Filomat 38:1 (2024), 211–220
https://doi.org/10.2298/FIL2401211M

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

A soft Dynkin system

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Abstract. Molodtsov defined the concept of a soft set, which is widely used in inference in case of vague, incomplete and imprecise information. The soft set theory became of interest to many researchers who defined new concepts such as soft measure, soft σ-algebra, soft premeasure, soft semiring, etc. In this paper, it is shown that starting from a soft premeasure defined on a soft semiring, we obtain a unique soft measure defined on a soft σ-algebra generated by a soft semiring. The terms soft $\sqcap$-stable, soft monotone class and soft Dynkin system were introduced as a necessary tool for realization.

1. Introduction

The field of soft set theory has been actively researched and developed in recent years. It was introduced by Molodtsov in 1999 (see [10] or [11]) as an alternative approach for dealing with uncertainty and imprecision in mathematical analysis. Soft sets provide a flexible framework for representing and processing uncertain, vague and incomplete information, making them a useful tool in a variety of fields such as computer science, decision making and pattern recognition (see [2], [12] and [14]).

Since its introduction, soft set theory has been widely adopted and expanded upon by researchers in various fields. Significant contributions have been made to the development of soft set theory (see [6], [7], [20] and [21]), including the introduction of new operations and algorithms (see [1], [3], [9], [16], [17], [19] and [21]), the extension of the theory to multi-valued and interval-valued soft sets, and the application of soft sets to real-world problems such as data mining, information retrieval and knowledge representation.

Overall, the field of soft set theory continues to evolve and grow, with new developments and applications being explored and proposed regularly. It remains an important and active area of research with the potential for significant impact in a wide range of domains.

One direction in the development of the soft set theory is the study of soft mappings such as soft measures and soft structures on which such mappings are defined. In the paper [9] Samanta and Majumdar introduced the notion of soft mappings and presented some of their properties, and measurable soft mappings were discussed in the paper [13], where some applications of soft set theory were also mentioned. The structure of the soft σ-algebra as well as the basic properties of such structure are presented in papers [4] and [13].

2020 *Mathematics Subject Classification*. Primary 06D72
Keywords. soft Dynkin system, soft monotone class, soft semiring, soft measure, soft premeasure
Received: 10 March 2023; Accepted: 02 July 2023
Communicated by Hari M. Srivastava
This work was supported by the Serbian Ministry of Education, Science and Technological Development (Agreement No. 451-03-47/2023-01/ 200122).
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The subject of research in paper [14] is soft measure and soft outer measure, where the mentioned terms are defined and the basic properties are examined, while in paper [18] terms like soft content and soft premeasure are defined. In the paper [5] the question was considered, whether a soft measure on a soft σ-algebra, with a certain extension, can be obtained by superimposing a mapping that is not a soft measure on a structure that is not a soft σ-algebra? The answer to the question is affirmative, and in the paper [5] it was shown that starting from the soft measure on the soft semiring, we can construct the soft measure on the obtained soft σ-algebra from the initial soft semiring. The question that naturally arises is under what conditions will such an expansion be unique?

With the aim of answering the imposed question, in this paper we define new collections of soft sets such as soft $\sqcap$-stable, soft monotone class and soft Dynkin system. The mentioned collections are helpful tool for answering some of the questions regarding the extension of soft premeasure to soft measure. Namely, in this paper we will prove that the extension of a soft premeasure, defined on a soft semiring, to a soft measure on a soft σ-algebra generated by the soft semiring is unique. The stated result represents the main result of the work, and in addition to it, the properties of the introduced collections and connections with collections such as soft σ-algebras were investigated.

2. Preliminaries

This section provides basic definitions and basic properties in soft set theory, as well as statements that are necessary in this paper and have been proven by many authors.

Let X be an initial universe set and E_X be the set of all possible parameters under consideration with respect to X. The power set of X is denoted by $\mathcal{P}(X)$ and A is a subset of E. Usually, parameters are attributes, characteristics, or properties of objects in X. In what follows, E_X (simply denoted by E) always means the universe set of parameters with respect to X, unless otherwise specified.

Definition 2.1. *[10] A pair (F, A) is called a soft set over X where $A \subseteq E$ and $F : A \to \mathcal{P}(X)$ is a set valued mapping. In other words, a soft set over X is a parameterized family of subsets of the universe X. For all $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A). It is worth noting that $F(e)$ may be arbitrary. Some of them may be empty, and some may have nonempty intersection.*

Definition 2.2. *[8] A soft set F_A on the universe X is defined by the set of ordered pairs $F_A = \{(e, f_A(e)) \mid e \in E, f_A(e) \in \mathcal{P}(X)\}$, where $f_A : E \to \mathcal{P}(X)$, such that $f_A(e) \neq \emptyset$ if $e \in A \subseteq E$ and $f_A(e) = \emptyset$, if $e \notin A$. Here, f_A is called an approximate function of the soft set F_A. The value of $f_A(e)$ may be arbitrary.*

Note that the set of all soft sets over X will be denoted by $\mathcal{S}(X, E)$.

Definition 2.3. *[2] Let $F_A \in \mathcal{S}(X, E)$. If $f_A(e) = \emptyset$, for all $e \in E$, then F_A is called an empty soft set, denoted by F_Φ or Φ. $f_A(e) = \emptyset$ means that there is no element in X related to the parameter $e \in E$. Therefore, we do not display such elements in the soft sets, as it is meaningless to consider such parameters.*

Definition 2.4. *[2] Let $F_A \in \mathcal{S}(X, E)$. If $f_A(e) = X$, for all $e \in A$, then F_A is called an A-universal soft set, denoted by $F_{\widetilde{A}} = \widetilde{A}$. If $A = E$, then the A-universal soft set is called a universal soft set, denoted by $F_{\widetilde{E}} = \widetilde{E}$.*

Definition 2.5. *[15] Let Y be a nonempty subset of X, then $\widetilde{Y}$ denotes the soft set Y_E over X for which $Y(e) = Y$, for all $e \in E$. In particular, X_E will be denoted by $\widetilde{X}$.*

Two soft sets can be compared in the following way.

