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The Methods for Synthesis and Analysis Controlled Time Delay System with Required Damping Factor

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***Abstract*: D-decomposition method in the area of relative stability , developed from Loo [7] in order to separate constant time settling area in parametric space which ensures the system having predefined settling time.[4].This paper develops the methods for synthesis and analysis of controlled-loop system with proportional regulator for transport and dosing devices with pre-defined damping factor. Here we presents and investigates the further expansion of last obtained results .**

Now it would be possible to separate the region in three-dimensional space(frequency ω (Hz), gain K=1/α and time delay constant (τ), so that adjustable parameters guarantee damping factor ξ of controlled system will have a priori defined value .

Useful of this researches is that checking of obtained results can be made with MATLAB software package and the simulation of dynamic behaviour will be done with this package at the end.The verification of the system model could be done with this method in order the system will have as much as possible accuracy of working.

***Keywords* : time delay system , relative stability , parametric plane, damping factor**

I. INTRODUCTION

The method for extracting the region in the parameter plane, which enables closed- loop system will have pre-defined damping factor was also particularly developed and explained [1], [2] and this paper will continue extend last mentioned results and their application.

The basis of mathematical equations and rules for shading parametric curves remain the same as in the case of-system without delay.We will discuss the case of closed-loop system with a single delay, when the adjustable parameters are non-linearly related to polynomial coefficients of quasicharacteristic equation [8] .

$$W_{ok} = \frac{N(s)}{\alpha D(s)} e^{-\tau s} \qquad (1)$$

so that quasicharacteristic equation has the following form:

$$f(s, e^{-\tau s}) = \alpha D(s) + N(s) e^{-\tau s} = 0 \qquad (2)$$

where K = 1/α is proportional regulator gain, so α is a regulator parameter linearly related to polynomial coefficients of quasicharacteristic polynomial. Pure time delay is τ, which in the case of transport and dosing device is ratio between length of transport belt and moving velocity of the belt, as described in the definition of a mathematical model of this system [1]. This system belongs to the class of time delay system, where the is adjustable time for transport delay by the moving velocity of the belt.

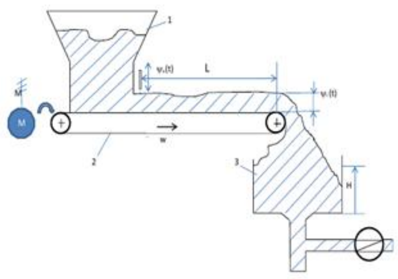

Fig.1.Functional sheme of transport and dosing device

II. EXTRACTION THE AREA OF PRE-DEFINED DAMPING FACTOR

The system will possess an appropriate damping factor only if all the roots of quasicharacteristic equation are within this contour shown on Fig.2.

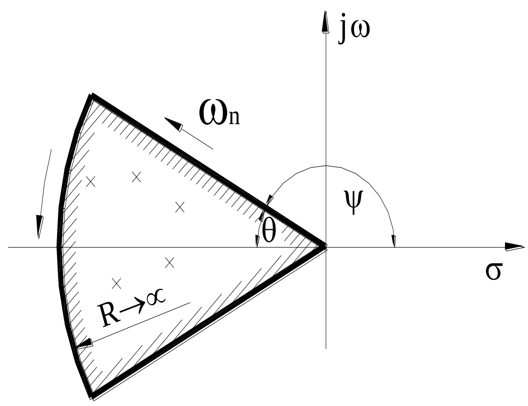

Fig.2.The method is transforming this contour from complex plane to parametric plane τ-α

