Predrag Mladenović
Teaching Assistant
University of Kragujevac
Faculty of Mechanical And Civil Engineering in Kraljevo

Marko Todorović
Teaching Assistant
University of Kragujevac
Faculty of Mechanical And Civil Engineering in Kraljevo

Nebojša Zdravković
Associate Professor
University of Kragujevac
Faculty of Mechanical And Civil Engineering in Kraljevo

Goran Marković
Associate Professor
University of Kragujevac
Faculty of Mechanical And Civil Engineering in Kraljevo

Mile Savković
Professor
University of Kragujevac
Faculty of Mechanical And Civil Engineering in Kraljevo

Goran Pavlović
Research Associate
University of Kragujevac
Faculty of Mechanical And Civil Engineering in Kraljevo

Cross section optimization of an auto crane articulated boom using metaheuristic optimization algorithms

The proliferation of different metaheuristic optimization algorithms in recent years introduced novel optimization algorithms offering fast and computationally inexpensive solutions for various optimization problems. Two such algorithms, the Marine Predator Algorithm (MPA) and the Search and Rescue optimization algorithm (SARO) were employed for the cross-section optimization of auto crane articulated boom. The cross-section within a single segment is constant along its length and takes the shape of the box. For optimization, a mathematical model of construction was created that enabled the execution of the optimization process. The construction was considered in the position that is the most unfavourable in terms of deflection. The optimization was done according to the criterion of permissible deflection, as well as the allowable level of normal stress. The optimization aimed to reduce the structure's mass as much as possible, thereby contributing to the reduction of the material that would have to be used for the production of the structure.

Keywords: *Cross-section optimization, auto crane, articulated boom, metaheuristic optimization algorithm.*

1. INTRODUCTION

The process of designing structures and machines consists of defining the basic technical characteristics and functions that a certain structure should have. With the growth of the world's population, the need for an increasing number of machines and structures that are used in all spheres of human life has also increased. Due to the lack of resources used in the production of these constructions, there was a need to save as much material as possible. In addition to the lack of resources, the problem of environmental pollution is currently one of the leading reasons why designers are faced with requirements that imply as little as possible the amount of material used, while constructions and machines will continue to perform the same function for which they were designed with the same degree of reliability. Simple dimensioning results in a construction that will satisfy the basic functions and criteria of strength and stability, while not paying much attention to the possibility of over-dimensioning the constructions. Because of this, there was a need for a process called engineering optimization.

The use of optimization during the design process enables the rational consumption of resources, improves the functionality of systems and structures, and long-term sustainability in terms of contributing to the sustainability of resources and reducing the negative impact on the environment.

Optimization is a key discipline applied in many fields, including engineering, economics, business, and everyday life. The goal of optimization is to find the best solution for a specific problem under given conditions. The goal of optimization can be to achieve the minimum or maximum of the objective function depending on the type of optimization problem considered.

In the optimization process itself, it is necessary to define the objective function, variable quantities and their limits, as well as the corresponding constraints that are important for the considered problem. It is very important to determine the limits of the optimization variables to ensure that a obtained solution is engineering feasible. More about the optimization process can be found in [1].

The subject of research in this paper is the optimization of the three-segment auto crane articulated boom. Articulated booms are devices that have a wide range of applications such as maintenance and repair - enabling access to hard-to-reach places such as electric poles, lamps, antenna towers, and construction works - where they are used for installation of windows and various installations, assembly of steel structures, fruit collection - in agriculture they have found application for fruit harvesting, film and media production - for recording demanding scenes at height and panoramic shots, tree work - removal of dry branches and tree pruning. Articulated boom cranes are very useful for all kinds of jobs that require access to high places or work in hard-to-reach locations, and as such, have found wide application in various fields.

Correspondence to: Marko Todorovic, teaching assistant
University of Kragujevac - Faculty of Mechanical And Civil Engineering in Kraljevo
Dositejeva 19, 36000 Kraljevo, Serbia
E-mail: todorovic.m@mfkv.kg.ac.rs

These devices consist of several segments, where each segment is connected to the adjacent segment by a joint connection, thus enabling the rotation of the segments and the movement of the entire construction in the appropriate directions. Each segment consists of profiles that can have different cross-sectional shapes. The cross-section of the segments, used in this paper has the appearance of a box profile and is shown in (Figure 1). The segments are kept in the proper position by the action of hydraulic cylinders, which are connected by their ends to the corresponding parts of the segments.