Definition 2.6. *[2] Let $F_A, G_B \in \mathcal{S}(X, E)$. Then,*

1. *F_A is a soft subset of G_B, denoted by $F_A \sqsubseteq G_B$, if $f_A(e) \subseteq g_B(e)$, for all $e \in E$.*
2. *F_A and G_B are soft equal, denoted by $F_A = G_B$, if and only if $f_A(e) = g_B(e)$, for all $e \in E$.*

Operations defined on soft sets, as well as their properties, have been the subject of study by many researchers.

Definition 2.7. *[2] Let $F_A, G_B \in \mathcal{S}(X, E)$. Then,*

1. *the soft union $F_A \sqcup G_B$ of F_A and G_B is defined by the approximate function $h_{A\cup B}(e) = f_A(e) \cup g_B(e)$, for all $e \in E$.*
2. *the soft intersection $F_A \sqcap G_B$ of F_A and G_B is defined by the approximate function $h_{A\cap B}(e) = f_A(e) \cap g_B(e)$, for all $e \in E$.*
3. *the soft difference $F_A \backslash G_B$ of F_A and G_B is defined by the approximate function $h_{A\backslash B}(e) = f_A(e) \backslash g_B(e)$, for all $e \in E$.*
4. *the soft complement F_A^c is defined by the approximate function $f_{A^c}(e) = f_A^c(e)$, where $f_A^c(e)$ is the complement of the set $f_A(e)$, i.e. $f_A^c(e) = X \backslash f_A(e)$, for all $e \in E$.*

Definition 2.8. *[21] Let I be an arbitrary index set and let $\{(F_A)_i\}_{i\in I}$ be a subfamily of $\mathcal{S}(X, E)$.*

- *The union of these soft sets is the soft set G_C, where $g_C(e) = \cup_{i\in I}(F_A)_i(e)$, for all $e \in E$. We write $G_C = \sqcup_{i\in I}(F_A)_i$.*
- *The intersection of these soft sets is the soft set H_D, where $h_D(e) = \cap_{i\in I}(F_A)_i(e)$, for all $e \in E$. We write $H_D = \sqcap_{i\in I}(F_A)_i$.*

As in the classical set theory, so in the soft set theory, a significant place takes the study of special collections of sets, i.e. a collection of sets with specific properties. For studing soft measure it is necessary to define collections of soft sets like soft semiring, soft σ-algebra, etc.

Definition 2.9. *[4] A collection $\widetilde{\mathcal{A}}$ of soft subsets of $\widetilde{X}$ is called a soft σ-algebra on $\widetilde{X}$ if and only if it satisfies the following conditions*

- $\Phi \in \widetilde{\mathcal{A}}$,
- *if $F_A \in \widetilde{\mathcal{A}}$, then $F_A^c = \widetilde{X} \backslash F_A \in \widetilde{\mathcal{A}}$,*
- *if $(F_A)_1, (F_A)_2, (F_A)_3 \ldots$ is a countable collection of soft sets in $\widetilde{\mathcal{A}}$, then $\bigsqcup_{i=1}^{\infty}(F_A)_i \in \widetilde{\mathcal{A}}$.*

  The pair $(\widetilde{X}, \widetilde{\mathcal{A}})$ is called a soft measurable space and $(F_A)_i \in \widetilde{\mathcal{A}}$ is called a measurable soft set.

As well as the classical set theory, the soft set theory defines the notion of measure [14], called soft measure.

Definition 2.10. *[14] Let $\widetilde{\mathcal{A}}$ be a soft σ-algebra of soft subsets of $\widetilde{X}$ and $\widetilde{\mu}$ be an extended soft real-valued mapping on $\widetilde{\mathcal{A}}$. Then $\widetilde{\mu}$ is called a soft measure on $\widetilde{\mathcal{A}}$, if*

- $\widetilde{\mu}(\Phi) = 0$,
- $\widetilde{\mu}(F_A) \geqslant 0$ *for each $F_A \in \widetilde{\mathcal{A}}$,*
- *$\widetilde{\mu}$ is countably soft additive, i.e.*

$$\widetilde{\mu}\left(\bigsqcup_{i=1}^{\infty}(F_A)_i\right) = \sum_{i=1}^{\infty}\widetilde{\mu}((F_A)_i),$$

  $(F_A)_i$'s being pairwise soft disjoint.

  If $\widetilde{\mu}$ is a soft measure on a soft σ-algebra $\widetilde{\mathcal{A}}$, then the triplet $(\widetilde{X}, \widetilde{\mathcal{A}}, \widetilde{\mu})$ is called a soft measure space.

Definition 2.11. *A collection $\widetilde{\mathcal{S}}$ of soft subsets of $\widetilde{X}$ is called a soft semiring on $\widetilde{X}$ if and only if it satisfies the following conditions*

- *$\Phi \in \widetilde{\mathcal{S}}$,*
- *if $F_A, G_B \in \widetilde{\mathcal{S}}$, then $F_A \sqcap G_B \in \widetilde{\mathcal{S}}$,*
- *if $F_A, G_B \in \widetilde{\mathcal{S}}$, then there exist soft disjoint $(C_H)_1, \dots, (C_H)_n \in \widetilde{\mathcal{S}}$, $n \in \mathbb{N}$, such that*

$$F_A \backslash G_B = \bigsqcup_{i=1}^{n} (C_H)_i.$$

In the paper [18], besides considering the terms of soft measure and soft outer measure, the terms of soft content and soft premeasure are defined. Also, some claims that are necessary for us to realize our idea have been proven.

3. A Soft Dynkin System

Observing the following simple example, we see that we cannot expect the extension of the soft premeasure $\widetilde{\mu}$ over the soft semiring $\widetilde{\mathcal{S}}$, to the soft measure defined on $\sigma(\widetilde{\mathcal{S}})$, to be unique.

Example 3.1. *Let $\widetilde{X} \neq \Phi$ be a soft set and let $\widetilde{\mathcal{S}} = \{\Phi\}$. It is clear that $\widetilde{\mathcal{S}}$ is a soft semiring, moreover $\widetilde{\mathcal{S}}$ is a soft ring on $\widetilde{X}$. Let $\widetilde{\mu} : \widetilde{\mathcal{S}} \to [0, \infty]$, $\widetilde{\mu}(\Phi) = 0$, so $\widetilde{\mu}$ is a soft premeasure. Then $\widetilde{\mathcal{A}} = \sigma(\widetilde{\mathcal{S}}) = \{\Phi, \widetilde{X}\}$ and for all $\alpha \in [0, \infty]$ the mapping $\widetilde{\mu}_\alpha : \widetilde{\mathcal{A}} \to [0, \infty]$, given by*

$$\widetilde{\mu}_\alpha(F_A) = \begin{cases} 0, & F_A = \Phi, \\ \alpha, & F_A = \widetilde{X}, \end{cases}$$

represents the extension of the soft premeasure $\widetilde{\mu}$ into the soft measure defined on $\widetilde{\mathcal{A}}$.