A. Decomposition curves

For ω_n-undamped frequency and ξ – damping factor,complex variable s has the form:

$$s = \omega_n e^{j\theta} = -\omega_n \xi + j\omega_n \sqrt{1-\xi^2}, \xi = \cos\theta \quad (3)$$

and for T_k and U_k, which are Chebishev's polynomials, its degrees are given in the form:

$$s^k = \omega_n^k T_k(-\xi) + j\omega_n^k \sqrt{1-\xi^2} U_k(-\xi) \quad (4)$$

By substituting (4) in (2), quasicharacteristic equation will also have real and imaginary part, i.e. in polar coordinates:

$$D(\omega_n,\xi) = r_D(\omega_n,\xi) e^{j\Phi_D};$$
$$N(\omega_n,\xi) = r_N(\omega_n,\xi) e^{j\Phi_N} \quad (5)$$

and then substituting (5) in (1) from (2) are followed next decomposition curves:

$$\alpha = \pm \frac{r_N(\omega_n,\xi)}{r_D(\omega_n,\xi)} e^{\tau\omega_n\xi} \quad (6)$$

$$\tau = \frac{1}{\omega_n\sqrt{1-\xi^2}}\left[\Phi_N(\omega_n,\xi) - \Phi_D(\omega_n,\xi) + 2k\pi + \frac{\pi}{2} \pm \frac{\pi}{2}\right] \quad (7)$$

$$k \in Z, \omega_n \in [0,+\infty)$$

Note:The upper sign of (6) correspondents to the upper sign of (7) and the lower sign of (6) correspondents to the lower sign of (7).

B. Curve shading

Shading of decomposed curves is determined by the sign of Jacobians (as in systems without delays). For complex s (3) quasicharacteristic equation is going to be:

$$f(\omega_n,\xi) = R_F(\omega_n,\xi,\tau,\alpha) + jI_F(\omega_n,\xi,\tau,\alpha)$$

$$R_F(\omega_n,\xi) = R_N(\omega_n,\xi) + \alpha e^{-\tau\omega_n\xi} \cdot \begin{bmatrix} R_D(\omega_n,\xi)\cdot\cos(\tau\omega_n\sqrt{1-\xi^2}) - \\ -I_D(\omega_n,\xi)\cdot\sin(\tau\omega_n\sqrt{1-\xi^2}) \end{bmatrix} \quad (8)$$

$$I_F(\omega_n,\xi) = I_N(\omega_n,\xi) + \alpha e^{-\tau\omega_n\xi} \cdot \begin{bmatrix} I_D(\omega_n,\xi)\cdot\cos(\tau\omega_n\sqrt{1-\xi^2}) + \\ +R_D(\omega_n,\xi)\cdot\sin(\tau\omega_n\sqrt{1-\xi^2}) \end{bmatrix} \quad (9)$$

Jacobians of the system as follows:

$$J = \begin{vmatrix} \frac{\partial R_F}{\partial \tau} & \frac{\partial R_F}{\partial \alpha} \\ \frac{\partial I_F}{\partial \tau} & \frac{\partial I_F}{\partial \alpha} \end{vmatrix} = \alpha\cdot\omega_n\sqrt{1-\xi^2}\cdot e^{-2\tau\omega_n\xi}\cdot r_D^2(\omega_n,\xi) \quad (10)$$

C.Singular lines

Singular lines, in the case of extracting area of pre-defined damping factor, is defined for boundary cases $\omega_n \to 0+$ and $\omega_n \to +\infty$, in (2), (6) and (7):

$$\alpha = \pm \lim_{\omega_n \to 0} \frac{r_N(\omega_n,\xi)}{r_D(\omega_n,\xi)} e^{\tau\omega_n\xi} \quad (11)$$

$$\tau = \lim_{\omega_n \to 0} \frac{1}{\omega_n\sqrt{1-\xi^2}}\left[\Phi_N(\omega_n,\xi) - \Phi_D(\omega_n,\xi) + 2k\pi + \frac{\pi}{2} \pm \frac{\pi}{2}\right]$$

$$\alpha = \pm \lim_{\omega_n \to +\infty} \frac{r_N(\omega_n,\xi)}{r_D(\omega_n,\xi)} e^{\tau\omega_n\xi} \quad (12)$$

$$\tau = \lim_{\omega_n \to +\infty} \frac{1}{\omega_n\sqrt{1-\xi^2}}\left[\Phi_N(\omega_n,\xi) - \Phi_D(\omega_n,\xi) + 2k\pi + \frac{\pi}{2} \pm \frac{\pi}{2}\right]$$

$$\alpha = \pm \lim_{\omega_n \to a} \frac{r_N(\omega_n,\xi)}{r_D(\omega_n,\xi)} e^{\tau\omega_n\xi} \quad (13)$$

$$\tau = \lim_{\omega_n \to a} \frac{1}{\omega_n\sqrt{1-\xi^2}}\left[\Phi_N(\omega_n,\xi) - \Phi_D(\omega_n,\xi) + 2k\pi + \frac{\pi}{2} \pm \frac{\pi}{2}\right]$$

