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## Optimization problem with the goal of minimizing the mass of gears

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**Abstract** *One of the existing problems with mechanical structures occurs in the form of oversizing components which lead to unnecessary, increased costs. This paper presents an application of the optimization problem with the goal of minimizing the mass of gears in a single stage geared speed reducer. Aside from cost savings through the use of less material, a reduction in mass also results in improvement of efficiency during operation. In this paper, this was achieved to a large extent and demonstrated using standardized examples from literature in order to have comparable result. With the help of the existing parameters, various combinations of possible solutions were developed and the corresponding optimal solution was selected. The solution was compared with the existing solutions from literature and appropriate conclusions were drawn. The Excel Solver tool was used for the purposes of this the research in order to show that practically applicable optimal results can be achieved without the use of additional specialized software*

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**Keywords** *Optimization, gears, mathematical model, Excel Solver, minimal mass*

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### 1. INTRODUCTION

To ensure successful market placement of a new design, it is necessary for it to have the best characteristics. Some parameters that fall under this category are product dimensions, weight, manufacturing process and technology, materials, manufacturing duration, and so on. Product optimization is often employed to choose the best possible values for these parameters, especially in the case of weight and dimensions.

For the purpose of this study, gears were selected as one of the main mechanical components with a wide range of applications in

mechanical engineering. Gears can be found everywhere and their main feature is power transmission, so it is necessary to manufacture them with great precision. Due to their complex geometry and shape, analytical calculations during gear design can often result in errors. While results obtained by computer analysis are only sometimes optimally calculated. Therefore, for this study, parameter analysis was chosen to minimize the weight of the gear to optimize it.

The increasing use of gears and higher demand requires the use of optimization in gear design. The main goal of this study is to highlight the advantages of parameter optimization that contribute to reducing the weight of gears. The obtained results are compared to the previously published results from the literature [1].

In order to perform parameter optimization in this study, Excel Solver tool was used.

In addition to the mentioned literature, several other papers were considered during the

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research process in order to have referent results. These papers dealt with similar or identical optimization problems, such as the paper by Golabi S. and Jafari J., which deals with weight optimization of gears based on the minimum required weight in design [2], and the paper that deals with practical optimization of cylindrical gears [3].

## 2. OPTIMIZATION WITH EXCEL SOLVER

Optimization problems are real-world problems that we encounter on a daily basis. These problems are often related to areas such as mathematics, engineering, science, business, economics, and accounting. The goal is to find the optimal or most efficient solution for use in a specific sphere for which optimization is being performed. Optimization achieves various benefits such as increasing profits, reducing costs, extending the lifespan of certain products, minimizing the overall time to complete a particular project, and so on. A mathematical model must be created to observe the situation for a given problem best and perform optimization in the most efficient way possible [4]. The model consists of the following:

1. Objective function, [5]

$$\max \{f(x)\} = -\min\{-f(x)\}$$

2. Constraints, [5]

$$g_i(x) = 1; \quad i=1,2,m_i$$

3. Decision variables, [5]

$$x = (x_1, x_2, \dots, x_n)$$

For the purpose of this study and optimization application for reducing the weight of gears, a tool called Excel Solver was used, which is a part of the Microsoft Excel program. It is often used for optimization purposes and is widely used because it is the most accessible tool of its kind. Creating and defining a mathematical model is done very simply, as shown in Fig 1.

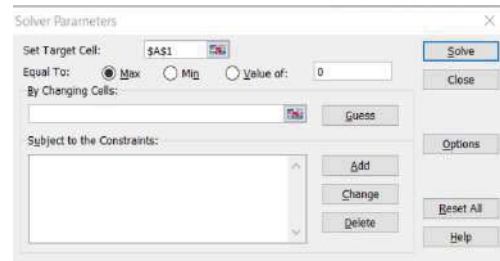


Fig. 1. The environment of the Excel Solver tool

The most important role in the overall optimization process is satisfying the objective function, primarily in a way that the obtained results are within the appropriate range of constraints. The optimization process is not a simple one-way process [5] because, in most cases, many conditions that are set can be unclear, ambiguous, or inaccessible. Due to a lack of information, this can significantly hinder the designer's ability to arrive at optimal solutions. Optimization requires precise definition in any sense to be calculated as accurately as possible for specific parameters. The purpose of optimization in technology is to reduce the time required to find a satisfactory solution. In this regard, the idea is to solve optimization problems that are already available, as well as those similar to them, but with minor differences, and to set new optimization problems using already developed models. In other words, everything that is available should be considered and used individually or collectively for new mathematical models [3].

## 3. EXAMPLE

This paper analyzes parameters influencing the minimization of mass, aiming to improve the solution by applying Excel Solver on the same optimization model as in the previously published paper by V. Savsani [1]. Figure 2 shows the models based on the obtained results in the paper to be analyzed, where the larger gear has 25 teeth.



Fig. 2. Gear with 25 teeth

The influential parameters for this study are

$x(b, d_1, d_2, m, z_1)$ , where since this is about minimizing mass, it is obtained as:

$$\text{Weight} = F(x) = (\pi\rho/4000) * [bm^2Z_1^2(1 + a^2) - (D_1^2 - d_0^2) * (1 - b_w) - nd_p^2b_w - (d_1^2 - d_2^2)b] \text{ [kg]} \quad [1]$$

