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Analysis of a Simple Algorithm for Phase Shift Measurement Between Sinusoidal Voltage Signals

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Abstract: *This paper presents the analysis of the problem of phase shift measurement error that arises in Simple Algorithm (SAL) when signals are not sampled synchronously. Data acquisition systems allow simultaneous multiple channel measurements by utilising one AD converter with multiplexer. Multiplexer introduces time latency between sampling of different channels, which makes sampling of signals non-simultaneous. By experimental measurements with NI 6009 data acquisition card and LabVIEW, as well as C++ simulations, two components of the measurement error will be identified: time dependant component which causes phase shift estimations to oscillate and constant component which introduces the offset in performed estimations.*

Keywords: *phase shift measurement; LabVIEW; Data acquisition system; measurement error*

1. INTRODUCTION

One of the problems when calculating electric power and energy in alternating current circuits is determining the phase difference between voltage and current signals [1, 2]. The current signal is easily transformed into a voltage signal by applying the shunt, so the task turns into estimating the phase difference between the two voltage signals. It is assumed that the harmonic distortions of these sinusoidal voltages are negligible, but realistic distortions and noise in the signals can also be considered.

High-accuracy measurements of phase are needed in the low-level RF systems [3]. Accurate and fast estimation of phase difference between the voltage and current of the electrical power system is required for calculation of its power factor [4]. It is also important in measurements of complex ratio of AC voltages [5]. The improvement of the precision of phase difference estimation method could be done by data extension and Hilbert transform [6].

This paper presents the application of the Simple Algorithm (SAL) for calculation of the phase shift between two signals [1, 2]. The analysis is performed both for measured and simulated signals. A focus is further given to the analysis of the measurement error identified during the measurements and simulations, caused by the imperfections in the signals or their sampling. Two independent components of this error are analysed in the paper.

The paper presents a short theoretical description of SAL, the obtained results and corresponding analysis, as well as the accompanying comments and conclusions.

2. THEORETICAL BACKGROUND

The problem of determining a phase shift between two signals can be mathematically described as follows: let two voltage signals be given in a form

$$u_1(t) = U_1 \sin(\omega_o \cdot t + \varphi_1), \quad (1)$$

$$u_2(t) = U_2 \sin(\omega_o \cdot t + \varphi_2), \quad (2)$$

where U_1 and U_2 are amplitudes, φ_1 and φ_2 are initial phases and ω_0 is the angular frequency which is known. The problem is finding the value of phase shift between two signals φ, given by:

$$\varphi = \varphi_1 - \varphi_2. \quad (3)$$

2.1. Mathematical base of Simple Algorithm

SAL is a fast algorithm for phase shift estimation between sinusoidal signals. It requires only two pairs synchronously and coherently obtained samples of signals to estimate their phase shift.

For solving a problem given by (3) two auxiliary signals should be introduced:

$$u_1'(t) = U_1 \sin(\omega_o \cdot t + \varphi_1 - \psi_1), \quad (4)$$

$$u_2'(t) = U_2 \sin(\omega_o \cdot t + \varphi_2 - \psi_2). \quad (5)$$

These auxiliary signals are phase-delayed for ψ_1 and ψ_2 compared to signals u_1 and u_2, respectively. Now, phase-delays can be set to:

$$\psi = \psi_1 = \psi_2 = \frac{\pi}{2}. \quad (6)$$

Finally, two parameters can be introduced:

$$m_a(t) = u_1(t)u_2(t) + u_1'(t)u_2'(t), \quad (7)$$

$$m_r(t) = u_1'(t)u_2(t) - u_1(t)u_2'(t). \quad (8)$$

It is easy to show that, when condition (6) is satisfied, solution of (3) is given by [1, 2]:

$$\varphi = \tan^{-1}\left(\frac{m_a(t)}{m_r(t)}\right). \quad (9)$$

Fig. 1 presents the signals and the corresponding samples needed for calculation of phase shift.