There are a lot of papers that have dealt with structures like this. Cross-section optimization of the mobile crane boom using Lagrange multiplier's method was shown in [2]. In [3] the inertial loads of a telescopic boom of a truck crane were determined. The finite element method was used on a virtual prototype in order to perform an analysis of the performance of the work platform in [4], and in [5] and [6] the analysis of the deflection and forces within the boom structure was completed where analytical expressions were obtained.

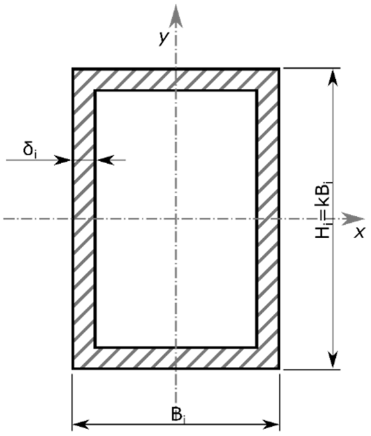

Figure 1. Cross-section of the articulated boom segment

A symbolic representation of the construction considered in this paper is shown in (Figure 2).

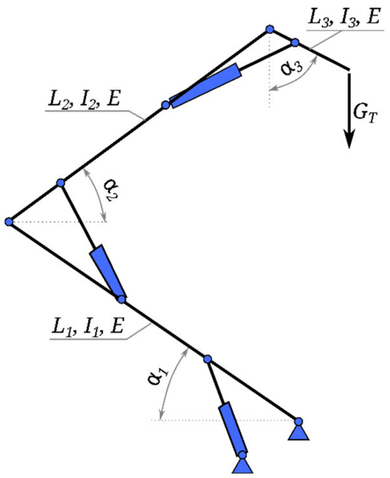

Figure 2. Illustration of the auto crane articulated boom

In many engineering constructions, in addition to the strength criteria, it is very important to meet the permissible deflection criteria. In many cases, the strength criterion is easily satisfied, while the problem arises with the allowable deflection criterion. That is why it sometimes happens that the stresses in the structure are very small, due to the over-dimensioning of the cross-sections of the structure. However, the oversizing of the cross-section is conditioned by the limitation of deflection that may occur in the construction. In this paper, the normal stresses in the segments, as well as the deflection of the tip of the boom of the structure, were taken as the criteria by which the structure is optimized. The goal of the optimization is to reduce the mass of the construction, which will be achieved by reducing the cross-sectional area of the construction segments. By reducing the mass of the structure, certain savings are made in the material used for its construction.

2. OPTIMIZATION ALGORITHMS

Earlier it was stated what are the many advantages of the optimization procedure and why it is used in almost all spheres of human life. Because of this, many optimization algorithms have been developed to help solve the most diverse problems. Among the many types of optimization algorithms, metaheuristic algorithms stood out and performed well when solving complex problems.

The proliferation of metaheuristic optimization algorithms explained by the simplicity of their use and effectiveness, also justified by the No Free Lunch Theory [7,8], in recent years introduced many novel optimization algorithms. In this paper, for the optimization procedure were used Marine Predator Algorithm (MPA) [9], and Search and Rescue optimization algorithm (SARO) [10]. Some of the key features of metaheuristic algorithms, which distinguishes them from other types of algorithms, are population-based approach - which implies that many of these algorithms use a population of solutions instead of a single solution, which increases the chance of finding a global optimum, adaptability – they provide great adaptability for different types of problems, robustness - they are effective in different situations and do not require much information about the problem being solved.

