COMETa 2020
5th INTERNATIONAL SCIENTIFIC CONFERENCE
26th - 28th November 2020
Jahorina, Republic of Srpska, B&H
University of East Sarajevo
Faculty of Mechanical Engineering
Conference on Mechanical Engineering Technologies and Applications

PARTICLE SWARM SIZING OPTIMIZATION OF PLANAR TRUSS STRUCTURES WITH BUCKLING CONSTRAINTS

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Abstract: This research examines the use of PSO sizing optimization for the planar truss mass minimization problem. In order to achieve practically applicable results, the examples used in the paper are also subjected to Euler buckling constraints. All examples use discrete sizing variables to generate models which can be constructed from bar element sizes available in the given material. The complexity added by implementing a dynamic constraint for buckling significantly influences the search space and makes finding global optima more difficult. Test example results are compared to results using GA from literature. As an additional parameter which influences the overall cost of a truss structure, the overall outer areas of all optimal models are compared.

Key words: buckling constraints, discrete variables, PSO, sizing optimization, truss optimization

1 INTRODUCTION

Structural optimization has become an important part of the design process in all industries. There is a need for more rational constructions in the design of trusses. One of the most common optimization goals is minimization of mass. This can be achieved by altering various truss parameters. The three types of optimization are sizing, shape and topology optimization. These types can also be combined simultaneously or sequentially to achieve better results than by just optimizing a single parameter. It has however been found that the most influential parameter in mass minimization is the optimization of cross-section dimensions and parameters, or sizing optimization.

Researchers have, in the past years, studied the effects of using various optimization algorithms to solve minimal mass problems. In most research to date the influence of buckling has not been considered, as is the case in [1-11]. More recently,

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papers have been published showing the differences and benefits of using dynamic buckling constraints in truss optimization [8-10]. Results show that the use of dynamic buckling constraints increases the total mass of optimal structures. It also proves that solutions which have not considered the use of this constraint cannot be considered acceptable for use, as they all have, at least one, if not almost all, bars which are subjected to compression forces which exceed critical buckling loads. The presence of even one bar element which does not meet buckling criteria results in an unacceptable solution.

Many papers differ in the use of continuous and discrete variables for sizing optimization. The use of continuous variables for sizing optimization has been compared to results using discrete variables in papers [12, 13] on standard truss examples. The use of discrete variables gives structures with marginally grater masses, however they are sized according to standardly available stock which ensures that their construction is possible.

This paper will present the use of particle swarm optimization algorithm for planar truss sizing using discrete sizing variables and dynamic buckling constraints.

An additional parameter which plays a significant part in overall construction costs is the total outer surface area which needs to have some form of coating. The premise for checking overall outer surface area has been found in the work of authors in [14] who checked areas of hollow, square bar elements in optimal solutions of a roof truss. This research will compare optimal models' total surfaces in order to get a better understanding of the influence mass minimization plays in decreasing the total area.

2 TRUSS STRUCTURAL OPTIMIZATION

Truss structural optimization generally implies the simultaneous optimization of sizing, shape, and topology, however that is not always possible. Depending on the initial design of the truss and practical conditions such as the location, installation limitations and loading of the truss optimization of one or a combination of two of these aspects of a structure are most commonly used.

In order to create a practically applicable optimal model it is necessary to create realistic loading cases and supports in the initial model. The variables for sizing must be a discrete set which should reflect available cross-section profiles typically used in such structures and have the bars to which they can be assigned grouped in order to ensure the possibility and ease of assembly [15].