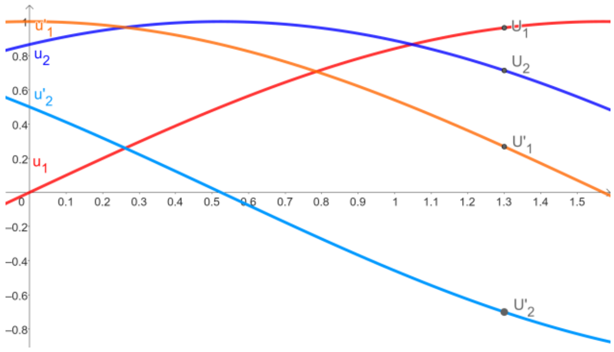

Figure 1. *An example of values necessary to estimate phase shift with SAL*

2.2. Sampling of original and auxiliary signals

By using any type of data acquisition system, a set of ordered pairs of time-discrete values of signals u_1 and u_2 can be obtained in the form of:

$$\{(u_1[nT_s], u_2[nT_s]) \mid n = 0, 1, 2, \ldots\}, \quad (10)$$

where T_s is a sampling period. From this set of time-discrete values of the original signals, values of auxiliary signals can be obtained by assigning them the values of the original signals shifted for appropriate time interval:

$$u_1'[nT_s] = u_1[(n-r)T_s], \quad (11)$$

$$u_2'[nT_s] = u_2[(n-r)T_s]. \quad (12)$$

This is illustrated in Fig. 2.

Integer r should be chosen to satisfy the equation:

$$r = \frac{f_s}{4f_0}, \quad (13)$$

where $f_s = 1/T_s$ is the sampling frequency and f_0 is the signal frequency, so that sampling is coherent. It is important to note that since r is an integer, sampling frequency cannot be chosen arbitrarily.

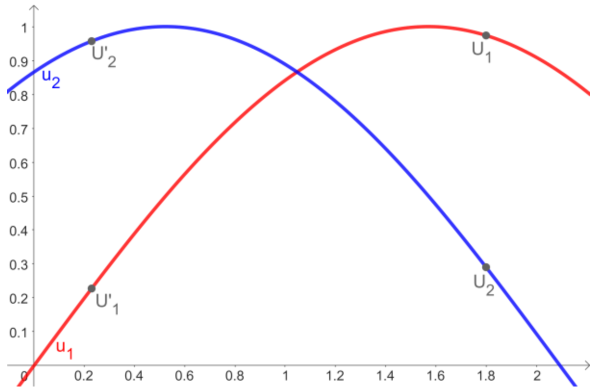

Figure 2. *Obtaining values of auxiliary signals from original signals*

After assigning values to auxiliary signals as described, equations (7) and (8) can be calculated, and therefore phase shift can be estimated using equation (9).

3. PHASE SHIFT MEASUREMENT ERROR ANALYSIS

From equations (7), (8) and (9) can be noticed that measurement error is dependent on the precision of measurement of signals u_1 and u_2 in terms of the current value and in terms of the time of sampling.

Measurement precision of current value of signals is directly determined by the resolution of AD converter used in data acquisition system.

If any pair of signal values is not sampled synchronously or if auxiliary signals are not determined from properly time-shifted original signals, the error will occur. In other words, if for samples of original signals $(u_1[t_1], u_2[t_2])$ and auxiliary signals $(u_1[t_3], u_2[t_4])$ equations:

$$t_1 = t_2, \quad (14)$$

$$t_3 = t_4, \quad (15)$$

$$t_3 = t_1 - r \cdot T_s, \quad (16)$$

do not hold true, algorithm produces the error in estimation.

It can be shown that errors of parameters m_a and m_r, denoted with δm_a and δm_r, respectively, are dependent of time and phase-delay ψ defined in equation (6), as [1, 2]:

$$\delta m_a = -\frac{\cos(\psi)}{\cos(\varphi)}\cos(2\omega_0 nT_s + \varphi_1 + \varphi_2 - \psi), \quad (17)$$

$$\delta m_r = \sin(\psi) - 1. \quad (18)$$

4. DATA ACQUISTION SYSTEM AND SIMULATIONS

Data acquisition system with NI 6009 data acquisition card based on LabVIEW software was created for utilisation of SAL in phase shift measurements [7]. C++ simulations of such measurements were then created for examining the measurement errors.

4.1. LabVIEW simulation

Before measurements with data acquisition card, LabVIEW simulation was created to verify the virtual instrument itself. A block diagram of the simulating application is presented in Fig. 3.

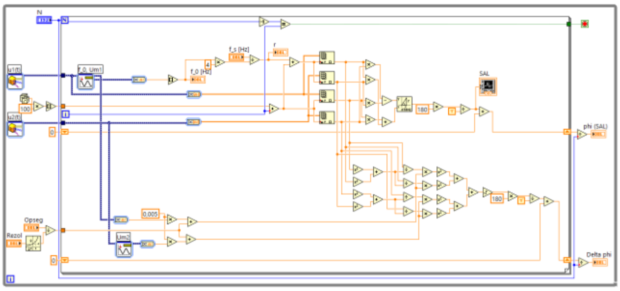

Figure 3. *Block diagram of simulated virtual instrument*

Existing functions (Express VI) of LabVIEW, "Simulate Signal", were used to create arrays of sampled values of signals u_1 and u_2. Using functions for array manipulation appropriate, the values for original and auxiliary signals were chosen, and then the phase shift was estimated. Obtained results, both with ideal and signals with white Gaussian noise, were satisfactory.

