.255 – 0.219 \cdot x_4$ semi-fin. product S_3

$d = 6.456 – 0.097 \cdot x_5$ semi-fin. product S_4

$e = 53.36 – 0.534 \cdot x_6$ semi-fin. product S_5

$f = 38.142 – 0.191 \cdot x_7$ semi-fin. product S_6

$g = 22.696 – 0.199 \cdot x_8$ semi-fin. product S_7

and the available quantities of parts $D_1, D_2, \ldots D_8$ cy $x_1, x_2, \ldots, x_8$.

It may happen in Phase 1 that a new part is obtained by additional treatment of a standard part. In that case, Table 4 can be extended by the category of standard parts (Table 4a) on the basis of which new constraints with respect to available quantities of standard parts can be written. There is not such a case in the given example so that these equations are not written.

Table 4a. Phase 1

			Products of Phase 1								Available quantities
			D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	
Phase 1	Semi-fin. products	S_1	4.888	3.459							10026.24
		S_2			0.646						38.733
		S_3				0.219					12.255
		S_4					0.097				6.456
		S_5						0.534			53.36
		S_6							0.191		38.142
		S_7								0.199	22.696
	Standard parts	GR_1									500
		GR_2									600
		GR_3									500
		GR_4									700
		GR_5									500
		GR_6									600
		GR_7									1000
		GR_8									700
	Quantity		x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	

In the second phase, the remaining quantity of semi-finished products, parts manufactured in the first phase and standard parts are used to form subassemblies *PS1*, *PS2* and *PS3* in quantities x_9, x_{10} and x_{11} (Table 5).

Table 5. Phase 2

			Products of Phase 2			Available quantities
			PS1	*PS2*	*PS3*	
Phase 2	Semi-fin. products	S_1				$a = 10026.24–4.888 \cdot x_1–3.459 \cdot x_2$
		S_2				$b = 38.733–0.646 \cdot x_3$
		S_3				$c = 12.255–0.219 \cdot x_4$
		S_4				$d = 6.456–0.097 \cdot x_5$
		S_5				$e = 53.36–0.534 \cdot x_6$
		S_6				$f = 38.142–0.191 \cdot x_7$
		S_7				$g = 22.696–0.199 \cdot x_8$
	Standard parts	GR_1		1		500
		GR_2		1		600
		GR_3			1	500
		GR_4			1	700
		GR_5	1			500
		GR_6	1			600
		GR_7				1000
		GR_8				700
	Products of Phase 1	D_1			1	x_1
		D_2		1		x_2
		D_3	1			x_3
		D_4	1			x_4
		D_5	1			x_5
		D_6	1			x_6
		D_7			1	x_7
		D_8			1	x_8
	Qty		x_9	x_{10}	x_{11}	

The constraint equations for Phase 2 are:
non-negativity conditions

$$x_9, x_{10}, x_{11} \geq 0 \quad (9)$$

and the constraints with respect to available quantities are:

$$x_9 \leq 500 \quad (10)$$
$$x_9 \leq 600 \quad (11)$$
$$x_{10} \leq 500 \quad (12)$$
$$x_{10} \leq 600 \quad (13)$$
$$x_{11} \leq 500 \quad (14)$$
$$x_{11} \leq 700 \quad (15)$$
$$x_{11} \leq x_1 \quad (16)$$
$$x_{10} \leq x_2 \quad (17)$$
$$x_9 \leq x_3 \quad (18)$$
$$x_9 \leq x_4 \quad (19)$$
$$x_9 \leq x_5 \quad (20)$$
$$x_9 \leq x_6 \quad (21)$$
$$x_{11} \leq x_7 \quad (22)$$
$$x_{11} \leq x_8 \quad (23)$$

As the constraints (10), (12) and (14) exclude constraints (11), (13) and (15), Equations (11), (13) and (15) can be excluded from further consideration. However, for the purpose of understanding the methodology which is proposed in the continuation, these constraints will be kept, and in the inequalities (16) through (23) the variables are moved to their left sides so that they now read:

$$-x_1 + x_{11} \leq 0 \quad (16')$$
$$-x_2 + x_{10} \leq 0 \quad (17')$$
$$-x_3 + x_9 \leq 0 \quad (18')$$
$$-x_4 + x_9 \leq 0 \quad (19')$$
$$-x_5 + x_9 \leq 0 \quad (20')$$
$$-x_6 + x_9 \leq 0 \quad (21')$$
$$-x_7 + x_{11} \leq 0 \quad (22')$$
$$-x_8 + x_{11} \leq 0 \quad (23')$$

After Phase 2, the remaining quantities of standard parts are:

$m = 500–x_{10}$

$n = 600–x_{10}$

$p = 500–x_{11}$

$q = 700 – x_{11}$

$r = 500 – x_9$

$s = 600 – x_9$

$t = 1000$

$u = 700$

In Phase 3, the remaining quantity of semi-finished products, standard parts, parts manufactured in Phase 1 and subassemblies manufactured in Phase 2 are used to form the main assembly *GS* in the quantity x_{12} (Table 6).