The minimum mass design problem, as given in most literature is defined in expression (1) which also gives the basic constraints used in truss structural optimization which are axial stress limits and displacement.

$$\begin{cases} minW\left(A\right)=\sum_{i=1}^{i=n}\rho_i\,A_i l_i \ \text{with}\ A=\left(A_1,\ldots,A_n\right) \\ \text{subjected to} \begin{cases} A_{min}\le A_i\le A_{max} & \text{for}\ i=1,\ldots,n \\ \sigma_{min}\le\sigma_i\le\sigma_{max} & \text{for}\ i=1,\ldots,n \\ u_{min}\le u_j\le u_{max} & \text{for}\ j=1,\ldots,k \\ \sigma_{Ai}^{P}\ \le\ \sigma_{ki} \end{cases} \end{cases} \tag{1}$$

In expression (1) is the number of truss elements, k is the number of nodes, l_i is the length of the i^{th} element, A_i is the area of the i^{th} element's cross-section profile, σ_i is the stress of the i^{th} element, u_j is displacement of the j^{th} node and σ_{Ai} is axial compression stress, and σ_{Ki} is critical buckling stress of the i^{th} element.

2.1 Buckling Constraints

In order to achieve practically applicable results this paper uses dynamic Euler buckling constraints. Testing compressed bars is done using the following expressions:

$$\sigma_{Ai}^{P} \leq \sigma_{ki}$$

$$\text{for } \sigma_{Ai}^{P} = \frac{F_{Ai}^{P}}{A_i} \quad \text{and} \quad \sigma_{ki} = \frac{F_{ki}}{A_i}, \tag{2}$$

$$F_{ki} = \frac{\pi^2 \cdot E_i \cdot I_i}{l_{ki}^2}$$

$$\left|F_{Ai}^{P}\right| \leq F_{ki} \text{ for } i = 1, ..., n. \tag{3}$$

Here, F_{Ai}^{comp} is axial compression force, F_{Ki} is Euler's critical load, E_i is modulus, and I_i is minimum area moment of inertia of the cross section of the of the i^{th} element.

The addition of any constraint increases the problems complexity, and makes the finding of global optima more difficult. By adding dynamic constraints such as Euler buckling this becomes an even greater problem, as the search-space is further discretized. Since these constraints change with each iteration of optimization, they are called dynamic constraints. In the case of truss sizing optimization, cross-section profile changes change the moment of inertia in each iteration, changing the constraint value each time.

These types of problems require non-linear optimization algorithms. Heuristic methods are generally used in engineering practice to solve these types of problems as they have favourable characteristics. This paper considers the use of the particle swarm optimization algorithm.

2.2 Particle Swarm Optimization

Particle swarm optimization (PSO) is classified as a swarm intelligent-based algorithm, and it searches the entirety of the acceptable domain. A great advantage of this algorithm is that it uses only one phase, which influences the algorithms performance and controllability. This method, due to its exceptional characteristics, has been widely used in previous years to solve complex engineering problems in various fields.

The main operation principle of the PSO algorithm is based on so-called particle acceleration, the distance from a particle position to the best value of a given particle (local best – $x_{p,i}$) and its position from the globally best particle (global best – $x_{g,i}$). The position of a particle in a given moment represents a potential solution. The best position is accepted and it is passed through an iterative optimization process. Each new solution is defined with two components, velocity and position. Position is defined as x_i, while the velocity is represented with v_i. The number of positions and accelerations is n depending on the total number of particles which is defined. Each new value is derived using the following formula:

$$x_{new,i} = x_{old,i} + \upsilon_{new,i} \quad , \tag{4}$$

where $i=1,2,\dots,N$, is the total size of the population,

$$\upsilon_{new,i} = \omega \cdot \upsilon_{old,i} + c_p \cdot r_p \left(x_{p,i} - x_{x,i} \right) + c_g \cdot r_g \left(x_{g,i} - x_{x,i} \right) \tag{5}$$

Constants c_p and c_g are defined by suggestions from literature to be 1.5 for both. Random values r_p and r_g are from the interval (0, 1). Current particle position is defined as $x_{x,i}$. Particle intensity is ω (inertia weight) and is defined as:

$$\omega = \omega_{max} - \frac{\omega_{max} - \omega_{min}}{Iteration_{max}} \cdot Iteration \, . \tag{6}$$

According to literature ω_{max}=0.9 and ω_{min}=0.4. Using the previous equations, solutions converge in an iterative approach in order to achieve an optimal solution.