A front panel of the simulating application is presented in Fig. 4.

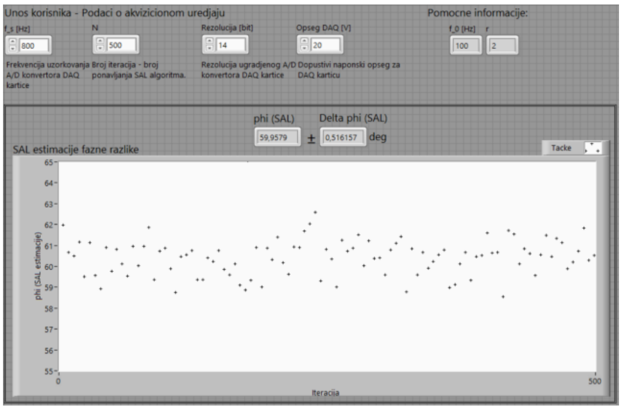

Figure 4. *Front panel of virtual instrument with simulated voltage signals with Gaussian white noise*

4.2. Data acquisition system

Signal generator (Siglent SDG 2082X), that can produce two voltage signals with predefined phase shift between them, was used as a signal source. NI 6009 data acquisition card connected to the laptop via USB was used as a measurement device. A photograph of the equipment is given in Fig.5.

Simulating LabVIEW application (Fig. 3) was modified to a virtual instrument to conduct measurement and analyse the results. Block diagram and front panel of this virtual instrument are presented in Figs. 6 and 7, respectively.

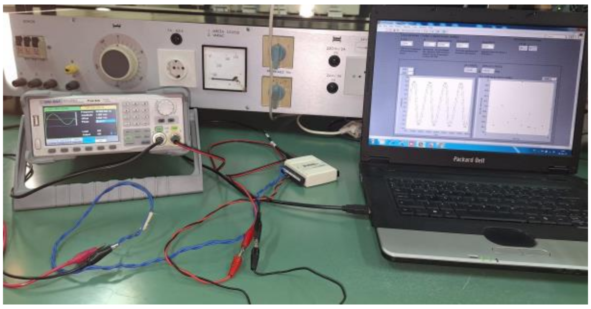

Figure 5. *Experimental equipment*

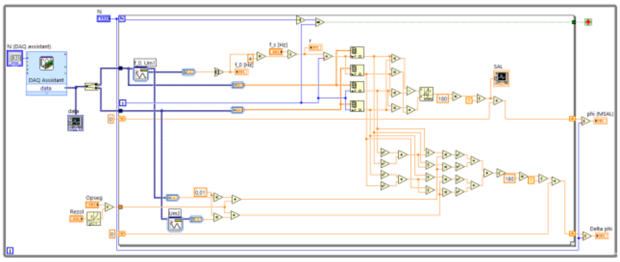

Figure 6. *Block diagram of virtual instrument for application of SAL algorithm*

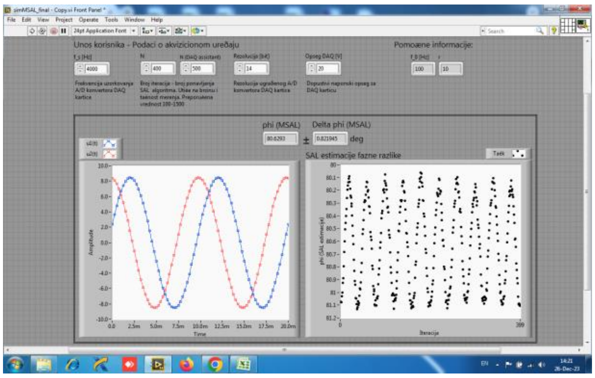

Figure 7. *Front panel of virtual instrument*

Shown virtual instrument utilises data acquisition card with "DAQ Assistant" function to collect time-discreet measurements of signals u_1 and u_2 into arrays. Further, using functions for array manipulation, appropriate values from arrays are used to calculate estimations of phase shift, as described in section 2.2. All estimations are then averaged, and average value is given as a measurement result. Bottom part of the diagram calculates an error of estimation, using a simplified approach which is acceptable for selected equipment and frequency range of signals.