Where **a** is every value of ω where (9) and (10) are not defined.
A special case boundaries of region is for $\xi = 1$ $\omega_n \to \infty$ and the curve of constant damping factor in that case becomes:

$$\alpha = \pm \lim_{\substack{\omega_n \to +\infty \\ \xi \to 1}} \frac{r_N(\omega_n,\xi)}{r_D(\omega_n,\xi)} e^{\frac{\xi}{\sqrt{1-\xi^2}}[\Phi_N(\omega_n,\xi) - \Phi_D(\omega_n,\xi) + (2k+1)\pi]} \quad (14)$$

$$\tau = \lim_{\substack{\omega_n \to +\infty \\ \xi \to 1}} \frac{1}{\omega_n\sqrt{1-\xi^2}}\left[\Phi_N(\omega_n,\xi) - \Phi_D(\omega_n,\xi) + 2k\pi + \frac{\pi}{2} \pm \frac{\pi}{2}\right]$$

That is the equation of singular lines in that case also.

It could be emphasized that the method of shading singular lines is the same as for systems without delay.The procedure of selection area of constant damping factor is defined by procedure of selection the area of absolute stability for required automatic control system.[2]. Area of pre-define damping factor is obtained from the section of area of required damping factor according to decomposition curves (6) and (7) in parametric space(α, τ, ω), according to rules of shading and area of absolute stability. Parametric plane α-τ is top view of that figure.

$$S = \bigcap_{k\to 0}^{k\to+\infty} S_k \cap S \text{ absolute stability} \qquad (15)$$

III. APPLICATION THE METHOD – TRANSPORT AND DOSING DEVICE

Application of the methods described here will be illustrated by the example of transport and dosing devices. A mathematical model is developed for control systems with proportional controller which gain is K = 1 / α and given object (Fig. 2) for some nominal parameter values, with time delay identical to ratio length of the transport belt and moving velocity of the bely and pre-defined damping factor ξ = 0,5. The open loop transfer of feedback system is:

$$W_{ok} = \frac{N(s)}{\alpha D(s)} e^{-\tau s} \qquad (16)$$

A. Synthesis of controlled-loop system

We can see that we can approach the synthesis by comparing (17) i (1) according to the methods described in chapter III. Then equations (6), (7) and (10) become:

$$\alpha = \pm\frac{0{,}11}{\omega_n} e^{0.5\tau\omega_n} \qquad (17)$$

$$\tau = \frac{1{,}778}{\omega_n}\left[\frac{\pi}{3} + 2k\pi + \frac{\pi}{2} \pm \frac{\pi}{2}\right] \qquad (18)$$

$$J = -\alpha\omega_n^3\sqrt{1-\xi^2}\, e^{-2\tau\omega_n\xi} \qquad (19)$$

Singular lines are determined from (11), (12) and (13) and (14) according to the rules of calculating boundaries value, equations are:

τ→∞ and α→∞ ,
τ=0 and α=0n

For the sign of α we take plus, because it is nominal sign of value for gain of propotional regulator.
Areas given from extraction curves (17), (18) and (19) by rules of shading which define that values of parameters for system with damping factor ξ = 0.5 **are not exist** , as it show on Fig.4 , Fig.5, Fig 6 .and Fig 7

