







Computational Analysis and Simulation of Geiger-Müller Counter Instrument F-factor Using Wolfram Mathematica Software: Case Study of LARA 10 as integrated part of Radiation Laboratory LR-M2

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Abstract: *This paper introduces an interactive simulation developed using the Wolfram Mathematica software package for determination of the device F-factor for the Geiger-Müller counter component LARA 10, within the context of the LR-M2 radiation laboratory. LR-M2, a military-origin laboratory established in the early 1980s, houses the LARA 10 device, which is crucial for radiation detection and measurement. The simulation focuses on computing the F-factor based on the registered radiation intensity with different radiation filters. Through an interactive interface, users can explore the impact of varying radiation levels and filter types on F-factor calculations, providing valuable insights into the device's performance characteristics. This interactive approach enhances understanding and facilitates experimentation, enabling users to optimize radiation detection processes in the LR-M2 laboratory setting. The findings contribute to the advancement of radiation monitoring technology, particularly in civilian applications, ensuring enhanced capabilities for effective radiation hazard management.*

Keywords: *Interactive simulation; Geiger-Müller counter; Wolfram Mathematica; Radiation Laboratory; Fission products*

1. INTRODUCTION

The accurate measurement and assessment of radiation levels are paramount in various fields, ranging from nuclear safety and environmental protection to public health. Geiger-Müller (GM) counters have long been employed as effective tools for detecting and quantifying ionizing radiation due to their sensitivity and versatility. In this article, we present a comprehensive analysis of the performance of the LARA 10 GM counter, integrated within the LR-M2 radiation laboratory, through the utilization of an interactive simulation developed using the Wolfram Mathematica software package. The LARA 10 radiometric laboratory is a mobile laboratory that is used for radiation-hygiene analysis of environmental samples, food samples of plant and animal origin, water, as well as animal feed that are radioactively contaminated with a Mixture of Fission Products (MFP), especially biologically dangerous radionuclides - ⁹⁰Sr, ¹³¹I and ¹³⁷Cs and others.

The utilization of interactive simulations, facilitated by the Wolfram Mathematica software package,

offers a dynamic approach to computing the instrument F-factor of the LARA 10 GM counter.

LR-M2, a radiation laboratory with military origins established in the early 1980s in the former Socialist Federal Republic of Yugoslavia, serves as a cornerstone facility for radiation monitoring and analysis. The LARA 10 GM counter is a vital component of LR-M2's instrumentation, specifically designed for the precise measurement of radiation intensity in various environmental samples. These samples include food, water, and soil contaminated with radionuclides resulting from fission products [1]. Additionally, the LARA 10 plays a critical role in civilian applications, aiding in the detection of radioactive elements in consumables and environmental samples.

The primary objective of this research is to develop an interactive simulation within the Wolfram Mathematica environment to calculate the instrument F-factor of the LARA 10 GM counter based on the registered radiation intensity for specific radiation filters. This simulation-based approach enables a thorough analysis of the F factor's dependency on various factors, such as

environmental conditions, radiation sources, and filter configurations [2, 3]. By incorporating user interaction, the simulation provides researchers and practitioners with a powerful tool for optimizing the performance of the LARA 10 device in diverse radiation monitoring scenarios.

To ensure the reliability and accuracy of the simulation results, this study relies on a comprehensive review of relevant literature and established methodologies for GM counter calibration and performance evaluation. References include studies on GM detector theory, radiation measurement techniques, and computational methods for radiation analysis. By drawing upon established principles and methodologies, we aim to validate the effectiveness of our simulation approach and contribute to the advancement of radiation detection technology.

1.1. Ecological significance of radionuclide presence in the environment

Radiation and radionuclide toxicity can have significant consequences on ecosystems, affecting biodiversity, genetic integrity, and overall ecological balance [4, 5]. The presence of radionuclides in the environment can lead to bioaccumulation and biomagnification, where organisms higher in the food chain accumulate higher concentrations of radionuclides, posing potential risks to human health through consumption of contaminated food and water [6]. In addition to its implications for human health and ecological integrity, radionuclide contamination can also have socio-economic ramifications, particularly in regions heavily reliant on agriculture and natural resources. Contaminated agricultural products and water sources can lead to economic losses and disruptions in food supply chains [7].

Therefore, understanding the dynamics of radiation and radionuclide contamination in the environment is essential for effective environmental management and mitigation strategies. By integrating computational simulations with ecological modeling, we can gain insights into the long-term effects of radiation contamination on ecosystems and develop strategies for remediation and restoration [8, 9, 10, 11, 12, 13].