The parameters must be within the range:

$$20 \leq b \leq 32$$

$$10 \leq d_1 \leq 30$$

$$30 \leq d_2 \leq 40$$

$$18 \leq z_1 \leq 25$$

$$m=(2.5, 2.75, 3, 3.5, 4)$$

While the constraints must range between [1]:

$$g_1(x) = F_s \geq b_1$$

$$g_2(x) = (F_s/F_p) \geq b_2$$

$$g_3(x) = d_1^3 \geq b_3$$

$$g_4(x) = d_2^3 \geq b_4$$

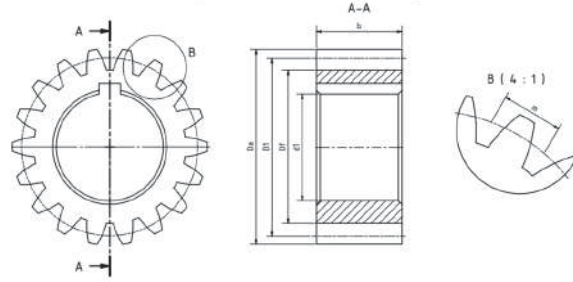
Given the calculated values shown in Table 1, where:

$$a = 4, \rho = 8, P = 7.5, n = 6, \sigma = 294.3, y = 0.102, b_2 = 0.193, \tau = 19.62, K_w = 0.8, K_v = 0.389, H = 7.5 \quad [6]$$

**Table 1.** Calculated parameter for optimization [1] [4]

exp No.	Needed dimensions	Calculated dimensions
1	$D_r = m(aZ_1 - 2.5)$	191,125
2	$l_w = 2.5m$	6,875
3	$D_i = D_r - 2l_w$	177,375
4	$b_w = 3.5m$	9,625
5	$d_0 = d_2 + 25$	55
6	$d_p = 0.25(D_i - d_0)$	30,593
7	$D_1 = mZ_1$	49,5
8	$D_2 = amZ_1$	198
9	$N_2 = N_1/a$	375
10	$Z_2 = Z_1D_2/D_1$	72
11	$\vartheta = \pi D_1 N_1 / 60.000$	3,887
12	$b_1 = 1000P/\vartheta$	1929,150
13	$b_3 = (48.68e6P)/(N_1\tau)$	0,202
14	$b_4 = (48.68e6P)/(N_2\tau)$	0,809
15	$F_s = \pi K_v K_w \sigma b m y$	1936,97
16	$F_p = (\pi K_v K_w D_1 b Z_2)/(Z_1 + Z_2)$	591,528

The Fig 3 shows some important parameters that affect the optimization of the gear.



**Fig. 3.** Parameters of gears

According to the obtained values in Table 1, which are necessary for calculating the mass from the main form and the set constraints, the results shown in Table 2 are obtained.

**Table 2.** Results calculated in this paper

Exp no.	Range of values	Calculated values	Constraints
m	2-2.5	2.5	0.145313
b	20-32	28.92455	3.081529
D1	10 do 30	30	14594.29
D2	30 do 40	36.75119	15.1762
Z1	18 - 25	18	/

When the results obtained in this paper are compared to the best results obtained in the paper by V. Savsani (table 3), it can be observed that the proposed method achieved a significant reduction in the weight of the gear while satisfying all constraints.

**Table 3.** Results calculated in paper V.Savsani [1]

Exp no.	Range of values	Calculated values	Constraints
m	2-2.5	2	1193,652926
b	20-32	32	2,164660844
D1	10 do 30	30	14594,29154
D2	30 do 40	36.75119	8,7799E-08
Z1	18 - 25	25	/

Based on the presented tables, it can be concluded that the minimization of the gear was carried out with greater success, under the same constraints and parameter ranges in this study, as the obtained mass was 2830.96 kg, as shown in Fig. 4.

Fig 4 show a graphical representation of the obtained parameters and their comparison,

as well as the representation of the obtained results

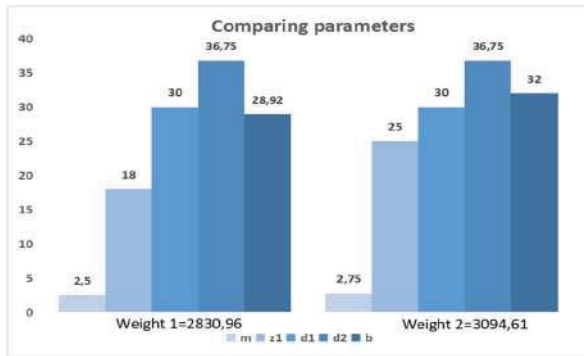


Fig. 4. Diagram of parameters from this research (left), from research [1] (right)

The gear model based on the obtained results of minimization is shown in Fig 5.



Fig. 5. Model of gears with 18 and 25 teehts

Figure 4 allows for a comparison of the appearance of the gear with 18 teeth (smaller gear) compared to the gear with a larger number of teeth.

#### 4. CONCLUSIONS

Considering the wide range of applications of gears, there is often a desire to improve them in various aspects. One of these aspects is the gear mass, which is examined in this paper, with the goal of minimizing it. The constraints that are set depend on strength, durability, bending, surface, etc., while the parameters being examined have the greatest impact on mass, such as module, width, number of teeth, and diameters. According to the optimization performed, the solutions obtained are better compared to the modified gear design carried out by V. Savsani, R. V. Rao, and D. P. Vakharia in their work [1]. Their analysis was conducted using two algorithms,

PSO and SA, but the results obtained and presented in this paper are significantly better.

Their obtained weight solutions are, in the best case, 3094.61[kg] . However, better solutions were obtained in this paper, with a mass of 2830.82[kg]at parameters:  $m=2.5$ ,  $b=28.9$ ,  $d_1=30$ ,  $d_2=36.756$ ,  $z_1=18$ , which is about 9.009% better. This shows that better results can be achieved using simpler optimization software such as Excel Solver. It can be concluded that, regardless of the optimization tool we use, the only important thing for successful optimization is to set appropriate constraints and examine all parameters to obtain the correct solution. This is just one optimization problem that could contribute to improvement in this field. However, there are still open problems that leave room for further exploration in the future.

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