The main feature of metaheuristic algorithms is that they have two phases when searching for a solution: exploitation and exploration. The exploration phase refers to the process of exploring new, unknown parts of the solution space to increase the variety of potential solutions. The goal is to discover new areas that may contain better solutions. Exploration helps prevent premature convergence of the algorithm to local optima. The exploitation phase focuses on intensive searching around already known good solutions to find even better solutions near them. The exploit uses information about the current best solutions to improve the efficiency of the search and increase the chances of finding the global optimum. The balance between exploration and exploitation is critical to the success of metaheuristic algorithms. If the algorithm explores too much, it may miss the opportunity to thoroughly explore areas with high potential. On the other hand, if it exploits too much, it can get stuck in the local optimum and miss the global optimum. Therefore, a good metaheuristic algorithm should have mechanisms that allow efficient switching between these two phases to achieve an optimal balance.

Not every optimization algorithm can be suitable for solving all kinds of problems [7,8]. For one optimization problem, completely different values of the objective function can be obtained, although the principle of functioning of the algorithms is similar. For this reason, one should not rely on only one optimization algorithm when solving a problem, but rather the same optimization problem should be solved with several different optimization algorithms.

2.1 Marine Predator Algorithm (MPA)

The Marine Predator Algorithm (MPA) is an optimization algorithm inspired by the behaviour of marine predators while hunting for prey. The basic idea is that the strategies used by marine predators to locate and capture prey can be translated into mathematical models that solve optimization problems. This algorithm consists of three phases: the search phase, the chase phase and the final attack phase.

In the search phase, predators search for prey in the vast ocean. This phase represents the exploration phase. This is simulated by randomly exploring the search space to find potential solutions. Mathematically, this may involve generating random positions within the search space and evaluating their suitability.

In the chase phase, once a potential prey is located, the predator chases and surrounds it. In the algorithm, this translates into narrowing the search around the best solutions found so far. This phase uses the best solution to guide the search and update the positions of other potential solutions.

The third phase i.e. the final attack is the moment when the predator catches the prey. In MPA, this is when the algorithm converges to the optimal solution. The positions are updated based on the best solution, and the algorithm iteratively improves the solutions until they meet the stopping criteria.

2.2 Search and Rescue optimization algorithm (SARO)

Search and Rescue Optimization (SAR) algorithm is a metaheuristic optimization method inspired by human search and rescue operations. It is designed to solve optimization problems in constrained engineering, imitating the exploratory behaviour of search and rescue teams. The optimization procedure in the Search and Rescue optimisation algorithm consists of four phases: initialization phase, exploration phase, exploitation phase and rescue phase.

In the first phase, the algorithm starts with an initial population of solutions, which represent search teams. Each solution or 'search agent' is a potential answer to an optimization problem.

The second phase implies that search agents explore the problem space to locate the optimal solution. This phase involves random movements and search patterns to cover a wide area.

In the third phase, once potential targets are identified, the algorithm focuses on those areas to refine the search. Search agents use information from their environment and previous experiences to narrow their search.

The final stage is the 'rescue' phase, where the best solution is identified and taken over.

3. MATHEMATICAL MODEL AND OBJECTIVE FUNCTION

The optimization problem can be categorized as a mass reduction optimization problem. As such, the goal is to minimize the total mass of an auto crane articulated boom consisting of three segments, as illustrated in (Figure 3), in such a way that the two boundary conditions are met: neither the deflection of the tip of the crane nor the maximal normal stress within each of the segments should cross previously set limits.

The objective function f, the function that needs to be minimized, is the sum of the segments masses in the following way:

$$f = \sum_{i=1}^{3} m_i + g \cdot 10^4 \qquad (1)$$

The masses of the segments are in direct relation with the geometry of the cross-section displayed in (Figure 1), and it can be calculated in the following way:

$$m_i = (B_\mathrm{i} + kB_\mathrm{i} - 2\delta_\mathrm{i}) \cdot 2L_i\delta_i \qquad (2)$$

The function g from (1) restricts the optimization algorithm and makes sure the conditions of deformation and stress are met. The general shape of the function g is as follows:

$$g = \sum_{i=1}^{5} g_i \,, g_i \geq 0 \qquad (3)$$

In this equation it can be seen that there are five boundary conditions that are being considered as part of the objective function:

- g_1 – prevents that the deflection of the tip of the articulated boom does not cross the permissible values;
- g_2 – prevents that maximal normal stress levels in each segment does not cross the permissible values;
- g_{3-5} – geometric constraints specific for the considered corss section displayed in (Figure 1).