Front panel of the virtual instrument shows an average value of estimations and two graphs. One graph shows measured signals, and the other shows estimated phase shift with respect to ordinal number of estimations.

Measurements were made for signals with frequencies of 50, 100 and 200 Hz, using the NI 6009 data acquisition card with one AD converter and multiplexer (multiplexed sampling). When current values of signals are obtained in this way, time latency occurs when multiplexer is switching between input channels, which means that equations (14)-(16) are not satisfied. It was observed, and is visible in Fig. 7, that values of phase shift estimations oscillate with time and have an offset from expected value.

For signals with the frequency of 100 Hz or less, error of average value of phase shift estimations was ±1°. For signals with frequency greater or equal to 200 Hz, error becomes equal or greater than ±3°.

4.3 C++ simulations

For purposes of additional error analysis, C++ simulations of acquisition of ideal signals for three cases were made:

First case: simulation of real measurements; when equations (14), (15) and (16) are not satisfied;

Second case: simulation when equations (14) and (15) are satisfied but (16) is not;

Third case: simulation when equations (14) and (15) are not satisfied but (16) is satisfied.

In all simulations values of sampled signals are obtained by calling a simple subfunction "double u" which is evaluated for input parameters: frequency (f), time (t) and phase (fi), as shown in Fig. 8.

```
//signal function
double u(float f, double t, double fi) {
    return sin(2*3.1415926535*f*t+fi);//sine signal
}
```

Figure 8. *Subfunction for calculating a value of sampled signal*

All relevant values: u_1, u_2, u'_1, u'_2, and ψ are initiated as double vector variables named "u1", "u2", "u1prim", "u2prim" and "fi", respectively, as shown in Fig. 9.

Simulation in the first case requires that none of the equations (14), (15) and (16) are satisfied. That calculation is realised by simple for loop, as shown in Fig. 9.

```
vector<double> u1, u2, u1prim, u2prim, fi; // signal values, parameters
int n=0;
for (int i=0; i<=N; i++) {
    u1.push_back(u(f_0, (i+n*k)*dt, fi1));
    u1prim.push_back(u(f_0, (r+i+(n+2)*k)*dt, fi1));
    u2.push_back(u(f_0, (i+(n+1)*k)*dt, fi2));
    u2prim.push_back(u(f_0, (r+i+(n+3)*k)*dt, fi2));
    fi.push_back(atan((u1prim.at(i)*u2.at(i)-u1.at(i)*u2prim.at(i))/
    (u1.at(i)*u2.at(i)+u1prim.at(i)*u2prim.at(i)))*57.2957795);
    n+=4;
}
```

Figure 9. *For loop for first case of phase shift measurement simulation*

Variable "f_0" corresponds with signal frequency f_0, "fi1" and "fi2" correspond with φ_1 and φ_2 respectively, "dt" corresponds with sampling period T_s, "k" is a value between 0 and 1 which represents time-latency due to multiplexing as a percent of sampling time T_s and "r" is the parameter defined by equation (13).

After calculations, C++ program creates a textual raw data file, formatted for further use in data analysis software.

Simulation for second case requires that equation (16) is not satisfied while equations (14) and (15) are satisfied. That can be easily accomplished by changing calculations in for loop as shown in Fig.10.

```
for (int i=0; i<=N; i++) {
    u1.push_back(u(f_0, i*dt, fi1));
    u1prim.push_back(u(f_0, (r+i)*dt, fi1));
    u2.push_back(u(f_0, (i+k)*dt, fi2));
    u2prim.push_back(u(f_0, (r+i+k)*dt, fi2));
    fi.push_back(atan((u1prim.at(i)*u2.at(i)-u1.at(i)*u2prim.at(i))/
    (u1.at(i)*u2.at(i)+u1prim.at(i)*u2prim.at(i)))*57.2957795);
    n+=4;
}
```

Figure 10. *For loop for second simulation of phase shift measurement*

Finally, for simulation of third case equations (14) and (15) are satisfied and (16) is not. For loop calculations for this case is given in Fig. 11.

```
for (int i=0; i<=N; i++) {
    u1.push_back(u(f_0, i*dt, fi1));
    u1prim.push_back(u(f_0, (r+i+k)*dt, fi1));
    u2.push_back(u(f_0, i*dt, fi2));
    u2prim.push_back(u(f_0, (r+i+k)*dt, fi2));
    fi.push_back(atan((u1prim.at(i)*u2.at(i)-u1.at(i)*u2prim.at(i))/
    (u1.at(i)*u2.at(i)+u1prim.at(i)*u2prim.at(i)))*57.2957795);
    n+=4;
}
```

Figure 11. *For loop for third simulation of phase shift measurement*

The results of simulations in all three cases were obtained with the following input data:

$\varphi = \varphi_1 - \varphi_2 = 40 - 0 = 40$; $f_0 = 50\ Hz$; $f_s = 800\ Hz$; $k = 0.01$; $N = 100$ iterations.