The problem in the previous example is that the soft measure $\widetilde{\mu}$ on the soft semiring $\widetilde{\mathcal{S}}$ is not a soft σ-finite. As a preparation for the proof of the uniqueness of the extension (proof of Theorem 3.13), we define some additional technical (auxiliary) tools. The introduced tools can be of great use in soft measure theory, not only as tools for proving the uniqueness of extensions.

Before the actual introduction of new terms, i.e. before the definition of certain classes of soft sets, let's introduce the following labels. Let F_A and $(F_A)_n$, $n \in \mathbb{N}$ be a soft sets. Let

$$(F_A)_n \uparrow F_A \Leftrightarrow \left(F_A = \bigsqcup_{n=1}^{\infty} (F_A)_n \ \wedge \ (F_A)_1 \sqsubseteq (F_A)_2 \sqsubseteq (F_A)_3 \sqsubseteq \dots \right),$$

$$(F_A)_n \downarrow F_A \Leftrightarrow \left(F_A = \bigsqcap_{n=1}^{\infty} (F_A)_n \ \wedge \ (F_A)_1 \sqsupseteq (F_A)_2 \sqsupseteq (F_A)_3 \sqsupseteq \dots \right).$$

Definition 3.2. *A collection $\widetilde{\mathcal{E}}$ of soft subsets of $\widetilde{X}$ is called*

(i) a soft $\sqcap$-stable if and only if $(\forall F_A, F_B \in \widetilde{\mathcal{E}})\ F_A \sqcap F_B \in \widetilde{\mathcal{E}}$.

(ii) a soft monotone class on $\widetilde{X}$ if and only if it satisfies the following conditions

1. *if $F_A \sqsubseteq \widetilde{X}$ and $(F_A)_n$, $n \in \mathbb{N}$ is a sequence in $\widetilde{\mathcal{E}}$ such that $(F_A)_n \uparrow F_A$, then $F_A \in \widetilde{\mathcal{E}}$,*
2. *if $F_A \sqsubseteq \widetilde{X}$ and $(F_A)_n$, $n \in \mathbb{N}$ is a sequence in $\widetilde{\mathcal{E}}$ such that $(F_A)_n \downarrow F_A$, then $F_A \in \widetilde{\mathcal{E}}$.*

(iii) a soft Dynkin system on $\widetilde{X}$ if and only if it satisfies the following conditions

1. *$\widetilde{X} \in \widetilde{\mathcal{E}}$,*

2. *if* $F_A \in \widetilde{\mathcal{E}}$, *then* $(F_A)^c \in \widetilde{\mathcal{E}}$,
3. *if* $((F_A)_n)_{n\in\mathbb{N}}$ *is a sequence of disjoint soft set in* $\widetilde{\mathcal{E}}$, *then* $\bigsqcup_{n=1}^{\infty}(F_A)_n \in \widetilde{\mathcal{E}}$.

Example 3.3. *Let* $X = \{h_1, h_2, h_3\}$ *and* $E = \{e_1, e_2\}$.

(*i*) *Let* $\widetilde{\mathcal{E}}_1 = \{(F_A)_i \mid i = 1, 2, 3\}$ *be a collection, where*
$(F_A)_1 = \Phi$,
$(F_A)_2 = \{(e_1, \{h_1\}), (e_2, \emptyset)\}$,
$(F_A)_3 = \{(e_1, \{h_1, h_2, h_3\}), (e_2, \{h_1, h_2\})\}$.
It is not difficult to see that the collection $\widetilde{\mathcal{E}}_1$ *is a soft* $\sqcap$*-stable on* $\widetilde{X}$.

(*ii*) *Let* $\widetilde{\mathcal{E}}_2 = \{(F_A)_n \mid n \in \mathbb{N}\}$ *be a collection, where*
$(F_A)_1 = \Phi$,
$(F_A)_2 = \{(e_1, \{h_1\}), (e_2, \emptyset)\}$,
$(F_A)_3 = \{(e_1, \{h_1, h_2, h_3\}), (e_2, \{h_1, h_2\})\}$, *and for* $n = 4, 5, ...$ *let* $(F_A)_n = (F_A)_3$.
Then, $(F_A)_n \uparrow F_A$, *where* $F_A = \{(e_1, \{h_1, h_2, h_3\}), (e_2, \{h_1, h_2\})\}$, *and since* $F_A = (F_A)_3$ *we also have that* $F_A \in \widetilde{\mathcal{E}}_2$, *so* $\widetilde{\mathcal{E}}_2$ *is a soft monotone class.*

Similarly, let $\widetilde{\mathcal{E}}_3 = \{(G_B)_n \mid n \in \mathbb{N}\}$ *be a collection, where*
$(G_B)_1 = \{(e_1, \{h_1, h_2, h_3\}), (e_2, \{h_1, h_2\})\}$,
$(G_B)_2 = \{(e_1, \{h_1\})\}$,
$(G_B)_3 = \Phi$, *and for* $n = 4, 5, ...$ *let* $(G_B)_n = (G_B)_3$.
Then, $(G_B)_n \downarrow G_B$, *where* $G_B = \{(e_1, \{h_1, h_2, h_3\}), (e_2, \{h_1, h_2\})\}$, *and since* $G_B = (G_B)_3$ *we also have that* $G_B \in \widetilde{\mathcal{E}}_3$, *so* $\widetilde{\mathcal{E}}_3$ *is a soft monotone class.*

(*iii*) *Let's consider the collection* $\widetilde{\mathcal{E}} = \{(F_A)_i \mid i = 1, 2, \ldots, 8\}$, *where*
$(F_A)_1 = \Phi$,
$(F_A)_2 = \{(e_1, \{h_1\}), (e_2, \{h_2\})\}$,
$(F_A)_3 = \{(e_1, \{h_2\}), (e_2, \{h_1\})\}$,
$(F_A)_4 = \{(e_1, \{h_3\}), (e_2, \{h_3\})\}$,
$(F_A)_5 = \{(e_1, \{h_2, h_3\}), (e_2, \{h_1, h_3\})\}$,
$(F_A)_6 = \{(e_1, \{h_1, h_3\}), (e_2, \{h_2, h_3\})\}$,
$(F_A)_7 = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\}$,
$(F_A)_8 = \widetilde{X}$.
Then, $\widetilde{\mathcal{E}}$ *is a soft Dynkin system over* $\widetilde{X}$.