Constraint g_3 refers to the fact that the profile widths of the segments must satisfy the following condition: $B_1 > B_2 > B_3$

Similar to the previous one, constraint g_4 refers to the fact that the profile heights of the segments must satisfy the following condition: $H_1 > H_2 > H_3$

Constraint g_5 implies that the following ratio between the height and width of each segment must be satisfied: $1.5 < \frac{H_i}{B_i} < 3$ where $i = 1,2,3$.

To calculate the values of the deflection and levels of the normal stress, the following mathematical model of an auto crane articulated boom illustrated in (Figure 3) will be presented.

The auto crane articulated boom illustrated in (Figure 3) consists of three segments of different lengths and cross-section geometry: the first segment - AB, the second segment - BC, and the third segment – CD. These segments are being moved by hydraulic cylinders: A'A", B'B", C'C". These segments are mutually connected with pinned joints at their ends – points C, B and A. Hydraulic cylinders are pinned on both ends which means they are subjected only to the axial load. At the tip of the articulated boom, the hoisted load G_T is applied at point D.

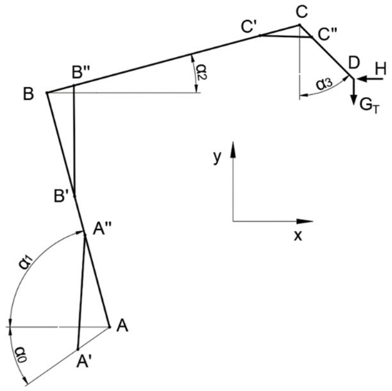

Figure 3. Mathematical model of the auto crane articulated boom

Second Castigliano's theorem can be used for the deflection calculation. The analytic expressions of reactions in joints should be derived in such way that the needed partial derivates can be done. If the articulated boom is loaded with vertical force G_T in point D, the partial derivative of transversal forces within the segments of the articulated boom will provide the value of vertical displacement v of point D. In order to calculate the hroziontal displacement h as well, the fictive horizontal force $H, H = 0$ is also introduced into the system. When these are taken into consideration, the second Castigliano's theorem takes the following form:

$$v = \frac{\partial A_d}{\partial G} = \frac{1}{E}\sum_{i=1}^{m}\frac{1}{I_i}\int_0^{l_i} M_i(s)\frac{\partial M_i(s)}{\partial G}\mathrm{d}s \qquad (4)$$

$$h = \frac{\partial A_d}{\partial H}\bigg|_{H=0} = \frac{1}{E}\sum_{i=1}^{m}\frac{1}{I_i}\int_0^{l_i} M_i(s)\frac{\partial M_i(s)}{\partial H}\mathrm{d}s|_{H=0} \qquad (5)$$

The total deflection is the geometric sum of the deflection in the vertical and horizontal directions:

$$f_{total} = \sqrt{v^2 + h^2} \qquad (6)$$

The first constraint g_1 can then be defined as the difference between already defined permissible deflection f_{prim} and the deflection obtained through the mathematical model:

$$g_1 = f_{perm} - f_{total} \qquad (7)$$

The normal stresses in the structural elements originating from the bending moment and axial stress were calculated using the moment equations for each of the segments.

The forces in joints of the segment as well as the forces in the hydraulic cylinders can be obtained out of the equilibrium principles out of which the system of static equations can be written. The structure illustrated in (Figure 3) can be disassembled as shown in (Figures 4-6). Based on these figures, the system of equations for the third segment CD takes the following form:

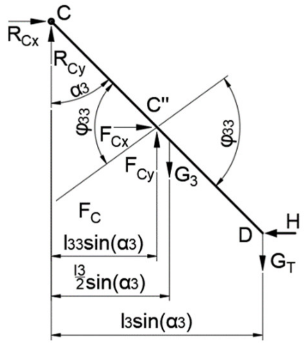

Figure 4. Representation of the reaction and active forces on the third segment

$$\sum X_i = R_{Cx} + F_{Cx} - H = 0 \qquad (8)$$

$$\sum Y_i = R_{Cy} + F_{Cy} - G_3 - G_T = 0 \qquad (9)$$

$$\sum M_C = -G_T l_3 \sin\alpha_3 - G_3\frac{l_3}{2}\sin\alpha_3 + F_{Cy} l_{33}\sin\alpha_3 + F_{Cx} l_{33}\cos\alpha_3 - H\cos\alpha_3 = 0 \qquad (10)$$