2. GENERAL CHARACTERISTICS OF LARA 10

The radiometric laboratory LARA 10 is intended for measuring the mass activity of A_z mixtures of fission products that are contaminated with radionuclides such as ^{90}Sr , ^{131}I and ^{137}Cs characteristic of nuclear incidents. Using this laboratory, it is possible to prepare and measure samples of the environment and objects of health surveillance (food, water, animal feed), as well as other samples (blood, urine and others).

The MFP activity of the given samples is measured from the raw sample in a thick layer (absolute A_z

measurement method) using the filter technique. This method also enables the determination of the age of the MFP, because this information is necessary for making a recommendation on the usability of the tested foods and foods.

Depending on the type of samples and the method of their collection, surface beta activity (A_p) and volume beta activity (A_z) can be measured. The units used to express these specific activities are: $[\text{kBq}\cdot\text{kg}^{-1}]$ of fresh sample – for solid samples, or $[\text{Bq}\cdot\text{l}^{-1}]$ for liquid samples.

The device is placed in two boxes whose total weight is about 100 kg (Fig. 1). In the first box there is a sampling kit with a capacity of 800 samples. They can be done continuously for 5-6 days. In the second box there are: a built-in lead housing, with a GM counter, a scaler with a timer, a scaler Svit 10 on which there is a display and control buttons, a built-in rectifier in case of power supply from the mains voltage, filters, radioactive radiation sources, etc.



Figure 1. Parts of LR-M2 radiation laboratory: housing, filters, radioactive calibration source, scaler Svit 10 with timer

The device can work in the conditions of the radiation field of the environment when the strength of the exposure dose is: $\dot{X} < 1.4 \text{ nC} / \text{kg} \cdot \text{s}$, that is, less than 20mR/h. The activity level measurement range is very large, which means that the speed of counting samples whose A_z is on the order of kBq can be measured. The permissible measurement error is less than 20%.

3. PRINCIPLE OF THE METHOD OF MEASURING BETA ACTIVITY OF MFP

The sample count rate R_{bf}^* is determined via a GM counter into which a sample contaminated with MFP is placed. The obtained value of the counting speed can be represented by the relation

$$R_{bf}^* = A_z \cdot m \cdot F_g \cdot F_{ras} \cdot F_{sa} \cdot F_{ra} \cdot F_{ap} \cdot F_{\tau} \cdot F_{\epsilon} \quad (1)$$

where: R_{bf}^* sample counting speed without filter (imp/s), A_z – mass/volumetric activity expressed in Becquerel (Bq), m - mass of measured sample (g), F_g – geometric efficiency factor, F_{ras} – correction factor for radioactive decay, F_{sa} - correction factor for the self-absorption effect, F_{ra} - correction factor for the scattering effect, F_{ap} - correction factor for the absorption effect in the air layer between the counter and the absorption source in the counter window, F_T - correction factor for the "dead time" of the counter, F_ϵ - correction factor for the efficiency of the counter as a function of energy. All the mentioned factors were determined experimentally and the obtained results were expressed by one F-factor. By using the filter method, some of the factors are eliminated and thus it is possible to get the result faster, with a slightly higher measurement error. It follows from equation (1) that if all corrections are represented by one factor F, the principle of determining A_z from MFP (^{90}Sr , ^{131}I and ^{137}Cs) in a sample is equal to the expression

$$A_z = R_{bf}^* \cdot F \quad (2)$$

where F is the factor that is expressed in kBq/kg and whose values are closely related to the choice of one of the offered five Aluminum beta radiation filters of different thickness (Fig. 2).



Figure 2. Aluminum beta radiation filters from 1 to 5 in radiation laboratory LARA 10

In order to be able to determine the factor F, it is necessary to determine the quotient between the sample counting speed without a filter corrected to the level of background radiation, i.e. R_{bf}^* and count rates with some filter R_{fx}^* also corrected for background radiation ($x=1, 2, 3, 4, 5$). Which type of filter is suitable for measuring the tested sample depends on the quotient

$$1.5 < \frac{R_{bf}^*}{R_{fx}^*} < 2.5. \quad (3)$$

After determining the value of R_{bf}^*/R_{fx}^* for a given filter, the device F-factor is determined from Table 1.

