10th International Scientific Conference
Technics, Informatics, and Education – TIE 2024
20-22 September 2024

Session: *Engineering Education and Practice*
Professional paper
DOI: 10.46793/TIE24.267R

Mathcad PTC Software Towards IM Static/Dynamic Characteristics Simulation

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Abstract: *The paper presents the advantages of Mathcad software for simulating both static and dynamic characteristics of induction machines based on known mathematical models. Mathcad allows for the direct entry of differential and other complex equations that mathematically describe any system or process, automatically calculating the results based on input parameters and conditions. The results can be easily visualized and adjusted by changing the input data, thanks to Mathcad's dynamic updating feature. This paper demonstrates Mathcad's use in visualizing the dynamic characteristics of induction machines (IM) during the startup, rotor resistance starter design and steady-state thermal analysis with varying duty types or machine losses.*

Keywords: induction machine, dynamic characteristic, Mathcad, rotor resistance starter, thermal analysis.

1. INTRODUCTION

The advancement of modern technologies has led to an increasing reliance on simulation tools in engineering disciplines, including the field of electrical machines and motor drives. Simulation plays a vital role in the design, analysis, and optimization of electrical machines' performance, allowing engineers to model and study dynamic states without the need for expensive prototypes. Electrical machines and motor drives are at the core of many industrial systems and applications. Their efficiency and reliability are critical factors in manufacturing processes, transportation, energy, and numerous other sectors. The dynamic behaviour of these systems, including starting, stopping, speed, and torque variations, requires thorough understanding and precise control. Simulation enables engineers to predict the behaviour of electrical machines under various operational conditions. This includes analysing transient states, such as motor startup and short-term disturbances, as well as steady states under different loads. By using simulation tools, potential problems can be identified and corrected before implementation in real-world conditions, resulting in significant time and resource savings [1].

Mathcad is a well-known software tool in technical and scientific calculations, recognized for its ability to combine text, mathematical expressions, and calculations in a single working environment. This software allows users to input and solve complex mathematical problems, create interactive graphics, and visualize data intuitively. One of the key advantages of Mathcad is its capability to simulate dynamic states in electrical machines. Dynamic simulation involves analysing how a system responds to changes in input parameters over time. In the context of electrical machines, this can include simulating motor startup, analysing transient states during load changes, and simulating various control strategies [2, 3 ,4].

Mathcad enables users to model and analyse differential equations, which are the basis of most dynamic models of electrical machines. It provides tools for solving these equations easily, including numerical methods for complex systems. Users can visualize simulation results through various graphical representations, allowing for detailed performance analysis of machines under different conditions. Mathcad can be integrated with other software tools and programming languages, offering flexibility and extended functionality. This is particularly useful when using specific algorithms for control and optimization. The intuitive user interface of Mathcad allows engineers to focus on analysis and design without needing deep programming knowledge [5].

The use of Mathcad in simulating electrical machines can be illustrated through several examples. Simulating the startup of an induction motor is a critical moment in its operation, often accompanied by large currents and torques. Mathcad allows modelling of this process, analysis of currents, torque, and speed over time, identification of potential issues, and optimization of startup characteristics. Variable load analysis is another example where variable load can cause fluctuations in motor performance. Mathcad's dynamic simulations enable engineers to study how the motor responds to load changes, and develop and test strategies for performance stabilization. Developing control algorithms often requires complex algorithms that can be modelled and tested in Mathcad, including PID controllers, adaptive controls, and other advanced techniques [6, 7].

Using software tools such as Mathcad is an essential component in the modern engineering approach to the design and analysis of electrical machines and motor drives. Mathcad offers powerful capabilities for dynamic state simulation, providing engineers with a tool for in-depth analysis and performance optimization. These capabilities enhance the efficiency of the design process and reduce risks and costs associated with the development of new technologies and systems.

In the following chapters, specific applications of Mathcad will be explored in more detail, present case studies, and analyse the advantages and challenges this tool brings to electrical machine simulation.

2. MATHCAD SOFTWARE ADVANGAES

Mathcad is a powerful software application for engineering calculations, combining a robust mathematics engine with an intuitive interface that allows users to solve, document, and share their calculations. Mathcad uses a worksheet interface where equations, text, and images can be input in a free-form manner. This format simplifies documenting calculations and adding explanatory notes and graphics. The worksheet allows for live, dynamic updating of results as changes are made, providing immediate feedback. Equations can be entered in standard mathematical notation, enhancing readability and comprehension of complex equations. Mathcad supports a wide range of mathematical functions, including algebra, calculus, differential equations, and linear algebra.