The system of equations for the second segment BC is:

$$\sum X_i = R_{Bx} - F_{Bx} - F_{Cx} - R_{Cx} = 0 \qquad (11)$$

$$\sum Y_i = R_{By} + F_{By} - G_2 - F_{Cy} - R_{Cy} = 0 \qquad (12)$$

$$\sum M_B = F_{By} l_{22}\cos\alpha_2 + F_{Bx} l_{22}\sin\alpha_2 - G_2\frac{l_2}{2}\cos\alpha_2 - F_{Cy}(l_2 - l_{32})\cos\alpha_2 + F_{Cx}(l_2 - l_{32})\sin\alpha_2 + R_{Cx} l_2\sin\alpha_2 - R_{Cy} l_2\cos\alpha_2 = 0 \qquad (13)$$

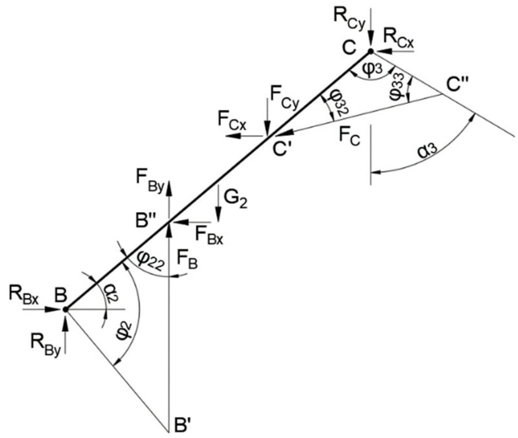

Figure 5. Representation of the reaction and active forces on the second segment

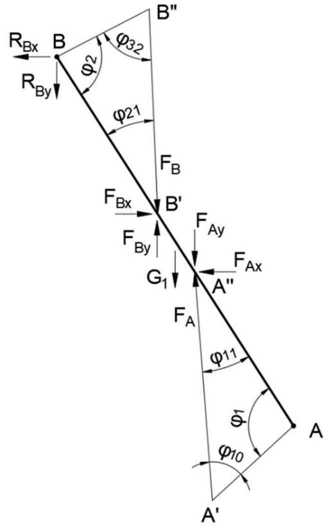

Figure 6. Representation of the reaction and active forces on the first segment

The system of equations for the first segment AB takes the following form:

$$\sum X_i = R_{Ax} - F_{Ax} + F_{Bx} - R_{Bx} = 0 \tag{14}$$

$$\sum Y_i = R_{Ay} + F_{Ay} - G_1 - F_{By} - R_{By} = 0 \tag{15}$$

$$\begin{aligned} \sum M_A &= F_{Ax} l_{11} \sin\alpha_1 - F_{Ay} l_{11} \cos\alpha_1 \\ &+ G_1 \frac{l_1}{2} \cos\alpha_1 + F_{By}(l_1 - l_{21}) \cos\alpha_1 \\ &- F_{Bx}(l_1 - l_{21}) \sin\alpha_1 + R_{Bx} l_1 \sin\alpha_1 \\ &+ R_{By} l_1 \cos\alpha_1 = 0 \end{aligned} \tag{16}$$

The forces G_1, G_2 and G_3 are dead load from the masses of the segments. Since the dead load will be neglected, it is assumed that these forces equal zero.