Table 6. Phase 3

			Products of Phase 3 *GS*	Available quantities
Phase 3	Semi-fin. products	S_1		$a = 10026.24–4.888·x_1–3,459·x_2$
		S_2		$b = 38.733–0.646·x_3$
		S_3		$c = 12.255–0.219·x_4$
		S_4		$d = 6.456–0.097·x_5$
		S_5		$e = 53.36–0.534·x_6$
		S_6		$f = 38.142–0.191·x_7$
		S_7		$g = 22.696–0.199·x_8$
	Standard parts	GR_1		$m=500–x_{10}$
		GR_2		$n=600–x_{10}$
		GR_3		$p=500–x_{11}$
		GR_4		$q=700–x_{11}$
		GR_5		$r=500–x_9$
		GR_6		$s=600–x_9$
		GR_7		$t=1000$
		GR_8		$u=700$
	Products of Phase 1	D_1		$x_1- x_{11}$
		D_2		$x_2- x_{10}$
		D_3		$x_3- x_9$
		D_4		$x_4- x_9$
		D_5		$x_5- x_9$
		D_6		$x_6- x_9$
		D_7		$x_7- x_{11}$
		D_8		$x_8- x_{11}$
	Products of Phase 2	PS_1	1	x_9
		PS_2	1	x_{10}
		PS_3		x_{11}
	Qty		x_{12}	

The constraint equations for Phase 3 are:
non-negativity conditions

$$x_{12} \geq 0 \tag{24}$$

and the constraints with respect to available quantities are:

$$x_{12} \leq x_9 \tag{25}$$

$$x_{12} \leq x_{10} \tag{26}$$

It is more suitable to write the inequalities (25) and (26) in the form:

$$-x_9+x_{12} \leq 0 \tag{25'}$$

$$-x_{10}+x_{12} \leq 0 \tag{26'}$$

In the last phase, i.e. Phase 4, the remaining quantity of semi-finished products, standard parts, parts manufactured in Phase 1, subassemblies manufactured in Phase 2 and the main assembly formed in Phase 3 are used to form the product *P* in the quantity X_{13} (Table 7).

Table 7. Phase 4

			Products of Phase 4 *P*	Available quantities
Phase 4	Semi-fin. products	S_1		$a = 10026.24–4.888·x_1–3.459·x_2$
		S_2		$b = 38.733–0.646·x_3$
		S_3		$c = 12.255–0.219·x_4$
		S_4		$d = 6.456–0.097·x_5$
		S_5		$e = 53.36–0.534·x_6$
		S_6		$f = 38.142–0.191·x_7$
		S_7		$g = 22.696–0.199·x_8$
	Standard parts	GR_1		$m=500–x_{10}$
		GR_2		$n=600–x_{10}$
		GR_3		$p=500–x_{11}$
		GR_4		$q=700–x_{11}$
		GR_5		$r=500–x_9$
		GR_6		$s=600–x_9$
		GR_7	2	$t=1000$
		GR_8	1	$u=700$
	Products of Phase 1	D_1		$x_1–x_{11}$
		D_2		$x_2–x_{10}$
		D_3		$x_3–x_9$
		D_4		$x_4–x_9$
		D_5		$x_5–x_9$
		D_6		$x_6–x_9$
		D_7		$x_7–x_{11}$
		D_8		$x_8–x_{11}$
	Prod. of Phase 2	PS_1		$x_9–x_{12}$
		PS_2		$x_{10}–x_{12}$
		PS_3	1	x_{11}
	Prod. of Phase 3	*GS*	1	x_{12}
	Quantities		x_{13}	

The constraint equations for Phase 4 are:
non-negativity conditions

$$x_{13} \geq 0 \tag{27}$$

and the constraints with respect to available quantities are:

$$2x_{13} \leq 1000 \tag{28}$$

$$x_{13} \leq 700 \tag{29}$$

$$x_{13} \leq x_{11} \tag{30}$$

$$x_{13} \leq x_{12} \tag{31}$$

It is more suitable to write the inequalities (30) and (31) in the following form:

$$-x_{11}+x_{13} \leq 0 \tag{30'}$$

$$-x_{12}+x_{13} \leq 0 \tag{31'}$$

The constraints (1) - (31) are summed in Table 8, which can be the basis for creation of the matrix for solving the given problem of a multiphase process by linear programming.