Theorem 3.4. *The soft intersection of any collection of soft monotone classes on* $\widetilde{X}$ *forms again a soft monotone class on* $\widetilde{X}$.

Proof. Let $\{\widetilde{\mathcal{E}}_i\}_{i\in I}$ be any collection of soft monotone classes over $\widetilde{X}$. Let $\widetilde{\mathcal{E}} = \sqcap_{i\in I}\widetilde{\mathcal{E}}_i$, and let $F_A \sqsubseteq \widetilde{X}$ be an arbitrary soft set. We'll prove that $F_A \in \widetilde{\mathcal{E}}$.

Let $(F_A)_n$, $n \in \mathbb{N}$ be a sequence in $\widetilde{\mathcal{E}}$ such that $(F_A)_n \uparrow F_A$. Since $\widetilde{\mathcal{E}} = \sqcap_{i\in I}\widetilde{\mathcal{E}}_i$, then $(F_A)_n$, $n \in \mathbb{N}$ is a sequence in $\widetilde{\mathcal{E}}_i$ such that $(F_A)_n \uparrow F_A$, for all $i \in I$. We have that $\widetilde{\mathcal{E}}_i$ are soft monotone classes for all $i \in I$, so we conclude that $F_A \in \widetilde{\mathcal{E}}_i$, for all $i \in I$, i.e. $F_A \in \sqcap_{i\in I}\widetilde{\mathcal{E}}_i$. Therefore, the collection $\widetilde{\mathcal{E}}$ is a soft monotone class. □

Theorem 3.5. *The soft intersection of any collection of soft Dynkin systems on* $\widetilde{X}$ *forms again a soft Dynkin system on* $\widetilde{X}$.

Proof. Let $\{\widetilde{\mathcal{E}}_i\}_{i\in I}$ be any collection of soft Dynkin systems on $\widetilde{X}$ and let $\widetilde{\mathcal{E}} = \bigsqcap_{i\in I}\widetilde{\mathcal{E}}_i$.

We know that $\widetilde{X} \in \widetilde{\mathcal{E}}_i$, for all $i \in I$, because $\widetilde{\mathcal{E}}_i$ are all soft Dynkin systems. Hence, $\widetilde{X} \in \widetilde{\mathcal{E}}$, which needed to be proven.

If $F_A \in \widetilde{\mathcal{E}}$, then $F_A \in \widetilde{\mathcal{E}}_i$, for all $i \in I$, and so $F_A^c \in \widetilde{\mathcal{E}}_i$, also because $\widetilde{\mathcal{E}}_i$ are all soft Dynkin systems. Thus, $F_A^c \in \widetilde{\mathcal{E}}$.

Finally, if $(F_A)_n \in \widetilde{\mathcal{E}}$, for all $n \in \mathbb{N}$, then $(F_A)_n \in \widetilde{\mathcal{E}}_i$ for all n, i, and $\bigsqcup_{n=1}^{\infty}(F_A)_n \in \widetilde{\mathcal{E}}_i$, for all i. This implies that $\bigsqcup_{n=1}^{\infty}(F_A)_n \in \widetilde{\mathcal{E}}$.

Therefore, the collection $\widetilde{\mathcal{E}}$ is a soft Dynkin system. □

Let $\widetilde{X}$ be a soft set and let $\widetilde{\mathcal{E}}$ be the collection of soft subsets of $\widetilde{X}$. By $\mu(\widetilde{X})$ we denote the soft intersection of all soft monotone classes over $\widetilde{X}$, such that $\widetilde{\mathcal{E}}$ is their subset. Similary, by $\delta(\widetilde{X})$ we denote the soft intersection of all soft Dynkin systems over $\widetilde{X}$, such that $\widetilde{\mathcal{E}}$ is their subset. Based on the previous two theorems, we know that $\mu(\widetilde{X})$ is a soft monotone class, and $\delta(\widetilde{X})$ is a soft Dynkin system.

Definition 3.6. *We say that the soft monotone class $\mu(\widetilde{X})$ is the smallest soft monotone class over $\widetilde{X}$ containing $\widetilde{\mathcal{E}}$ and we call it the soft monotone class generated by $\widetilde{\mathcal{E}}$.*

Definition 3.7. *We say that the soft Dynkin system $\delta(\widetilde{X})$ is the smallest soft Dynkin system over $\widetilde{X}$ containing $\widetilde{\mathcal{E}}$ and we call it the soft Dynkin system generated by $\widetilde{\mathcal{E}}$.*

Theorem 3.8. *Let $\widetilde{X}$ be a soft set and let $\widetilde{\mathcal{E}}$ be the collection of soft subsets of $\widetilde{X}$. Then $\widetilde{\mathcal{E}}$ is a soft Dynkin system on $\widetilde{X}$ if and only if the following conditions (i)-(iii) hold*

(i) $\widetilde{X} \in \widetilde{\mathcal{E}}$.

(ii) For all $F_A, G_B \in \widetilde{\mathcal{E}}$, $G_B \sqsubseteq F_A$ implies $F_A \setminus G_B \in \widetilde{\mathcal{E}}$.

(iii) $\widetilde{\mathcal{E}}$ is a soft monotone class.

Proof. ($\leftarrow$) Let conditions (*i*) – (*iii*) hold for the collection $\widetilde{\mathcal{E}}$. Let's prove that the collection of soft sets $\widetilde{\mathcal{E}}$ is a soft Dynkin system. How property 1. (part (*iii*)) of Definition 3.2. holds, it's necessary to prove properties 2. and 3.