One of Mathcad's significant features is its built-in support for units, enabling users to perform calculations with units and handle unit conversions automatically. This is particularly beneficial in engineering and scientific applications where units are crucial.

Mathcad supports both symbolic (exact) and numeric (approximate) calculations, allowing users to solve equations either symbolically for exact solutions or numerically for practical, approximate solutions. The software can perform symbolic manipulation of equations, such as simplification, differentiation, and integration.

The software can generate 2D and 3D plots and graphs, allowing users to visualize data and functions, which is useful for analysing trends and patterns. These graphs and plots are interactive and update automatically when the underlying data or equations change.

Mathcad can import and export data from various sources, including Excel and other data formats, allowing for seamless integration with other tools and datasets. It provides tools for data fitting, interpolation, and statistical analysis.

Additionally, Mathcad includes a simple programming language for writing scripts and functions within worksheets, automating repetitive tasks, and extending the software's capabilities. Users can create custom functions and use control flow statements like loops and conditionals.

Key Benefits of Using Mathcad can be formulated as follows: **Ease of Use:** The intuitive interface and natural math notation make it accessible to engineers and scientists who may not be programming experts. **Integration of Documentation and Calculation:** Combining both documentation and calculations in one place ensures clarity and reduces the risk of errors. **Dynamic Updating:** Automatic updates of results in response to changes help maintain accuracy and save time.

3. IM MATH. MODEL AND CORRESPONDING DYNAMIC CHARACTERISTICS

To develop the mathematical model of an Induction Machine (IM), Clarke and Park transformations should be applied to the 3-phase stator and rotor voltage equations [2]. After applying these coordinate transformations, the IM model related to the stator and rotor circuit can be represented by a set of equations in the stationary αβ reference frame (1).

$$\begin{aligned} u_{s\alpha} &= R_s \cdot i_{s\alpha} + \frac{d\psi_{s\alpha}}{dt} \\ u_{s\beta} &= R_s \cdot i_{s\beta} + \frac{d\psi_{s\beta}}{dt} \\ u_{r\alpha} &= R_r \cdot i_{r\alpha} + \frac{d\psi_{r\alpha}}{dt} + p \cdot \omega \cdot \psi_{r\beta} \\ u_{r\beta} &= R_r \cdot i_{r\beta} + \frac{d\psi_{r\beta}}{dt} - p \cdot \omega \cdot \psi_{r\alpha} \end{aligned}, \quad (1)$$

where: $u_{s\alpha\beta}/u_{r\alpha\beta}$ are stator/rotor voltages, $i_{s\alpha\beta}/i_{r\alpha\beta}$ stator/rotor currents and $\psi_{s\alpha\beta}/\psi_{r\alpha\beta}$ machine stator/rotor flux linkages in stationary αβ frame. R_s/R_r stands for stator and rotor resistances, p number of pole pairs and ω machine speed.

Corresponding expressions for stator and rotor flux linkages can be given by set of equations (2):

$$\begin{aligned} \psi_{s\alpha} &= L_s i_{s\alpha} + L_m i_{r\alpha} = \Lambda_s i_{s\alpha} + L_m \left(i_{s\alpha} + i_{r\alpha}\right) \\ \psi_{s\beta} &= L_s i_{s\beta} + L_m i_{r\beta} = \Lambda_s i_{s\beta} + L_m \left(i_{s\beta} + i_{r\beta}\right) \\ \psi_{r\alpha} &= L_r i_{r\alpha} + L_m i_{s\alpha} = \Lambda_r i_{r\alpha} + L_m \left(i_{r\alpha} + i_{s\alpha}\right) \\ \psi_{r\beta} &= L_r i_{r\beta} + L_m i_{s\beta} = \Lambda_r i_{r\beta} + L_m \left(i_{r\beta} + i_{s\beta}\right) \end{aligned}. \quad (2)$$

Additionally, mechanical part of the corresponding machine model can be described by (3):

$$\frac{J_m}{p}\frac{d\omega}{dt}=\frac{3}{2}p\left(\Psi_{s\alpha}i_{s\beta}-\Psi_{s\beta}i_{s\alpha}\right)-m_l-m_{fv}\,, \qquad (3)$$

where: L_s/L_r are stator and rotor inductances, L_m mutual inductance, λ_s/λ_r stator and rotor leakage inductances, J_m moment of inertia, m_l load torque, m_{fv} friction torque and $M_t=m_l+m_{fv}$.

To simulate the machine dynamic characteristics in Mathcad, it is necessary to define all the drive parameters as well as the supply conditions. After defining the motor parameters (given in Table 1), the load and friction torque profile, it is also necessary to define in Mathcad the complete mathematical model of the IM machine given by the voltage equations (1), the flux-current relationships of the machine (2), and the mechanical equation (3).