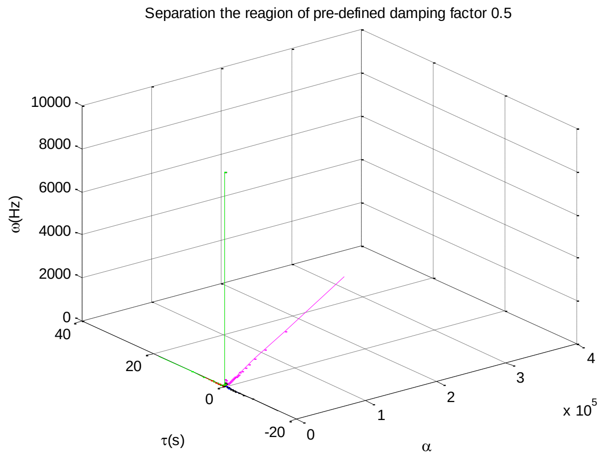

Fig. 3. Separation the region of constant damping factor ξ=0.5

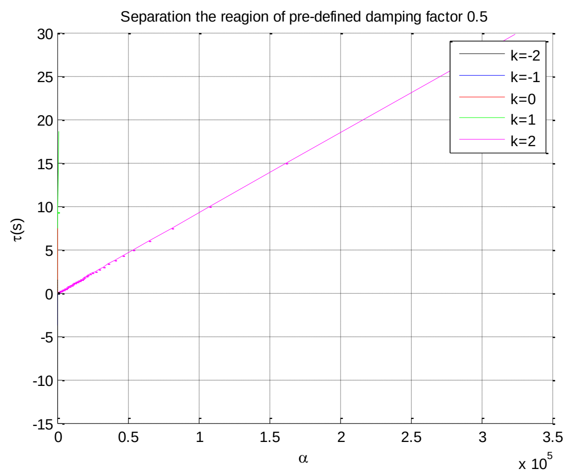

Fig. 4 Top view of Fig 4.

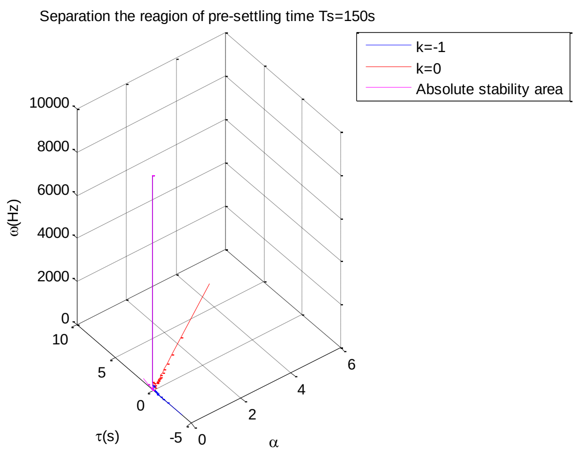

Fig. 5 Comparing with region of absolute stability

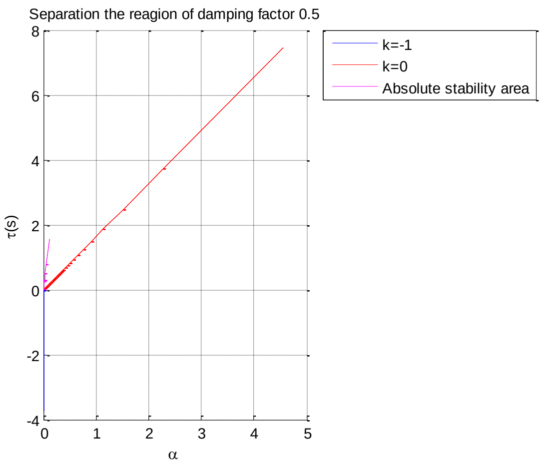

Fig.6 Top view of Fig. 5

It could be possible to define according to (15) that

$$S = \bigcap_{k\to 0}^{k\to+\infty} S_k \cap S \text{ absolute stability} = 0, \quad \textit{empty group}$$

So for this system it is not capable to separate the region of damping factor ξ=0.5. If k→-∞ than the decomposition curve separate region which is one line α=0. If k→+∞ than the decomposition curve separate region which is one line τ=0.

IV.CONCLUSION

New software package MATLAB enables to obtain more precocious D-decomposition method applied to adoption new principle of separation the field of pre-specified damping factor in the parametric plane of α-τ [1]. The results presenting here together with previous two papers for conference SAUM2012 (author and co-author V.Brasic)overview variation and dependences of parameters in three-dimensional form(α, τ, ω_n) where could be possible to choose from the given area the values for parameters which guarantee absolute and relative stability of specific class of time –delay systems [1], [2].

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