Let $G_B \in \widetilde{\mathcal{E}}$ be an arbitrary soft set, and we know that $\widetilde{X} \in \widetilde{\mathcal{E}}$. Using the condition (*ii*) of the theorem, follows that $\widetilde{X} \setminus G_B \in \widetilde{\mathcal{E}}$, i.e. $(G_B)^c \in \widetilde{\mathcal{E}}$.

Let $((F_A)_i)_{i\in\mathbb{N}}$ be a sequence of soft disjoint sets from the collection $\widetilde{\mathcal{E}}$. Consider the soft sets $(F_A)_1, (F_A)_2 \in \widetilde{\mathcal{E}}$. Then $(F_A)_1 \sqcup (F_A)_2 \in \widetilde{\mathcal{E}}$, because $(F_A)_2 \sqsubseteq (F_A)_1^c$, and further $(F_A)_1^c \setminus (F_A)_2 \in \widetilde{\mathcal{E}}$ based on the condition (*ii*). Since the collection $\widetilde{\mathcal{E}}$ is closed for soft complements, we have that $((F_A)_1^c \setminus (F_A)_2)^c = (F_A)_1 \sqcup (F_A)_2 \in \widetilde{\mathcal{E}}$.

Now, we show by mathematical induction that for all $n \in \mathbb{N}$ holds $(G_B)_n = \bigsqcup_{i=1}^{n}(F_A)_i \in \widetilde{\mathcal{E}}$.

Proof by induction on n. The base case holds, since $(G_B)_1 = (F_A)_1 \in \widetilde{\mathcal{E}}$. For the induction step, let be valid that $(G_B)_n = \bigsqcup_{i=1}^{n}(F_A)_i \in \widetilde{\mathcal{E}}$, for some fixed $n \in \mathbb{N}$.

Let's prove that $(G_B)_{n+1} = \bigsqcup_{i=1}^{n+1}(F_A)_i = (\bigsqcup_{i=1}^{n}(F_A)_i) \sqcup (F_A)_{n+1} \in \widetilde{\mathcal{E}}$.

Based on the induction step, we have that $\bigsqcup_{i=1}^{n}(F_A)_i \in \widetilde{\mathcal{E}}$ and also $(F_A)_{n+1} \in \widetilde{\mathcal{E}}$. It is shown that the collection $\widetilde{\mathcal{E}}$ is closed for the soft union of two soft sets, so $(G_B)_{n+1} \in \widetilde{\mathcal{E}}$, and that's what should have been shown.

Hence, for all $n \in \mathbb{N}$, holds $(G_B)_n = \bigsqcup_{i=1}^{n}(F_A)_i \in \widetilde{\mathcal{E}}$.

So, we have that $(G_B)_n \uparrow G_B = \bigsqcup_{i=1}^{\infty}(F_A)_i$, so then $G_B = \bigsqcup_{i=1}^{\infty}(F_A)_i \in \widetilde{\mathcal{E}}$ based on condition (*iii*).

Finally, based on the proven properties, we conclude that $\widetilde{\mathcal{E}}$ is a soft Dynkin system.

($\rightarrow$) Let the collection $\widetilde{\mathcal{E}}$ be a soft Dynkin system, it suffices to show the conditions (*ii*) and (*iii*). Let $F_A, G_B \in \widetilde{\mathcal{E}}$ such that $G_B \sqsubseteq F_A$, then the soft sets $(F_A)^c, G_B, \Phi, \Phi, ...$ are soft disjoint soft sets of the collection $\widetilde{\mathcal{E}}$. Using the property 3. (part (*iii*)) of Definition 3.2. we have that $(F_A)^c \sqcup G_B \sqcup \Phi \sqcup \Phi \sqcup ... \in \widetilde{\mathcal{E}}$, i.e. $F_A \setminus G_B \in \widetilde{\mathcal{E}}$, so the property (*ii*) holds.

Let $F_A \sqsubseteq \widetilde{X}$ and let $((F_A)_i)_{i\in\mathbb{N}}$ be a sequence of soft sets from the collection $\widetilde{\mathcal{E}}$. If $(F_A)_i \uparrow F_A$, then $F_A = (F_A)_1 \sqcup (\bigsqcup_{i=2}^{\infty}((F_A)_i \setminus (F_A)_{i-1})) \in \widetilde{\mathcal{E}}$. And if $(F_A)_i \downarrow F_A$, then $(F_A)^c = \bigsqcup_{i=1}^{\infty}(F_A)_i^c$, i.e. $(F_A)_i^c \uparrow (F_A)^c \in \widetilde{\mathcal{E}}$. Hence, $F_A \in \widetilde{\mathcal{E}}$, so $\widetilde{\mathcal{E}}$ is a soft monotone class, and the proof of the theorem is complete. □

Theorem 3.9. *Let $\widetilde{X}$ be a soft set and let $\widetilde{\mathcal{E}}$ be a soft Dynkin system over $\widetilde{X}$. Then the following statements are equivalent*

(i) $\widetilde{\mathcal{E}}$ *is a soft $\sqcap$-stable.*

(ii) $\widetilde{\mathcal{E}}$ *is a soft σ-algebra.*

Proof. As every soft σ-algebra is soft $\sqcap$-stable, it only remains to show that (i) implies (ii).

If $F_A, G_B \in \widetilde{\mathcal{E}}$ and $\widetilde{\mathcal{E}}$ is soft $\sqcap$-stable, then $F_A \sqcup G_B = (F_A \sqcap G_B^c) \sqcup (G_B \sqcap F_A^c) \sqcup (F_A \sqcap G_B) \in \widetilde{\mathcal{E}}$.

Let $((F_A)_n)_{n\in\mathbb{N}}$ be a sequence of soft sets in $\widetilde{\mathcal{E}}$. By induction on $n \in \mathbb{N}$ we prove that $(G_B)_n = \bigsqcup_{i=1}^{n}(F_A)_i \in \widetilde{\mathcal{E}}$ for all $n \in \mathbb{N}$.

For $n = 1$, trivially, holds $(G_B)_1 = \bigsqcup_{i=1}^{1}(F_A)_i = (F_A)_1 \in \widetilde{\mathcal{E}}$.

Suppose that the statement is valid for some fixed natural number n, i.e. $(G_B)_n = \bigsqcup_{i=1}^{n}(F_A)_i \in \widetilde{\mathcal{E}}$.