Table 1. *3-ph IM 1ZK 160 L-6 parameters*

U_n **[V]**	400	R_s **[Ω]**	1.2	L_s **[H]**	0.148
I_n **[A]**	25.7	R_r **[Ω]**	1.3	L_r **[H]**	0.149
P_n **[kW]**	11	n_n **[min⁻¹]**	957	L_m **[H]**	0.142

After entering the described mathematical model of the machine, it is necessary to select an ODE solver (Ordinary Differential Equations solver) in Mathcad. The chosen ODE solver is the function **rkfixed**, which uses the fourth-order Runge-Kutta fixed-step method and provide satisfactory results with small computational burden needed. The definition of the differential matrix, as well as the initial conditions, simulation time frame, and calculation step, is given by the following system of equations (4):

$$D(t,x):=\begin{bmatrix}\sqrt{2}U_f\cos(\omega_1 t)-R_s i_{s\alpha}(x_0,x_2)\\ \sqrt{2}U_f\sin(\omega_1 t)-R_s i_{s\beta}(x_1,x_3)\\ -R_r i_{r\alpha}(x_0,x_2)-p x_4 x_3\\ -R_r i_{r\beta}(x_0,x_2)+p x_4 x_3\\ \frac{1}{J_m}\left[\frac{3p}{2}\left(x_0 i_{s\beta}(x_1,x_3)-x_1 i_{s\alpha}(x_0,x_2)\right)-m_l(x_4)-m_{fv}(x_4)\right]\end{bmatrix} \qquad (4)$$

$$t_0:=0 \qquad t_k:=2 \qquad N:=30000$$

$$z:=\text{rkfixed}(x,t_0,t_k,N,D)$$

where variables $x_0 \ldots x_4$ represent $\Psi_{s\alpha}$, $\Psi_{s\beta}$, $\Psi_{r\alpha}$, $\Psi_{r\beta}$ *and* ω respectively and *N* number of samples.

For a graphical representation of the results of characteristic dynamic quantities of the machine within a given time frame during acceleration (t_0=0 - t_k=2s), it is necessary to define a 2D graph X-Y plot by simply defining the quantities for the abscissa and ordinate. The obtained results are shown in Fig. 1 (in SI units).