3 EXAMPLES

In order to demonstrate the use of PSO algorithm with implemented buckling constraints, for the purposes of this research two standard test examples most commonly found in literature have been used.

3.1 The 10 Bar Truss Problem

The 10 bar truss is the most frequently found example in truss structural optimization papers. This cantilever truss, shown in figure 1, has 10 independent cross-sectional sizing variables. The material used is Aluminum 6063-T5 with the following characteristics: Young modulus 68947MPa, and a density of 2.7g/cm^3.

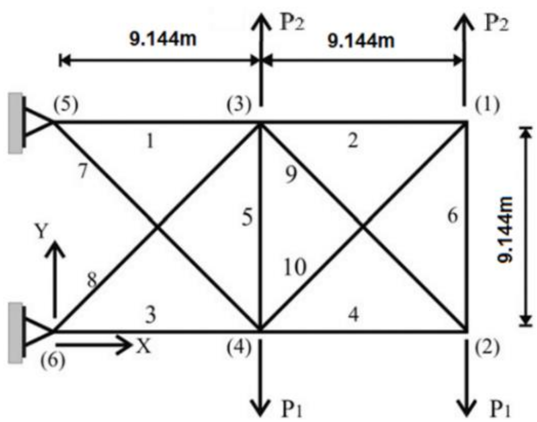

Figure 1. *Layout of the 10 bar truss example*

Point loads are P_1=444.82kN, P_2=0kN for the first loading case (LC1). For the second loading case (LC2), loads are P_1=667.233kN, and P_2=222.411kN as shown in figure 1. The model is limited to a maximal displacement of ±0.0508m of all nodes in all directions, axial stress of ±172.3689MPa for all bars.

3.2 The 17 Bar Truss Problem

The 17 bar truss problem's initial model layout is given in figure 2. For this example the material characteristics are: Young modulus 206842.719MPa, and a density of 7.4g/cm^3. Loading is in a single node (9) with a force of 444.82kN.

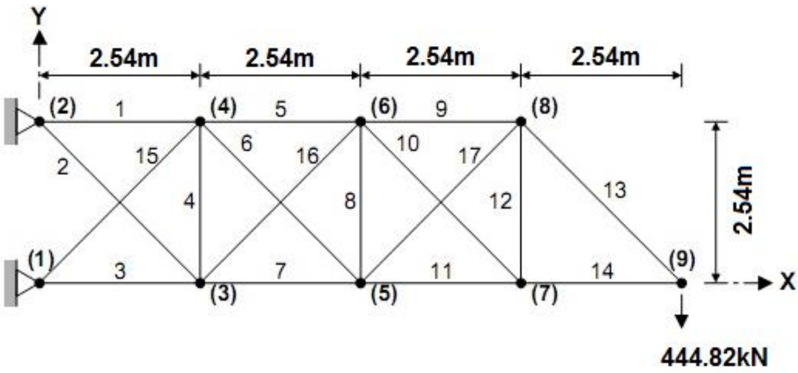

Figure 2. *Layout of the 17 bar truss example*

This example does not have a stress constraint, the only constraint is a displacement limitation for all nodes of ±0.0508m of all nodes in both x and y directions.

Both examples use the following list of cross-section diameters as variables: 12, 16, 20, 25, 30, 34, 35, 40, 45, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 105, 110, 115, 120, 125, 130, 140, 145, 150, 152, 160, 165, 170, 175, 178, 180, 190, 200, 210, 220, 230, 240, 250, 254, 260, 270, 278, 280, 300, 305, 356, given in mm.

4 RESULTS

For the purposes of this research a newly developed original software was used. The software is based in the Rhinoceros 6's Grasshopper visual programming environment. Finite element analysis was done using operators from the Karamba 3D plugin and optimization was done with Silvereye's PSO algorithm.