Table 1. Relationship between filter serial number and value of R_{bf}^*/R_{fx}^*

| R_{bf}^*/R_{fx}^* | Serial number of the filter | | | | |
|---------------------|-----------------------------|------|------|-----|-----|
| | 1. | 2. | 3. | 4. | 5. |
| 1.5 | 39.2 | 16.3 | 6.7 | 2.7 | 1.2 |
| 1.55 | 43.3 | 17.8 | 7.3 | 2.9 | 1.3 |
| 1.6 | 48.1 | 19.3 | 7.8 | 3.1 | 1.4 |
| 1.65 | 50.8 | 20.7 | 8.4 | 3.4 | 1.5 |
| 1.7 | 56.3 | 22.2 | 9 | 3.6 | 1.6 |
| 1.75 | 61.7 | 23.7 | 9.6 | 3.8 | 1.7 |
| 1.8 | 65.9 | 25.2 | 10.1 | 4.1 | 1.8 |
| 1.85 | 70.3 | 26.7 | 10.6 | 4.3 | 1.8 |
| 1.9 | 76.2 | 28.5 | 11.1 | 4.6 | 1.9 |
| 1.95 | 81.4 | 30 | 11.7 | 4.8 | 2 |
| 2 | 85.8 | 31.5 | 12.2 | 5 | 2.1 |
| 2.05 | 92.1 | 33.3 | 12.6 | 5.2 | 2.2 |
| 2.1 | 96.2 | 34.8 | 13.3 | 5.3 | 2.3 |
| 2.15 | 102.1 | 36.6 | 13.7 | 5.6 | 2.4 |
| 2.2 | 106.2 | 37.7 | 14.1 | 5.7 | 2.5 |
| 2.25 | 111 | 39.2 | 14.8 | 5.8 | 2.5 |
| 2.3 | 116.6 | 41 | 15.2 | 5.9 | 2.6 |
| 2.35 | 122.1 | 42.9 | 15.5 | 6.1 | 2.6 |
| 2.4 | 126.9 | 44.8 | 15.9 | 6.3 | 2.7 |
| 2.45 | 132.8 | 46.6 | 16.7 | 6.4 | 2.7 |
| 2.5 | 136.9 | 48.1 | 17 | 6.4 | 2.8 |

From equation (3), it becomes evident that a significant challenge in measurements with the LARA 10 device is the identification of a suitable filter that reliably satisfies the condition described in relation (3), acknowledging that finding such a filter is not always feasible. In practice, it often happens that the first selected filter is not suitable, which is why it is necessary to repeat the measurements with another filter. This procedure is repeated until the R_{bf}^*/R_{fx}^* value is between 1.5 and 2.5. Then we know we got the right filter.

The next problem is to find the appropriate value of the F-factor in Table 1. In this table, the R_{bf}^*/R_{fx}^* quotient values for different types of filters are shown in different columns. The problem is that the R_{bf}^*/R_{fx}^* value does not change continuously but in certain steps. This requires the user of the device to round the obtained quotient to the nearest value that he can find in the column in Table 1. During this procedure, due to the need to round the obtained division result, an additional measurement error is introduced.

4. MATHEMATICA SOFTWARE SIMULATION CODE FOR F-FACTOR CALCULATION

Our application presented in the software package Wolfram Mathematica avoids this quotient rounding step. This problem is solved by moving the locator, in the application which is an interactive simulation of R_{bf}^*/R_{fx}^* values, along the curve marked in red until the calculated R_{bf}^*/R_{fx}^* value is found on the fitted curve (Fig. 3). The curve connecting the F-factor in the function R_{bf}^*/R_{fx}^* is linear and can be represented in the form

$$F = a_x \cdot \frac{R_{bf}^*}{R_{fx}^*} + b_x \quad (4)$$

where the coefficients a_x and b_x depend on the ordinal number of the filter selected. In the next step, the value of the instrument F-factor is read. It should be noted that the measurement error is significantly reduced with this kind of procedure.

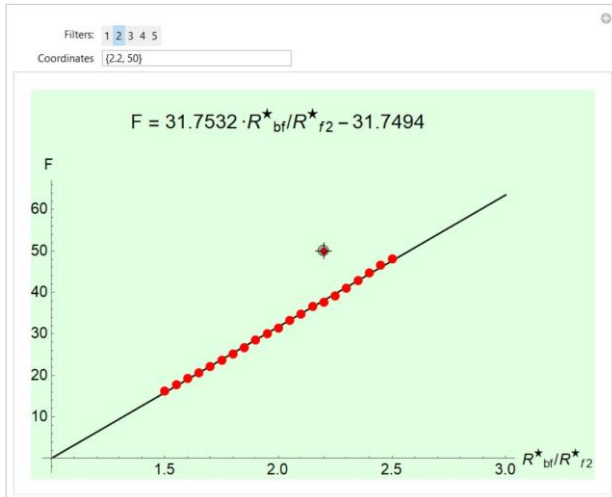


Figure 3. Layout of the F-factor calculation application window with filter number two selected.