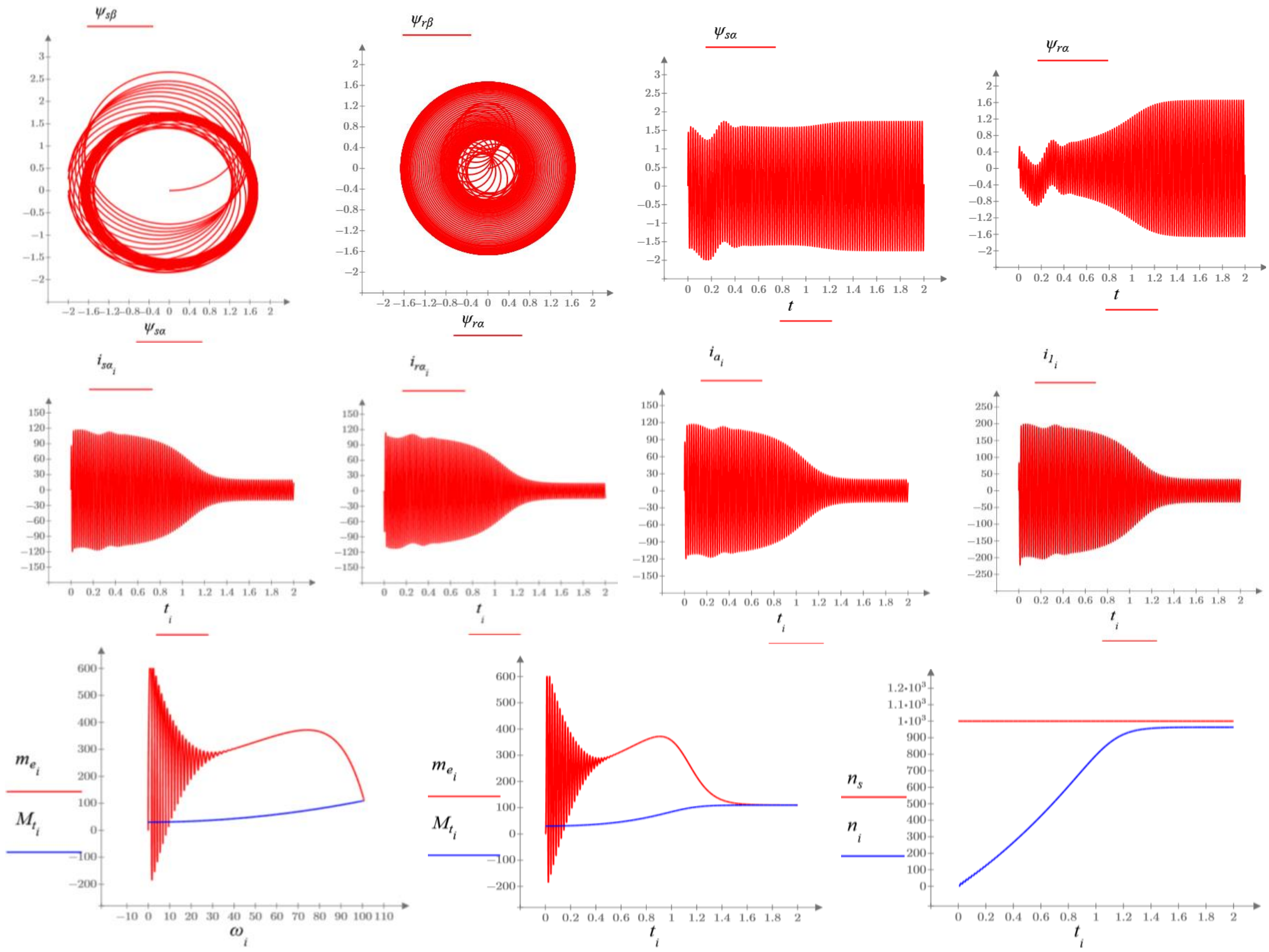

Figure 1. *IM characteristic during the startup: machine stator and rotor flux in αβ plane and corresponding time shapes (top row); stator/rotor (isα/irα) current in α axis, phase/line (isa/il) stator currents (middle row); torque-speed characteristic, electromagnetic (me), load torque (Mt) and machine speed (bottom row)*

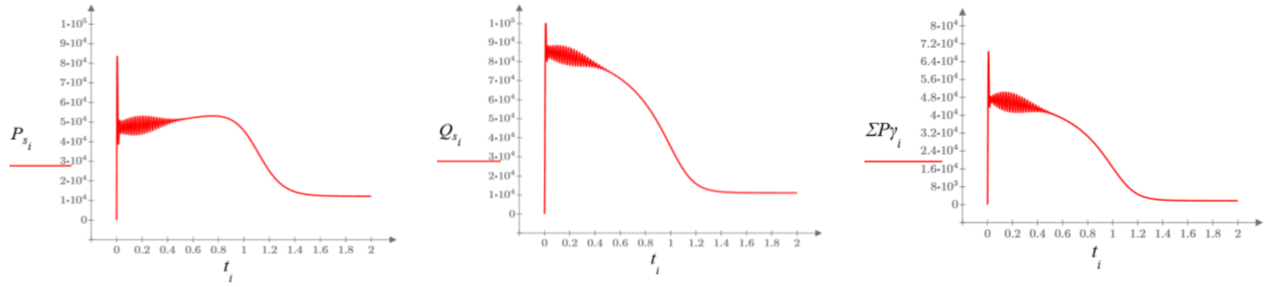

Figure 2. *Active, reactive power and overall machine losses during the startup*

Besides the quantities showed in Fig. 1 other important machine variables during the described time frame can be visualised and analysed. One of these are active/reactive power and overall machine losses during the startup shown in Fig. 2.

The time-domain representation of the displayed quantities is significant for the power flow analysis within the machine, total losses, as well as the change in reactive power that can be efficiently compensated by appropriate power factor correction devices. A significant advantage of using Mathcad software is its ability to dynamically update results and corresponding graphical displays with each change in input parameters or simulation conditions. This advantage of Mathcad software enables faster analysis of the dynamic behaviour of IM compared to similar software.

4. DESIGNING ROTOR RESISTANCE STARTER OF HIGH POWER IM

When it comes to high-power machines, rotor resistance starters (Fig. 3) are usually used for starting instead of appropriate high-power converters as a more economical solution. Sizing these rotor resistance starters typically requires a detailed iterative mathematical analysis and consideration of a large number of parameters that affect the limitation of the starting currents of the stator and rotor of the machine (I_s and I_p respectively). Here is described design procedure of the IM rotor resistance starter used during the start-up of the high-power IM by Mathcad. Namely, a 2.1 MW 3-ph wounded rotor (slip-ring) induction machine is analysed with and without rotor resistance starter applied during startup. Detailed IM parameters are given in Table 2.