In the first case, graph of phase shift estimations is shown in Fig. 12. Signal samples are taken asynchronously (in the manner of multiplexed sampling) and time delay between original and auxiliary signals is different from condition (6).

Figure 12. *Results of the simulation for the first case*

It can be observed that estimations in this case behave in the same way as a physical measurement: they oscillate periodically with time around the average value and their average value has an offset from the true value of phase shift. This is consistent with equations (17) and (18). Error of parameter m_a is dependent from time (nT_s) of sampling and phase-delay ψ according to the cosine function, and error of parameter m_r is constant and dependent only from ψ.

For second case, ordered pairs of signal samples are taken synchronously, but time delay between original and auxiliary signal values is inaccurate. In other words, condition (6) is not satisfied. Graph of phase shift estimations with respect to ordinal number of estimations is given in Fig. 13.

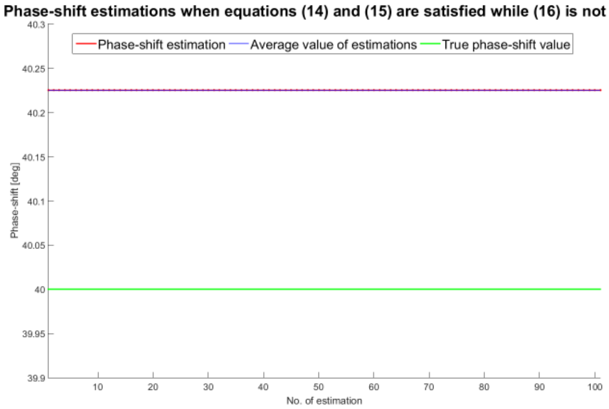

Figure 13. *Results of the simulation for the second case*

In this case δm_a is equal to zero, while δm_r is not, and its value is constant. This error component cannot be reduced by averaging estimations.

For the third case, ordered pairs of signals u_1 and u_2 are sampled asynchronously, but condition (6) is satisfied. Then δm_r is equal to zero, and only non-zero error component is δm_a. Graph of phase shift estimations with respect to ordinal number of estimations is given in Fig. 14.

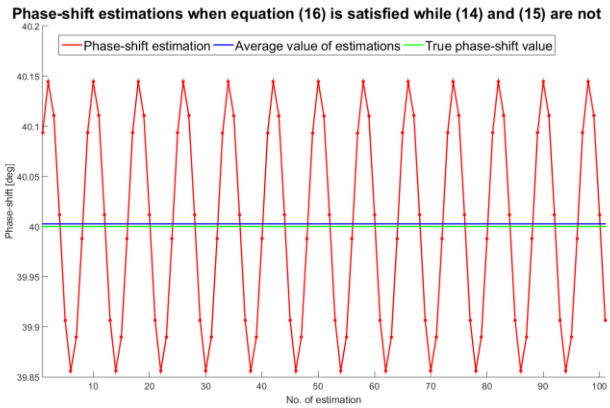

Figure 14. *Results of the simulation for the third case*

It is noticeable that the estimations oscillate around average value, which is consistent with equation (17). It is also observable that the average value of estimations tends to true phase shift value as number of estimations *N* tends to infinity. That means that this error component can be reduced by increasing the number of estimations.

In second case, where samples are taken synchronously and condition (6) is not satisfied, errors of parameters m_a and m_r can be described as: $\delta m_a = 0$, $\delta m_r \neq 0$. In third case, condition (6) was satisfied and samples were taken asynchronously, $\delta m_r = 0$ and $\delta m_a \neq 0$. Therefore, the first case can be interpreted a "superposition" of the other two.

5. CONCLUSION

The estimation of the phase shift between two sinusoidal voltage signals is performed using SAL, and the analysis of the error introduced by this algorithm is performed in this paper.

Calculated error was divided into two components and their separate analysis was performed using C++ simulations. It was shown by the experiment that the overall error increases with the frequency of the signals, with the satisfying level of the error (below 1%) at frequencies below 100 Hz. The main cause of the error was non-simultaneous sampling of the signals, which produces offset and oscillations in the phase shift estimation.

Further research would be focused on the analysis of the measurement error during the simultaneous sampling of the signals, especially at the increased signal frequencies, up to several kHz.

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