The application is made to be interactive, that is, the user can choose the type of filter that is used in the device. In doing so, a linear fit of the data from Table 1 for the given filter is displayed in a separate window. Also, the application window shows the slope of the curve as well as a locator that can be moved on the graphic and read its coordinates (Fig. 4).

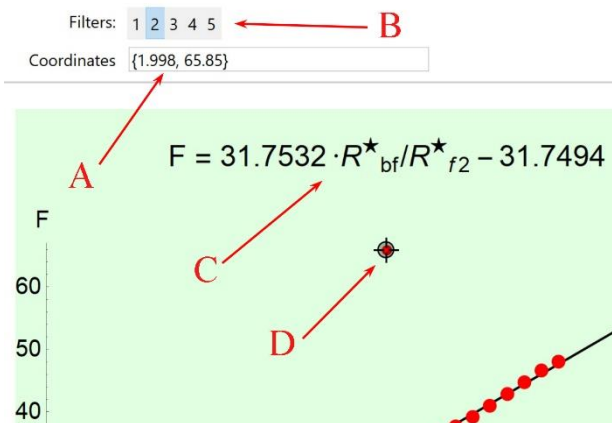


Figure 4. A - coordinate values x, y - axis of the current position of the locator; B - window for selection of the serial number of the filter; C - coefficient of the straight line for the selected filter; D - current position of the locator

The x-coordinate represents the quotient R_{bf}^*/R_{fx}^* , while the value of the F factor is read on the y-coordinate.

Another advantage of using the application is that it is not necessary to use a large number of filters, but in case the quotient R_{bf}^*/R_{fx}^* is outside the allowed limits (i.e. between 1.5 and 2.5), the locator can be moved along the fitted curve and for values significantly below 1.5 and for values significantly above 2.5. This means that A_z can most often be measured using the first choice of filter. In this way, significant savings in time are achieved and the capacity of the LARA 10 measuring device is increased.

5. MATHEMATICA SOFTWARE SIMULATION CODE FOR F-FACTOR CALCULATION

The Mathematica code of the application used to calculate the instrument F-factor is as follows:

```

filter={{1.5`,39.2`,16.3`,6.7`,2.7`,1.2`},
{1.55`,43.3`,17.8`,7.3`,2.9`,1.3`},
{1.6`,48.1`,19.3`,7.8`,3.1`,1.4`},
{1.65`,50.8`,20.7`,8.4`,3.4`,1.5`},
{1.7`,56.3`,22.2`,9.3.6`,1.6`},
{1.75`,61.7`,23.7`,9.6`,3.8`,1.7`},
{1.8`,65.9`,25.2`,10.1`,4.1`,1.8`},
{1.85`,70.3`,26.7`,10.6`,4.3`,1.8`},
{1.9`,76.2`,28.5`,11.1`,4.6`,1.9`},
{1.95`,81.4`,30.11.7`,4.8`,2.`},
{2.05`,92.1`,33.3`,12.6`,5.2`,2.2`},
{2.1`,96.2`,34.8`,13.3`,5.3`,2.3`},
{2.15`,102.1`,36.6`,13.7`,5.6`,2.4`},
{2.2`,106.2`,37.7`,14.1`,5.7`,2.5`},
{2.25`,111.39.2`,14.8`,5.8`,2.5`},
{2.3`,116.6`,41.15.2`,5.9`,2.6`},
{2.35`,122.1`,42.9`,15.5`,6.1`,2.6`},
{2.4`,126.9`,44.8`,15.9`,6.3`,2.7`},
{2.45`,132.8`,46.6`,16.7`,6.4`,2.7`},
{2.5`,136.9`,48.1`,17.6.4`,2.8`}};
novalista=List[filter];
MatrixForm[filter]
Manipulate[LocatorPane[Dynamic[filter],
resenjel=FindFit[novalista[[1,All,{1,faktor}]],b*x-a,{a,b,c},x];
a1=a/.resenjel[[1]];
b1=b/.resenjel[[2]];
c1=c/.resenjel[[3]];
Show[Plot[b1*x-a1,{x,1,3},
LabelStyle->{Black,FontSize->16},
PlotStyle->Black,ImageSize->Large,
Background->LightGreen,
AxesLabel->{"R*_{bf}/R*_{fx}"^faktor-1,Row[{Style["F",Black,FontSize->16]}]},
PlotLabel->Row[{Style["F=",Black,FontSize->20],
Style[b1" * R*_{bf}/R*_{fx}"^faktor-1-a1,Black,FontSize->20}]}],
ListPlot[{novalista[[1,All,{1,faktor}]}],
AxesOrigin->{1,Automatic},
PlotStyle->Red,
LabelStyle->Black,PlotStyle->Black,
ImageSize->Large,Background->LightGreen],
Epilog->{Red,PointSize[Large],
Point[Dynamic[filter]]}]]],
{faktor,2,"Filters:"},
{2->"1",
3->"2",4->"3",5->"4",6->"5"}},
{{filter,{2.2,50},"Coordinates"}}]
    
```