Table 2. *IM 3 AKVh 1407-8 KONCAR parameters*

U_n **[kV]**	3.46	R_s **[Ω]**	0.030	L_s **[H]**	0.879
I_n **[kA]**	0.24	R_r **[Ω]**	0.016	L_r **[H]**	0.797
P_n **[MW]**	2.1	n_n **[min⁻¹]**	743	L_m **[H]**	0.796

The number of starter sections *k* and corresponding starter resistance values r_k are calculated by Mathcad using a predefined loop given by (5), where R_0 is overall rotor resistance with starter and s_1 is machine slip at the end first resistance starter level, that is the ratio of I_{p_max}/I_{p_min} during startup.

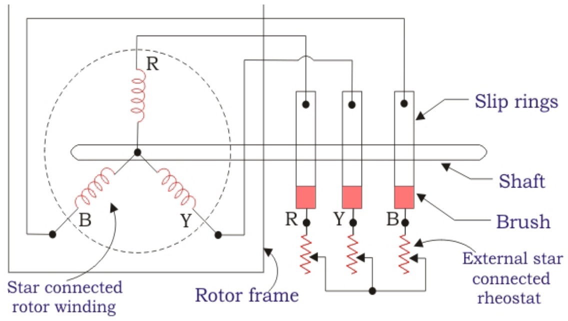

Figure 3. *IM rotor resistance starter operation*

$$r_k := \begin{cases} \text{if } k \le 8 \\ \quad \left(R_0 \cdot s_1^{(k-1)} \cdot (1 - s_1)\right) \\ \text{if } Z < k \le 9 \\ \quad 0 \end{cases} \quad . \qquad (5)$$

The short ***if*** loop defined by (5) is based on relation describing machine slip and rotor resistance R_r (6) allowing automatic calculation of the appropriate rotor resistances depending on the desired number of rotor resistance levels *Z*.

$$s_1 = \sqrt[Z]{\frac{R_r}{R_0}} \quad . \qquad (6)$$

Two sets of rotor resistance starters are analysed, one with 6 and other with 8 resistance levels. Resulting resistance values for both cases are shown in Table 3.

Table 3. *6-level rotor resistance starter values*

r_1 **[s]**	r_2 **[s]**	r_3 **[s]**	r_4 **[s]**	r_5 **[s]**	r_6 **[s]**
0.463	0.234	0.119	0.06	0.03	0.015

8-level rotor resistance starter values

r_1 **[s]**	r_2 **[s]**	r_3 **[s]**	r_4 **[s]**	r_5 **[s]**	r_6 **[s]**	r_7 **[s]**	r_8 **[s]**
0.95	0.56	0.33	0.19	0.11	0.06	0.028	0.011

In Fig. 4, the starting currents of the stator and rotor are shown for 6 (left) and 8 (right) added rotor resistance values. The initial stator and rotor currents exceed 1200A and 3600A, respectively, and must be limited during the machine's acceleration. The torque/speed characteristics at the bottom of Fig. 4 are compared with the constant load torque characteristic.