We prove that $(G_B)_{n+1} = \bigsqcup_{i=1}^{n+1}(F_A)_i \in \widetilde{\mathcal{E}}$. Indeed, $(G_B)_{n+1} = (\bigsqcup_{i=1}^{n}(F_A)_i) \sqcup (F_A)_{n+1}$, and as $\bigsqcup_{i=1}^{n}(F_A)_i \in \widetilde{\mathcal{E}}$ and $(F_A)_{n+1} \in \widetilde{\mathcal{E}}$, based on the property shown above, $(G_B)_{n+1} \in \widetilde{\mathcal{E}}$, which should have been shown.

Since $(G_B)_n \uparrow G_B = \bigsqcup_{i=1}^{\infty}(F_A)_i$ and the soft Dynkin system $\widetilde{\mathcal{E}}$ is a soft monotone class, $G_B \in \widetilde{\mathcal{E}}$, showing $\widetilde{\mathcal{E}}$ to be a soft σ-algebra. □

Example 3.10. *Let $X = \{h_1, h_2, h_3\}$ and $E = \{e_1, e_2\}$. Let $\widetilde{\mathcal{E}} = \{(F_A)_i \mid i = 1, 2, \dots, 6\}$ be a collection, where*
$(F_A)_1 = \Phi$,
$(F_A)_2 = \{(e_1, \{h_1\}), (e_2, \{h_2\})\}$,
$(F_A)_3 = \{(e_1, \{h_1\}), (e_2, \{h_1\})\}$,
$(F_A)_4 = \{(e_1, \{h_2, h_3\}), (e_2, \{h_1, h_3\})\}$,
$(F_A)_5 = \{(e_1, \{h_2, h_3\}), (e_2, \{h_2, h_3\})\}$,
$(F_A)_6 = \widetilde{X}$.

The collection $\widetilde{\mathcal{E}}$ is a soft Dynkin system, but it is neither a soft σ-algebra nor a soft $\sqcap$-stable. However, in example 3.3. we have a soft Dynkin system in which both conditions of Theorem 3.9. hold.

Theorem 3.11. *Let $\widetilde{X}$ be soft set and let $\widetilde{\mathcal{E}}$ be soft $\sqcap$-stable over $\widetilde{X}$. Then $\delta(\widetilde{\mathcal{E}}) = \sigma(\widetilde{\mathcal{E}})$.*

Proof. Since every soft σ-algebra is a soft Dynkin system, holds that $\widetilde{\mathcal{D}} = \delta(\widetilde{\mathcal{E}}) \sqsubseteq \sigma(\widetilde{\mathcal{E}})$.

We will prove that $\widetilde{\mathcal{D}}$ is a soft σ-algebra. Based on the Theorem 3.9. it suffices to show that $\widetilde{\mathcal{D}}$ is soft $\sqcap$-stable.

For all $G_B \in \widetilde{\mathcal{D}}$ notice the collection of soft sets

$$\widetilde{\mathcal{D}}_{G_B} = \{F_A \sqsubseteq \widetilde{X} \mid F_A \sqcap G_B \in \widetilde{\mathcal{D}}\}.$$

We prove that for all G_B over $\widetilde{\mathcal{D}}$ holds

$$\widetilde{\mathcal{D}} \sqsubseteq \widetilde{\mathcal{D}}_{G_B}.$$

First, let us prove that, for all G_B , $\widetilde{\mathcal{D}}_{G_B}$ is a soft Dynkin system.

Indeed, $\widetilde{X} \in \widetilde{\mathcal{D}}_{G_B}$, since $\widetilde{X} \sqcap G_B = G_B \in \widetilde{\mathcal{D}}$. Further, if $F_A \in \widetilde{\mathcal{D}}_{G_B}$, then $F_A^c \in \widetilde{\mathcal{D}}_{G_B}$, following from the fact that $F_A \sqcap G_B \in \widetilde{\mathcal{D}}$ and $F_A^c \sqcap G_B = G_B \setminus (F_A \sqcap G_B) \in \widetilde{\mathcal{D}}$ using property (ii) of the Theorem 3.8.

Let $((F_A)_n)_{n\in\mathbb{N}}$ be a sequence of disjoint soft sets over $\widetilde{\mathcal{D}}_{G_B}$ and let $F_A = \bigsqcup_{n=1}^{\infty}(F_A)_n$. Then, $((F_A)_n \sqcap G_B)_{n\in\mathbb{N}}$ is a sequence of disjoint soft sets over $\widetilde{\mathcal{D}}$, and because $\widetilde{\mathcal{D}}$ is a soft Dynkin system, holds $F_A \sqcap G_B = \bigsqcup_{n=1}^{\infty}((F_A)_n \sqcap G_B) \in \widetilde{\mathcal{D}}$. Hence, we concluded that $\bigsqcup_{n=1}^{\infty}(F_A)_n \in \widetilde{\mathcal{D}}_{G_B}$, so the proof that the sequence $\widetilde{\mathcal{D}}_{G_B}$ is a soft Dynkin system is completed.

Since $\widetilde{\mathcal{E}}$ is a soft $\sqcap$-stable, then $\widetilde{\mathcal{E}} \sqsubseteq \widetilde{\mathcal{D}}_{H_D}$, for all $H_D \in \widetilde{\mathcal{E}}$, where

$$\widetilde{\mathcal{D}}_{H_D} = \{F_A \sqsubseteq \widetilde{X} \mid F_A \sqcap H_D \in \widetilde{\mathcal{D}}\}.$$

Hence, for all $H_D \in \widetilde{\mathcal{E}}$

$$\widetilde{\mathcal{D}} \sqsubseteq \widetilde{\mathcal{D}}_{H_D},$$

as $\widetilde{\mathcal{D}}_{H_D}$ is a soft Dynkin system.

Thus, if $H_D \in \widetilde{\mathcal{E}}$ and if $G_B \in \widetilde{\mathcal{D}}$ is arbitrary, then $G_B \in \widetilde{\mathcal{D}}_{H_D}$ i.e. $G_B \sqcap H_D \in \widetilde{\mathcal{D}}$, which means that $H_D \in \widetilde{\mathcal{D}}_{G_B}$, and further $\widetilde{\mathcal{E}} \sqsubseteq \widetilde{\mathcal{D}}_{G_B}$.