By solving this system of equations the reactions in all joints and hydraulic cylinders are known, and static diagrams can be drawn. The maximal normal stress in each element can be calculated. Since the bending moment is the dominant load in the articulated boom segments, the moment criteria can be applied. In each segment, the highest value of bending moment $M_{max,i}$ can be found and the location of the maximum can be found. For that location, the intensity of axial force $F_{a,i}$ for the same position can be obtained. The maximal normal stress value for each segment is:

$$\sigma = \frac{M_{max,i}}{W_{x,i}} + \frac{F_{a,i}}{A_i} \tag{17}$$

The constraint g_2 can then be defined as the difference between the already defined permissible stress value σ_{perm} and the maximal normal stress value for the given segment σ_i:

$$g_2 = \sum_{i=1}^{3} \left(\sigma_i - \sigma_{perm}\right) \tag{18}$$

If the values of g_i is lower than 0, then it is taken to be equal to 0. However, if it is higher than 0, the value is taken as calculated.

4. NUMERICAL EXAMPLE

The cross-section optimization of elements that make up the articulated boom of an auto crane defined with the parameters displayed in Table 1 and 2 and loaded with $G_T = 2$ kN will be performed using MPA and SAR optimization algorithms. Permissible total deflection is set to be $f_{perm} = 0{,}05$ m, and permissible level of normal stress is taken to be $\sigma_{perm} = 16\ \text{kN/cm}^2$ and Young's elasticity modulus $E = 2{,}1 \cdot 10^{11}$Pa for S235 structural steel which makes the structure.

Table 1. Nomenclature and segment lengths

Segment label	The symbolic label of the segment	Segment length
$\|AB\|$	l_1	7602 mm
$\|BC\|$	l_2	8210 mm
$\|CD\|$	l_3	2400 mm
$\|AA'\|$	l_{10}	1215 mm
$\|AA''\|$	l_{11}	2987 mm
$\|BB'\|$	l_{21}	3361 mm
$\|BB''\|$	l_{22}	876 mm
$\|CC'\|$	l_{32}	1288 mm
$\|CC''\|$	l_{33}	540 mm

Table 2. The angles between structure segments and the global axis

Angle mark	Angle value
α_0	35°
α_1	75°
α_2	15°
α_3	95°

The values of the angles in Table 2 are the values for which the deflection takes the maximal value out of all possible positions the articulated boom can physically make within a plane. When these parameters are defined, the static diagrams can be drawn, as shown in (Figure 7).

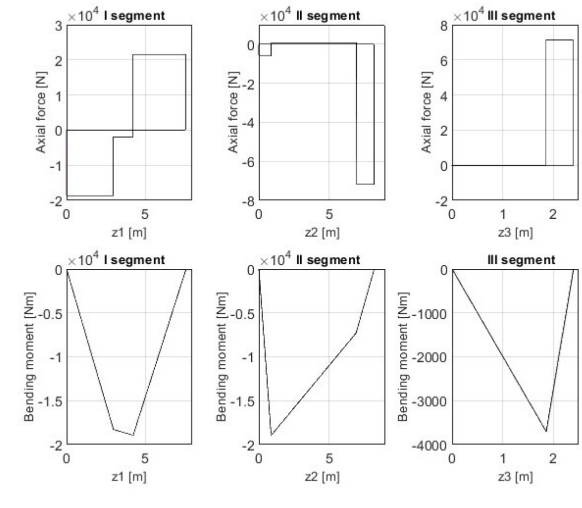

Figure 7. Static diagrams

The variables are the geometric parameters of the cross-section (Figure 1) of each element:

- $\delta_1, \delta_2, \delta_3$ – the thickness of the box cross-section wall;
- B_1, B_2, B_3 – the width of the cross-section;

- k – the parameter that defines the ratio between the height and the width of the cross-section.

Since there are seven variables, the dimension of the problem is $dim = 7$. The lower and upper boundaries of these variables are shown in Table 3.

Table 3. Boundaries of variables

	Lower boundary	Upper boundary
δ_1 [m]	0,003	0,006
δ_2 [m]	0,003	0,006
δ_3 [m]	0,003	0,006
B_1 [m]	0,15	0,4
B_2 [m]	0,15	0,4
B_3 [m]	0,15	0,4
k [-]	1,2	3

Both algorithms were set in the following way:

- number of searching agents $N = dim \cdot 10$;
- maximal number of iterations $T_{max} = 500$.

After the maximal number of iterations for both algorithms was exceeded, the results illustrated in (Figure 8) were obtained.