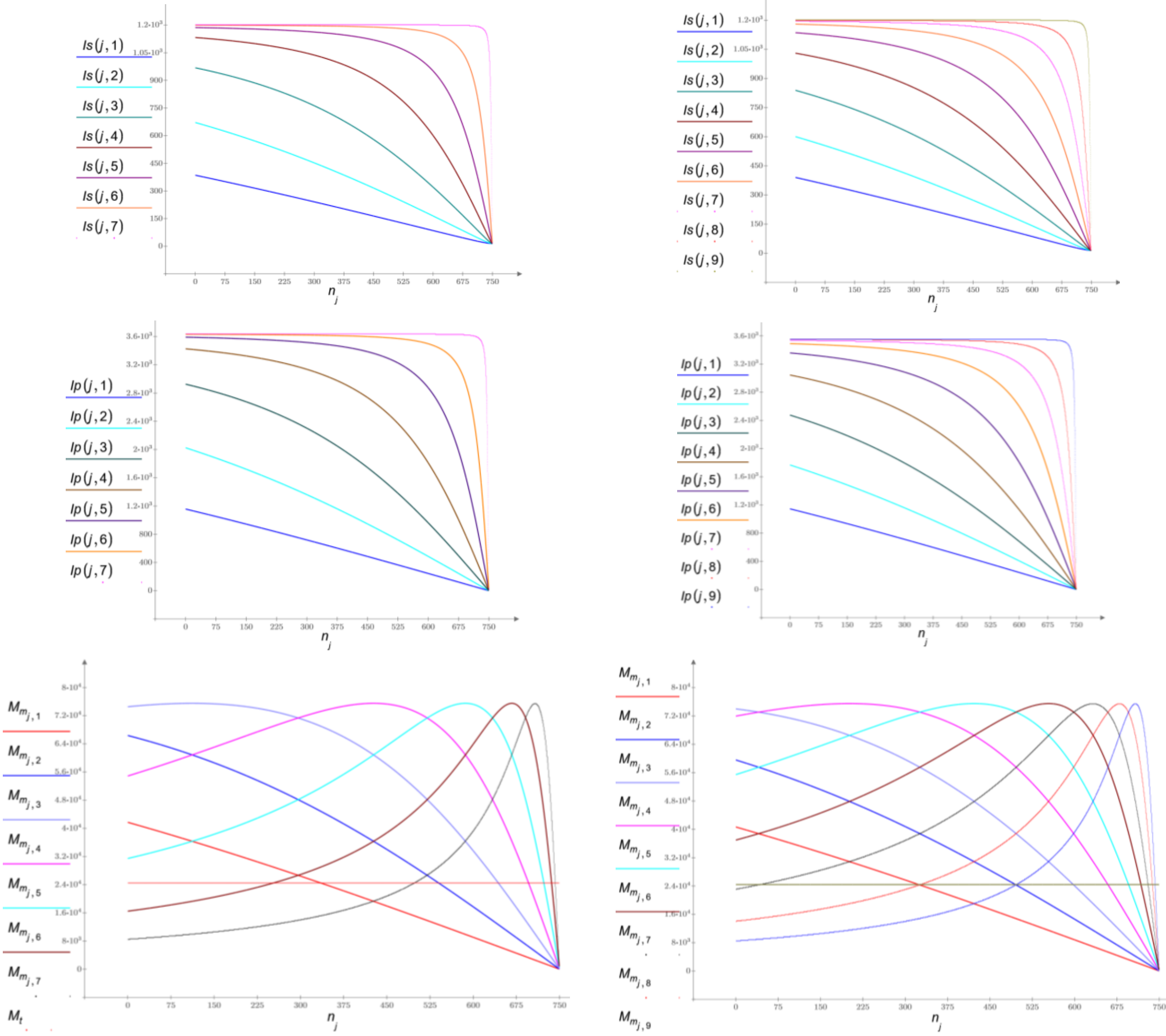

Figure 4. *IM stator (Is) and rotor (Ip) currents, and corresponding torque vs speed characteristics for different values of added rotor resistances: 6 values (left), 8 values (right)*

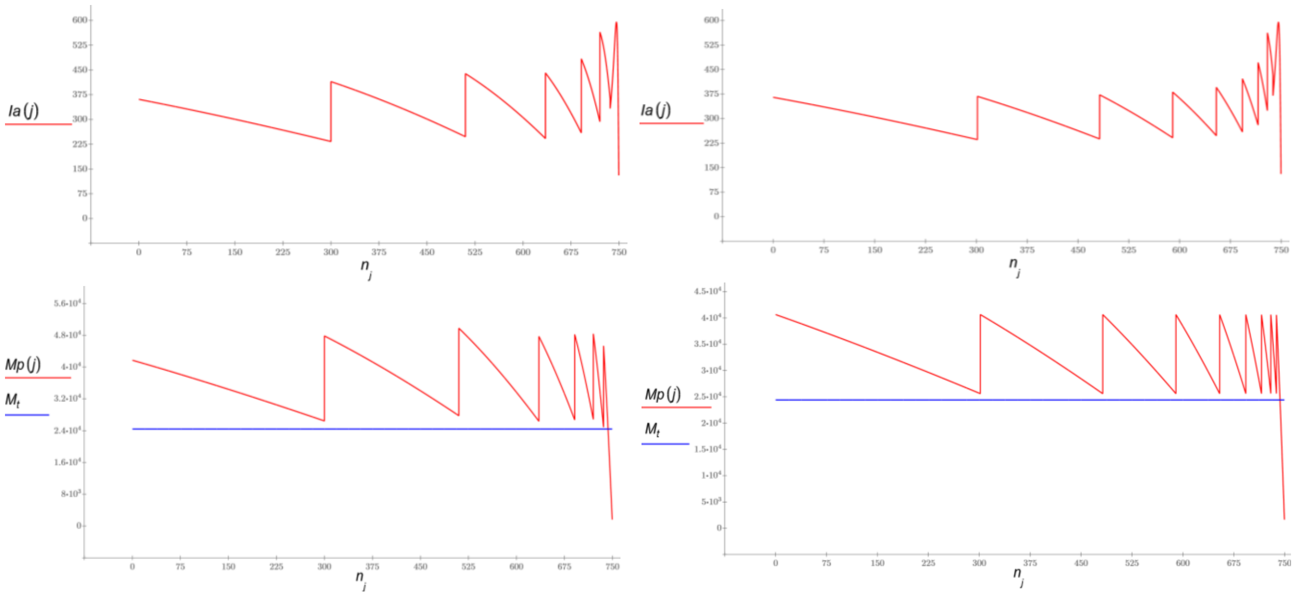

Figure 5. *Resulting IM stator current in a phase (Ia) and torque (Mt - load and Mp – starting machine torque) with 6 level (left) and 8 level (right) resistance starter*