Thus, for all $G_B \in \widetilde{\mathcal{D}}$, $\widetilde{\mathcal{D}} \sqsubseteq \widetilde{\mathcal{D}}_{G_B}$.

Finally, let's check if the collection $\widetilde{\mathcal{D}}$ is closed for soft intersection. Namely, let $F_A, G_B \in \widetilde{\mathcal{D}}$. Then $F_A \in \widetilde{\mathcal{D}}_{G_B}$. Based of the structure of the collection $\widetilde{\mathcal{D}}_{G_B}$, holds $F_A \sqcap G_B \in \widetilde{\mathcal{D}}$. □

Example 3.12. *Let $X = \{h_1, h_2, h_3\}$ and $E = \{e_1, e_2\}$. Let $\widetilde{\mathcal{E}} = \{(F_A)_i \mid i = 1, 2, 3, 4\}$ be a collection, where*

$(F_A)_1 = \Phi$,
$(F_A)_2 = \{(e_1, \{h_1\}), (e_2, \emptyset)\}$,
$(F_A)_3 = \{(e_1, \{h_1, h_3\}), (e_2, \{h_1, h_2, h_3\})\}$,
$(F_A)_4 = \widetilde{X}$.

The collection $\widetilde{\mathcal{E}}$ is a soft $\sqcap$-stable.

Let's determine the collection $\delta(\widetilde{\mathcal{E}})$. As the collection $\widetilde{\mathcal{E}}$ is not a soft Dynkin system, we need to extend it to the collection $\delta(\widetilde{\mathcal{E}}) = \{(F_A)_i \mid i = 1, 2, \ldots, 8\}$, where

$(F_A)_5 = \{(e_1, \{h_2, h_3\}), (e_2, \{h_1, h_2, h_3\})\}$,
$(F_A)_6 = \{(e_1, \{h_2\}), (e_2, \emptyset)\}$,
$(F_A)_7 = \{(e_1, \{h_1, h_2\}), (e_2, \emptyset)\}$,
$(F_A)_8 = \{(e_1, \{h_3\}), (e_2, \{h_1, h_2, h_3\})\}$.

On the other hand, if we extend the collection $\widetilde{\mathcal{E}}$ to the soft σ-algebra, we get $\sigma(\widetilde{\mathcal{E}}) = \{(F_A)_i \mid i = 1, 2, \ldots, 8\}$.

Hence, $\delta(\widetilde{\mathcal{E}}) = \sigma(\widetilde{\mathcal{E}})$.

Theorem 3.13. *Let $(\widetilde{X}, \widetilde{\mathcal{A}})$ be a soft measurable space and let $\widetilde{\mu}, \widetilde{\nu} : \widetilde{\mathcal{A}} \to [0, \infty]$ be a soft measures. Let $\widetilde{\mathcal{E}}$ be a collection of soft subsets of $\widetilde{X}$ with the following properties*

(i) $\sigma(\widetilde{\mathcal{E}}) = \widetilde{\mathcal{A}}$,

(ii) $\widetilde{\mu} \upharpoonright_{\widetilde{\mathcal{E}}} = \widetilde{\nu} \upharpoonright_{\widetilde{\mathcal{E}}}$,

(iii) $\widetilde{\mathcal{E}}$ *is soft $\sqcap$-stable,*

(iv) there exists a sequence $((G_B)_i)_{i \in \mathbb{N}}$ of soft sets over $\widetilde{\mathcal{E}}$ such that

$$((\forall i \in \mathbb{N}) \widetilde{\mu}((G_B)_i) = \widetilde{\nu}((G_B)_i) < \infty) \quad \wedge \quad \widetilde{X} = \bigsqcup_{i=1}^{\infty} (G_B)_i.$$

Then $\widetilde{\mu} = \widetilde{\nu}$.

Proof. For all $G_B \in \widetilde{\mathcal{E}}$ such that $\widetilde{\mu}(G_B) = \widetilde{\nu}(G_B) < \infty$, let's show that

$$\widetilde{\mathcal{A}} = \widetilde{\mathcal{D}}_{G_B} = \{F_A \in \widetilde{\mathcal{A}} \mid \widetilde{\mu}(F_A \sqcap G_B) = \widetilde{\nu}(F_A \sqcap G_B)\}.$$

For all $G_B \in \widetilde{\mathcal{E}}$ such that $\widetilde{\mu}(G_B) = \widetilde{\nu}(G_B) < \infty$, $\widetilde{\mathcal{D}}_{G_B}$ is a soft Dynkin system, because of the following properties

- $\widetilde{X} \in \widetilde{\mathcal{D}}_{G_B}$, since $\widetilde{X} \sqcap G_B = G_B \in \widetilde{\mathcal{E}}$.
- If $F_A \in \widetilde{\mathcal{D}}_{G_B}$, then $F_A^c \in \widetilde{\mathcal{D}}_{G_B}$, since $\widetilde{\mu}((F_A)^c \sqcap G_B) = \widetilde{\mu}(G_B \backslash ((F_A) \sqcap G_B)) = \widetilde{\mu}(G_B) - \widetilde{\mu}(F_A \sqcap G_B) = \widetilde{\nu}(G_B) - \widetilde{\nu}(F_A \sqcap G_B) = \widetilde{\nu}(G_B \backslash ((F_A) \sqcap G_B)) = \widetilde{\nu}((F_A)^c \sqcap G_B)$.

- Let $((F_A)_i)_{i\in\mathbb{N}}$ be a sequence of disjoint soft sets over $\widetilde{\mathcal{D}}_{G_B}$. Then $F_A = \bigsqcup_{i=1}^{n}(F_A)_i \in \widetilde{\mathcal{D}}_{G_B}$, since $\widetilde{\mu}(F_A \sqcap G_B) = \widetilde{\mu}(\bigsqcup_{i=1}^{n}((F_A)_i \sqcap G_B)) = \sum_{i=1}^{n}\widetilde{\mu}((F_A)_i \sqcap G_B) = \sum_{i=1}^{n}\widetilde{\nu}((F_A)_i \sqcap G_B) = \widetilde{\nu}(\bigsqcup_{i=1}^{n}((F_A)_i \sqcap G_B)) = \widetilde{\nu}((F_A) \sqcap G_B)$.