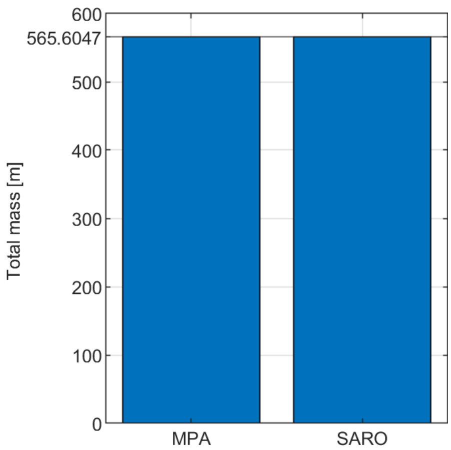

Figure 8. The optimal value of objective function obtained through optimization using MPA and SAR optimization algorithm

In (Figure 8) it can be noticed that both MPA and SAR algorithms managed to reach the same minimum. The values of the variables for which both algorithms reached the minimal value are shown in Table 4.

Table 4. Obtained optimization results

	MPA	SARO
δ_1 [m]	0,003	0,003
δ_2 [m]	0,003	0,003
δ_3 [m]	0,003	0,003
B_1 [m]	0,1735	0,1735
B_2 [m]	0,1657	0,1657
B_3 [m]	0,1500	0,1500
k [-]	3	3

5. CONCLUSION

Mass reduction of an auto crane articulated boom is a complex optimization problem with seven variables that can successfully be solved using metaheuristic optimization algorithms. Both MPA and SAR optimization algorithms reached the same solution which can be seen in (Figure 8) and Table 4.

Considering the conclusions of the No free lunch theorem which states that there is not a single optimization algorithm that can solve every single optimization problem equally well, the same mathematical model for calculating the deflection and normal stress levels in elements can be utilized with another optimization algorithm.

ACKNOWLEDGMENT

This work has been supported by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia, through the Contracts for the scientific research financing in 2024, 451-03-65/2024-03/200108 and 451-03-66/2024-03/200108.

REFERENCES

[1] Martins J. R. R. A., Ning A., *Engineering Design Optimization*, Cambridge University Press England, 2021.

[2] Stephen, J.D.G., Banerjee, A., Lahiri, A., Mehta, I. Optimization of cross section of mobile crane boom using lagrange multiplier’s method, IOP Conference Series: Materials Science and Engineering, 402, 2018.

[3] Volianiuk, V.O., Gorbatyuk, Ie.V., Mishchuk, D.O. The inertial loads of a telescopic boom of a truck crane. Common problems of automobile transport, Automobile transport, 49, 54-62, 2021.

[4] Guo, J., He, H., Sun, C. Analysis of the performance of areal work platform working device based on virtual prototype and finite element method. CUE2016-Applied Energy Symposium and Forum: Low carbon cities & urban energy systems, Energy Procedia, 104, 568-573. 2016.

[5] Zdravković, N., Gašić, M., Savković, M., Marković, G.: Load analysis of the articulating boom sections of the mobile elevating work platform in relation to the operator basket position. 10th International Conference „RaDMI 2010“, 16-19. September 2010, Donji Milanovac, Serbia, 2010.

[6] Zdravković, N., Petković, Z., Gašić, M., Savković, M. Research of deflection – payload dependence of the auto crane articulated boom. Proceedings of the XX International Conference MHCL’12, , 101-106, 2012.

[7] Wolperth, D.H., Macready, W.G. No free lunch theorems for optimization. IEEE transactions on evolutionary computation, 1(1), 67-82, 1997.

[8] Belot, G. Absolutely no free lunches! Theoretical Computer Science, 845, 159-180., 2020.

[9] Faramarzi, A., Heidarinejad, M., Mirjalili, S., Gandomi, A.H. Marine Predators Algorithm: A nature-inspired metaheuristic. Expert Systems with Applications,152, 2020.

[10] Shabani, A., Asgarian, B., Gharebaghi, S.A., Salido, M.A., Giret, A. A new optimization algorithm based on search and rescue operations. Mathematical Problems in Engineering, 1-23 2019.