Table 8. Summary table of constraints

Phase	1								2			3	4	***Available quantities***
	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	PS_1	PS_2	PS_3	*GS*	*P*	**B**
S_1	4.888	3.459												10026.24
S_2			0.646											38.733
S_3				0.219										12.255
S_4					0.097									6.456
S_5						0.534								53.36
S_6							0.191							38.142
S_7								0.199						22.696
GR_1										1				500
GR_2										1				600
GR_3											1			500
GR_4											1			700
GR_5									1					500
GR_6									1					600
GR_7													2	1000
GR_8													1	700
D_1	-1										1			0
D_2		-1								1				0
D_3			-1						1					0
D_4				-1					1					0
D_5					-1				1					0
D_6						-1			1					0
D_7							-1				1			0
D_8								-1			1			0
PS_1									-1			1		0
PS_2										-1		1		0
PS_3											-1		1	0
GS												-1	1	0
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	

Instead of Tables 4, 5, 6 and 7, which can serve as the basis for forming the constraint equations, it is possible to initially form only one summary table for all production phases. In table 8 the phases are shown in different colours. In order to keep the available quantities of semi-finished products from Table 2 and available quantities of standard parts from Table 3 (column B- available quantities), it is necessary, in the part of the matrix which refers to the same category (e.g. parts-parts), to add the number -1 on the diagonal (parts of the table that are grey shaded).

The procedure of writing the constraint equations in the mathematical model is thus shortened and it is possible to write the equations in their matrix form directly from the summary table 8.

3. MATHEMATICAL MODEL

3.1. Objective function

If the maximum profit from the sale of product is desired, the objective function can be written in the form:

$$\max f(x_{13}) = d \cdot x_{13} \qquad (32)$$

where: *d – the profit gained by the sale of 1 piece of product*

x_{13} – the optimim quantity of products which should be manufactured

If it is assumed that the profit per product piece is d=100*нј*, the objective function reads:

$$\max f(x_{13}) = 100 \cdot x_{13} \qquad (33)$$

3.2. Constraints

The constraints (1) - (31) hold for all phases and include the non-negativity conditions of the variables:

$$x_1, x_2, x_3, \ldots, x_{13} \geq 0$$

The mathematical model in its matrix form reads:

It is necessary to maximize the objective function:

$$\mathbf{\max F(X) = d\ X} \qquad (34)$$

with satisfying the constraints with respect to available quantities:

$$\mathbf{M\ X \leq B} \qquad (35)$$

and the non-negativity condition:

$$\mathbf{X \geq 0} \qquad (36)$$

For the given example, the matrices **M** , **X** and **B** read:

$$
\mathbf{M} = \left[\begin{array}{cccccccc|ccc|c|c}
4.888 & 3.459 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.646 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.219 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.097 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.534 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.191 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.199 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\hline
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1
\end{array}\right]
$$

$$
X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \end{bmatrix}
\qquad
B = \begin{bmatrix} 10026.24 \\ 38.733 \\ 12.255 \\ 6.456 \\ 53.36 \\ 38.142 \\ 22.694 \\ 500 \\ 600 \\ 500 \\ 700 \\ 500 \\ 600 \\ 1000 \\ 700 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

4. CONCLUSION

Due to the limited space, the paper has analyzed the phase production problem reduced only to the constraints of available semi-finished products and standard parts. Real problems are much more complex in practice because products are more complex, they require a larger number of production phases and the mathematical model can include some other constraints, such as: labour constraints, available capacities of machines, market constraints, available funds, etc.

In addition, phase processes are not always linear or single criterion processes so the mathematical models which describe this problem are much more complex.

The advantages of the presented methodology for solving problems of a multiphase process by linear programming are as follows:

- Instead of forming several tables which are the basis for defining the mathematical model of a phase process by linear programming, it is possible to shorten the procedure by forming only one summary table,
- In contrast to the classical way of forming a mathematical model in which constraints equations are defined for each phase, the mentioned methodology uses the summary table, which defines forming products per phase, in order to directly define the matrices **X**, **M** and **B**.

- The possibility of occurrence of errors in the process of forming the mathematical model of the problem is considerably reduced.

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