The list of numbers defined by the name filter represents the data presented in Table 1.

5.1. Explanations for parts of the code

Here are some explanations for parts of the code written in the Wolfram Mathematica programming language, and also for the commands that we used:

- `MatrixForm[filter]`: This command uses the `MatrixForm` function to format and display the `filter` matrix as a table.
- `Manipulate[...]`: This command defines an interactive manipulation that allows the user to control simulation parameters. The manipulation consists of `LocatorPane` and `Show` functions.
- `LocatorPane[Dynamic[filter], ...]`: The `LocatorPane` function creates an interactive surface where the user can place points (locators). The dynamic attribute `Dynamic[filter]` ensures that changes in the locator's location are reflected in the overall interactive display.
- `resenje1=FindFit[novalista[[1,All,{1,faktor}]],b*x-a,{a,b,c},x]`: It is used to find the parameters a , b , and c that best fit the `novalista` data using the `FindFit` function to fit an exponential model of the form $b \cdot x - a$. The `FindFit` function uses a method of parameter estimation in regression analysis based on minimizing the sum of squared residuals - the Least Squares Method.
- `a1=a/.resenje1[[1]];`
`b1=b/.resenje1[[2]];`
`c1=c/.resenje1[[3]];`: The values of the parameters a , b , and c are extracted from the solution `resenje1` and assigned to the variables `a1`, `b1`, and `c1`.
- `Show[...]`: This command combines several plots into one display. It includes a plot of the fitted model function, a plot of the `novalista` data points, and an epilog of points that the user sets interactively.
- `Plot[...]`: Displays a plot of the fitted model function with the corresponding parameters `a1` and `b1`.
- `ListPlot[...]`: Displays a plot of the `novalista` data points that the user can interactively place.
- `Epilog->{Red,PointSize[Large],Point[Dynamic[filter]]}`: This option adds an epilog to the plot, consisting of red points with a dynamic location corresponding to the location of the locators set by the user.
- `{{faktor,2,"Filteri:"}, {2->"1",3->"2",4->"3",5->"4",6->"5"}}`: Defines an interactive controller that allows the user to select the value of the `faktor`

parameter from the list of filters, labelled with numeric values and corresponding names.

- `{{filter,{2.2,50}}}`: Defines the initial value for the `filter` locator as `{2.2, 50}`.

6. CONCLUSION

The advantages of using interactive simulation in determining the instrument F-factor in the Wolfram Mathematica software application are multiple. Using the application, it is possible to determine the quotient R_{bf}^*/R_{fx}^* and calculate the instrument F-factor in the first measurement, which avoids the need for multiple measurements and the use of multiple filters. This is especially important if it is known that one measurement per one sample can take at least 15 minutes. By using interactive simulation, the whole procedure becomes much more efficient and faster, thus increasing the capacity of the radiometric laboratory.

Furthermore, as mentioned, the allowed measurement error on the device must be less than 20%. However, since now we can read the data continuously because we have a linear dependence of F on R_{bf}^*/R_{fx}^* via Eq. (4), the precision of the measured data is greatly increased because the rounding of the data presented in Table 1 is avoided. This means that now the data can be continuously read in a wide range of R_{bf}^*/R_{fx}^* coefficients, which significantly reduces the measurement error and increases the accuracy of measured radiation values.

Also, our research offers a novel approach to analyzing and simulating the performance of the LARA 10 GM counter within the LR-M2 radiation laboratory context. Through the integration of computational simulation and interactive visualization, we provide a valuable tool for enhancing radiation monitoring capabilities and ensuring public safety.

Our findings contribute to the ongoing efforts to improve radiation detection and measurement technologies for both military and civilian applications.

ACKNOWLEDGEMENTS

This study was supported by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia, and these results are parts of the Grant No. 451-03-66/2024-03/200132 with University of Kragujevac – Faculty of Technical Sciences Čačak.

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