Using the previous result and Teorem 3.11, we have that $\widetilde{\mathcal{A}} = \sigma(\widetilde{\mathcal{E}}) = \widetilde{\mathcal{D}} \sqsubseteq \widetilde{\mathcal{D}}_{G_B}$, and since it is obvious that $\widetilde{\mathcal{D}}_{G_B} \sqsubseteq \widetilde{\mathcal{A}}$, we have $\widetilde{\mathcal{A}} = \widetilde{\mathcal{D}}_{G_B}$.

Let $((G_B)_i)_{i\in\mathbb{N}}$ be a sequence from the condition (*iv*) and define a sequence of soft sets $(C_D)_0 = \Phi$, $(C_D)_n = \bigsqcup_{i=1}^{n}(G_B)_i$, for all $n \in \mathbb{N}$. Then $(C_D)_n \uparrow \widetilde{X}$. For all $n \in \mathbb{N}$, holds $(C_D)_n = \bigsqcup_{i=1}^{n}((G_B)_i \sqcap (C_D)_{i-1}^c)$, hence for all $n \in \mathbb{N}$, holds

$$(\forall F_A \in \widetilde{\mathcal{A}})\ \widetilde{\mu}(F_A \sqcap (C_D)_n) = \sum_{i=1}^{n}\widetilde{\mu}(F_A \sqcap (C_D)_{i-1}^c \sqcap (G_B)_i) = \sum_{i=1}^{n}\widetilde{\nu}(F_A \sqcap (C_D)_{i-1}^c \sqcap (G_B)_i) = \widetilde{\nu}(F_A \sqcap (C_D)_n)$$

and

$$(\forall F_A \in \widetilde{\mathcal{A}})\ \widetilde{\mu}(F_A) = \lim_{x\to\infty}\widetilde{\mu}(F_A \sqcap (C_D)_n) = \lim_{x\to\infty}\widetilde{\nu}(F_A \sqcap (C_D)_n) = \widetilde{\nu}(F_A),$$

which should have been proved. □

Theorem 3.14. *Let $\widetilde{X}$ be a soft set and let $\widetilde{\mathcal{S}}$ be a soft semiring over $\widetilde{X}$. If $\widetilde{\mu} : \widetilde{\mathcal{S}} \to [0,\infty]$ is a σ-finite soft premeasure, then there exists a unique soft measure $\widetilde{\nu} : \sigma(\widetilde{\mathcal{S}}) \to [0,\infty]$ extending $\widetilde{\mu}$.*

Proof. The existence of the soft measure was shown in the paper [5], while the uniqueness is valid due to Theorem 3.13. in which $\widetilde{\mathcal{E}} = \widetilde{\mathcal{S}}$, as the soft semiring $\widetilde{\mathcal{S}}$ is a soft $\sqcap$-stable. □

Example 3.15. *Let $X = \{h_1, h_2, h_3\}$ and $E = \{e_1, e_2\}$. Let $\widetilde{\mathcal{S}} = \{(F_A)_i \mid i = 1,2,3\}$ be a collection, where*
$(F_A)_1 = \Phi$,
$(F_A)_2 = \{(e_1, \{h_1\}), (e_2, \emptyset)\}$,
$(F_A)_3 = \{(e_1, \{h_2, h_3\}), (e_2, \{h_1, h_2\})\}$.
As we can notice, the collection $\widetilde{\mathcal{S}} = \{(F_A)_i \mid i = 1,2,3\}$ is a soft semiring. Mapping $\widetilde{\mu} : \widetilde{\mathcal{S}} \to [0,\infty]$, given by

$$\widetilde{\mu}(F_A) = \begin{cases} 0, & F_A = (F_A)_1, \\ 1, & F_A = (F_A)_2, \\ 2, & F_A = (F_A)_3. \end{cases}$$

is a soft σ-finite soft premeasure.

Starting from the collection $\widetilde{\mathcal{S}} = \{(F_A)_i \mid i = 1,2,3\}$ we can easily find the collection $\sigma(\widetilde{\mathcal{S}}) = \{(F_A)_i \mid i = 1,2,3,4,5,6,7,8\}$ that is a soft σ-algebra such that
$(F_A)_1 = \Phi$,
$(F_A)_2 = \{(e_1, \{h_1\}), (e_2, \emptyset)\}$,
$(F_A)_3 = \{(e_1, \{h_2, h_3\}), (e_2, \{h_1, h_2\})\}$,
$(F_A)_4 = \{(e_1, \{h_1, h_2, h_3\}), (e_2, \{h_1, h_2\})\}$,
$(F_A)_5 = \{(e_1, \{h_2, h_3\}), (e_2, \{h_1, h_2, h_3\})\}$,
$(F_A)_6 = \{(e_1, \{h_1\}), (e_2, \{h_3\})\}$,
$(F_A)_7 = \{(e_1, \emptyset), (e_2, \{h_3\})\}$,
$(F_A)_8 = \widetilde{X}$.
If we try to expand $\widetilde{\mu}$ to the soft measure $\widetilde{\nu}$ defined over soft σ-algebra $\sigma(\widetilde{\mathcal{S}})$, we get that the expansion must be unique and that the soft measure is given by

$$\widetilde{\nu}(F_A) = \begin{cases} 0, & F_A = (F_A)_1, \\ 1, & F_A \in \{(F_A)_2, (F_A)_7\}, \\ 2, & F_A \in \{(F_A)_3, (F_A)_6\}, \\ 3, & F_A \in \{(F_A)_4, (F_A)_5\}, \\ 4, & F_A = (F_A)_8, \end{cases}$$

which was expected considering the Theorem 3.14.

4. Conclusion

Molodtsov defined and presented several possible applications of soft set theory. Many researchers have continued studying the possible applications of the soft set theory, and an important segment is the introduction of the concept of soft measures, as an unavoidable term in the applications of soft sets in various fields. In this work, research in the field of soft measure theory was continued, and new collections of soft sets were defined, where the properties of soft measures and soft premeasures, as well as their possible extensions, were studied. Defined soft $\sqcap$-stable collections, soft monotone classes and soft Dynkin systems are studied in this paper in the context of soft measure, and certainly such collections can serve as a good tool in the theory of soft sets, in general. In this context, this work provides an opportunity for further research and more detailed studies of the mentioned collections. We hope that the defined concepts and properties given in this paper will help many researchers to improve and promote the theory of soft sets, and especially the soft measure.

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