By adding appropriate resistances to the rotor circuit, the maximum starting currents in the stator and rotor can be effectively controlled. This control is achieved by adjusting the switching times of individual resistors in the rotor circuit. Since changes in stator currents correlate with machine speed and slip, the correct resistance values and their switching times can be determined (Table 4).

Defining the switching times for the rotor starter levels results in the stator's starting currents for both cases with 6 and 8 resistor levels, shown in Fig. 5. The stator currents are significantly reduced, ranging from 230A to 600A. Higher starting stator currents correspond to higher speeds and shorter activity periods of the respective resistor levels. The specific switching times for each of the 6 or 8 levels are provided in Table 4.

Table 4. *6-level rotor resistance starter times*

t_1 [s]	t_2 [s]	t_3 [s]	t_4 [s]	t_5 [s]	t_6 [s]
13.054	6.209	4.113	1.879	0.944	0.838

8-level rotor resistance starter times

t_1 [s]	t_2 [s]	t_3 [s]	t_4 [s]	t_5 [s]	t_6 [s]	t_7 [s]	t_8 [s]
16.3	9.73	5.82	3.48	2.08	1.24	0.744	0.445

In the upper part of Fig. 5, the resulting starting current of one stator phase is shown, while in the lower part, the resulting torque-speed characteristic of the machine with the load characteristic is displayed. It should be noted that the resulting torque characteristic during startup does not take values lower than the load torque to ensure a successful startup. Since the activation times of individual starter steps are different, by knowing the current and the resulting resistance values, Mathcad enables a simple calculation of the voltage change at the ends of the starter as well as the thermal losses on individual steps needed for the proper sizing of the starter power.

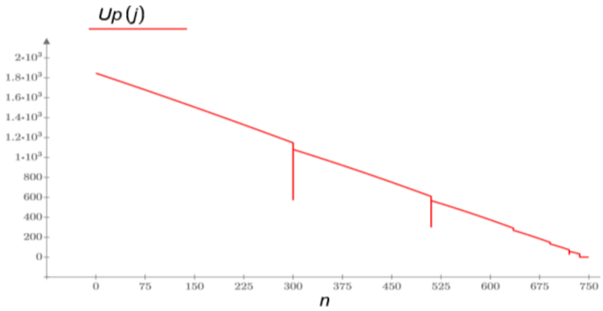

Figure 6. *Resulting voltage across resistance starter during the startup*

Fig. 6 shows that maximal voltage across rotor starter (*Up*) correspond to low speeds and decreases as machine speeds up. This voltage profile together with corresponding starter currents and times form Table 4. allows an engineer to determine energy dissipation and thermal analysis of starter resistance during its designing procedure.

5. THERMAL STRESS ANALYSIS OF IM

International Electrotechnical Commission standard for rotating electrical machines IEC 60034-1 (corresponding Serbian version SRPS IEC 60050-411) [8]) defines different electric drive classification in terms of their operation regime (duty type) and corresponding heating during the defined operation.

Heating of an electric machines within the drive depends on overall losses and electric drive duty type. A well-known conclusion from Montsinger's thermal degradation model [9] indicates that operating just 8°C above the allowed insulation temperature of the windings halves the working life of an electric machine. Therefore, it is necessary to ensure that the maximum allowable temperature of the machine's windings does not exceed the permissible temperature specified by the insulation class of the machine windings. Having said that, it's very important to monitor or estimate machine temperature during different machine operating regimes. This kind of machine thermal analysis and corresponding thermal modelling and simulation is mandatory during first stages of the machine design. Moreover, during the machine exploitation additional thermal analysis should be performed if machine duty type is changed or higher number of startups per hour is expected.