Table 1 gives a comparison of optimal results using genetic algorithm (GA) with the same problem setup from [MD, FAMENA] with this paper's PSO results for both load-cases of the 10 bar truss problem. As an additional comparison metric, overall outer surface area is given for all models, which is not a parameter in the optimization process. Most published literature compares bar areas, as opposed to bar diameters, therefore such a comparison is also given in this research.

Table 1. *Optimal solutions of the 10 bar truss problem with buckling constraints for LC1 and LC2.*

Area of bar [cm^2]	**GA LC1 [13]**	**PSO LC1**	**GA LC2 [16]**	**PSO LC2**
1	*78.540*	*103.869*	*103.869*	*95.033*
2	*15.904*	*1.131*	*12.566*	*9.621*
3	*415.475*	*380.133*	*415.476*	*415.475*
4	*240.528*	*283.529*	*254.469*	*283.528*
5	*1.131*	*1.131*	*3.142*	*1.131*
6	*15.904*	*1.131*	*33.183*	*28.274*
7	*122.718*	*50.265*	*122.718*	*95.033*
8	*415.47.6*	*425.389*	*314.159*	*346.361*
9	*103.869*	*95.033*	*63.617*	*70.882*
10	*181.458*	*4.909*	*113.097*	*1.131*
Mass [kg]	***4795.734***	***4028.775***	***4195.899***	***3869.938***
Area [m^2]	***42.706***	***34.600***	***40.407***	***35.646***

Table 2 shows optimal results for the 17 bar truss problem comparing this paper's PSO results with GA results from literature. As with the previous example, apart from the areas and optimal mass, the overall outer surface areas are also given in the table.

Table 2. *Optimal solutions of the 17 bar truss problem with buckling constraints*

Area of bar [cm²]	**GA [9]**	**PSO**
1	*44.179*	*70.882*
2	*23.758*	*31.172*
3	*86.590*	*103.869*
4	*23.758*	*6.158*
5	*56.745*	*95.033*
6	*32.758*	*2.011*
7	*86.590*	*63.617*
8	*0.283*	*8.042*
9	*38.485*	*8.042*
10	*38.485*	*38.485*
11	*56.745*	*63.617*
12	*38.486*	*44.179*
13	*38.486*	*33.183*
14	*44.179*	*44.179*
15	*50.265*	*44.179*
16	*56.745*	*70.882*
17	*50.265*	*9.621*
Mass [kg]	***1571.875***	***1536.741***
Area [m²]	***11.044***	***10.649***

All results are taken as the best of 10 repeated optimizations with the same initial parameters.

5 CONCLUSION

This research presents the benefits of using a more modern optimization method in order to achieve results closer to global optima values for truss structural optimization. Sizing optimization was considered as it has been found to be the parameter with the greatest influence on mass with these types of structures. The PSO algorithm shows better and faster convergence than the use of GA. The algorithm also shows a good ability to work with dynamic constraints such as was the case in this paper with using dynamic constraints for buckling. This addition to the complexity of the problem allows for practically applicable results.

Optimal models show a ~16% decrease in optimal mass solutions for the 10 bar truss problems first load-case, and a 7.8% decrease for the second load-case. The improvement on the 17 bar truss example is less at 2%, however this just shows that both solutions are close to the global optimum.

When comparing the parameters of overall outer surface area the decreases in mass correlate to the decreases in area. This was to be expected as the profiles are full cross-sections. Using PSO, the optimal models for the 10 bar truss example show a decrease of 19% and 11.8% for the two load-cases respectively, and a 3.5% decrease for the 17 bar truss problem.

Further research in this field will include a comprehensive analysis of the influence of different optimization types to the overall surface area, as well as the influence when using hollow cross-sections.

ACKNOWLEDGMENT

This paper is a result of the TR32036 and TR33015 projects, which are investigations of the Technological Development of the Republic of Serbia. The projects are titled “Development of software for solving the coupled multi-physical problems” and “Research and development of a Serbian net-zero energy house” respectively. We would like to thank to the Ministry of Education, Science and Technological Development of the Republic of Serbia for their financial support during this investigation.

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