Starting from thermal equilibrium relation (7) it is possible to deliver relations defining thermal heating (8) and cooling of an electrical machine (9).

$$Q \cdot dt = A \cdot \theta \cdot dt + C \cdot d\theta \,, \tag{7}$$

$$\theta = \frac{Q}{A}\left(1 - e^{-\frac{A}{C}t}\right) + \theta_0 e^{-\frac{A}{C}t} \,, \tag{8}$$

$$\theta = \theta_n e^{-\frac{t}{T_0}} \,, \tag{9}$$

where, Q represents machine power losses responsible for heat, θ machine temperature, A specific heat dissipation or emissivity, C specific heat capacity of the material of the machine body.

By defining this set of equations in Mathcad and entering the basic parameters of the machine along with the necessary data related to the insulation class and heating/cooling constants (Table 5), it is possible to obtain all the necessary information related to the thermal stress of the machine during changes in the duty cycle. Specifically, when changing the machine's load or the intermittence of the drive, the machine's (windings) heating profile will differ over time. It is necessary to check the thermal stress on the windings depending on the insulation class (F=100°C).

Table 5. *3ph IM 30kW, DT: S3 parameters*

P_n [kW]	30	*C* [kWs/kg°C]	0.48
η [%]	90	*A* [kW/C]	0.0284
P_n [kW]	30	*Th. cl. (F)*	100°
Tz [min]	45	*Th* [min]	60

In this particular task of thermal stress analysis, the focus was on machine load increase by 10% with an intermittence of 70% of S3 duty type drive. The result of comparing the thermal stress during continuous operation and the described change at an unusually low ambient temperature of θ_a=-15°C is shown in Fig. 7.

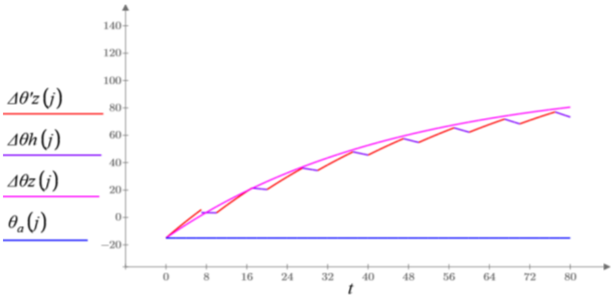

Figure *7.* *Machine heating diagram in case of continuous duty and intermittent duty (ε=70%) with 10% overload ($\Delta\theta_z$ heating temp., $\Delta\theta_h$ cooling temp., $\Delta\theta'_z$ overall temperature)*.

Fig. 7 shows that 10% machine overload with ε=70% will not jeopardise machine windings in terms of overheating since maximal temperature doesn't reach temperature defined by insolation class F. Moreover, the resulting machine temperature with 10% overload and ε=70% (S3) doesn't exceeds the temperature in comparison with resulting machine temperature during the continuous duty (S1) which proves that changing machine operating regime in this way is safe form thermal aspect.

Simple changes to input parameters, such as intermittent duty, load, or changes in the machine's loss values, lead to the automatic recalculation of all other defined equations and graphs in the Mathcad file. Fig. 8 shows the case of a change in the percentage of losses in the machine, i.e., the efficiency level, resulting in faster heating of the machine at the same load and duty cycle.

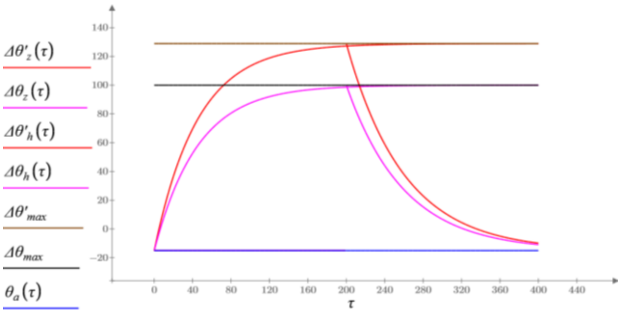

Figure 8. *Machine heating diagram in case of increased machine losses*

It can be noted form Fig. 8 that increased losses or reduced efficiency cause the machine to reach significantly higher temperatures. This consequently necessitates either a reduction in the machine's load or a change in the duty cycle.

6. CONCLUSION

The presented paper provides a brief overview of Mathcad software's capabilities in determining and visualizing the static and dynamic characteristics of induction machines. It showcases the specific characteristics of an IM during the startup period and the procedure for selecting the appropriate resistance levels for the rotor resistance starter for high power IM. Moreover, Mathcad possibilities are presented in terms of thermal aspect analysis of the machine in case of changed duty cycle and machine efficiency followed by appropriate discussion.

ACKNOWLEDGEMENTS

This study was supported by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia, and these results are parts of the Grant No. 451-03-66/ 2024-03/200132 with University of Kragujevac – Faculty of Technical Sciences Čačak.

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