OPTIMIZATION OF GEAR PAIRS IN THE TWO STAGE PLANETARY GEARBOX USING AHP AND TOPSIS METHOD

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Resume

The optimization of gear pairs in a two-stage planetary gearbox was performed using the Analytic Hierarchy Process (AHP) and the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). A total of 18 combinations of input parameters were analyzed, varying: gear width (20, 22, 24 mm), module (2.25, 2.5, 2.75 mm) and material (16MnCr5, 34CrNiMo6). Safety factor from pitting (Sh) and safety factor from tooth fracture (Sf) were numerically obtained and those two parameters were chosen as criteria for optimization. TOPSIS identified alternative 1 (20 mm width, 2.25 mm module, 16MnCr5 material) as the optimal solution, demonstrating the highest safety factor values. Additionally, the optimization achieved satisfactory results regarding the mass of gear pairs. After optimization, the mass of gear pairs decreased by 20% compared to the pre-optimization version.

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1 Introduction

Mechanical transmissions, as the name suggests, mechanically transmit power and motion from the driving machine to the driven machine. The development of mechanical transmissions dates back to ancient civilizations, where people crafted various mechanisms from objects found in their environment to facilitate their work. As society progressed, these transmissions evolved [1-2].

In a modern history of mechanical transmissions, with the development of industry and capitalism, efforts have been made to reduce the production and maintenance costs, while emphasizing increased efficiency and reliability. Mechanical transmissions have paralleled the development of industry and science, advancing in terms of load capacity calculations, material selection and geometry. With the advance of automation, monitoring and control of their operation has been greatly facilitated, while robotics development has enabled the implementation of these transmissions in devices performing specific tasks.

In modern engineering practice, the optimization of the design of mechanical transmissions becomes crucial to achieve high efficiency, reliability and cost reduction. With ever-increasing demands for energy-efficient and long-lasting systems, engineers face the challenge of optimizing the transmission characteristics, while minimizing production and maintenance costs, which sets new standards in the field of design and application of these systems.

Continuous investment in development of advanced software allows engineers to model and test transmissions in a virtual environment before the physical production, minimizing the possibility of errors during their manufacture. One such mechanical transmission that has evolved from a basic to a complex geometric element is the gear pair, which consists of two meshing gears. Gear pairs have undergone numerous modifications in terms of geometry and material selection to align their characteristics with their intended applications.

The need to optimize the space and energy utilization, as well as the demand for high transmission ratios, have led to the emergence and development of planetary gear transmissions. Planetary gear transmissions are gear transmissions consisting of the two central gears and satellites. This type of transmission represents a challenging endeavor in terms of design, production, and assembly.

Over the past decades, manufacturing technologies have advanced significantly, with one direction being the development and application of new materials for gear transmissions, including the use of polymers, ceramics, and composites. The use of polymer gears has experienced a significant increase primarily due to numerous advantages compared to steel gears. Mass production is cheaper when manufactured by injection molding, they can operate without additional lubrication, absorb noise and vibrations better and are resistant to corrosion and other chemical reactions. Zorko et al. [3] presented the possibility of replacing the fossil-based polymers with bio-based polymers, such as polyoxymethylene (POM) and polyamide 66 (PA 66). The potential of bio-based polymers was studied based on gear lifetime testing. The results show that gears made of bio-based polyamide have a lifespan three and a half times longer than POM gears and ten times longer than PA66 gears. Ceramic materials, due to their higher pressure resistance compared to tensile strength, are not a good choice for gear manufacturing. However, due to several advantages such as low weight, wear resistance, high-temperature resistance and chemical resistance, considerable attention is being given to them and possible solutions for their application in gears are being developed. Vasileiou et al. [4] explored the application of ceramic gears and structural solutions in their work. They also compared the performance of ceramic gears against steel gears.

The use of composite materials for gear manufacturing is becoming challenging in modern engineering due to numerous advantages. Gears made of composite materials are lighter compared to steel gears, have better corrosion resistance, fatigue resistance, etc. Hybrid steel-composite gears, composite-bodied gears with a steel gear rim, represent a technology that is rapidly developing. The concept of such gears was introduced in 2012 by the NASA Research and Development Center, replacing conventional steel gears in aviation. This prototype successfully operated for three hundred million cycles and was 20% lighter than a steel gear [5]. Under the normal operating conditions, these gears performed well, but under unlubricated conditions, they exhibited shortcomings. Hybrid steel-composite gears were tested in unlubricated conditions in 2017 and the results showed that the composite softened at high temperatures (above 232 °C), necessitating the discovery of composites capable of withstanding operation at such high temperatures. Waller et al. [5] addressed this issue in their work, studying and researching composite materials specifically for the manufacture of hybrid steel-composite gears for aviation. The research was based on experimentation involving different types of matrices and fiber orientations. Various samples were first examined under a microscope and then tested for pressure resistance using a testing device. Subsequently, they created 3D models tested in Abaqus CAE software. They concluded that Carbon/BMI composite provides greater pressure resistance than carbon-epoxy at temperatures above 204 °C, and the thermal resistance of multidirectional laminate layers is identical to steel, reaching a balance between the resistance and thermal conductivity.

The application of new materials for gear transmissions is a key aspect of modern engineering focused on improving system performance and efficiency. Efforts are directed towards finding materials that can meet and satisfy all the requirements of modern technical systems, while providing better performance compared to conventional materials. By employing modern analysis and testing methods, new materials are being developed that can meet expectations regarding reliability and longer lifespan, while reducing production costs and thereby reducing costs. Additionally, the use of these materials can have a positive environmental impact compared to steel, reducing electricity consumption during production, enabling recycling and reuse, reducing emissions of harmful gases, biodegradability, etc.

There is extensive research on the complex dynamic characteristics of planetary gear transmissions, employing various methodologies to optimize system performance in terms of vibration efficiency [6-7], power transmission efficiency [6, 8-9], geometric characteristics [10]. Furthermore, another group of researchers has studied the gear tooth wear and its impact on system dynamics Li et al. [11], Tian et al. [12]. Li et al. [11], modeled the planetary gear system with helical gears in a wind turbine gearbox with tooth surface damage, concluding that the tooth damage leads to abnormal vibrations and lateral stripes in frequency signals. Tian et al. [12] investigated the impact of angular misalignment on the prediction of wear in planetary gear sets with rotating carriers, finding that the combined effect of angular misalignment and rotating carrier results in a greater wear at the ends of the tooth face width. Another group of researchers, Huangfu et al. [13], predicted the gear wear in planetary gear transmissions using a developed dynamic model employing geometrically adaptively loaded tooth contact analysis (GA-LTCA) method. They validated their developed model through the finite element analysis and failure testing, demonstrating accurate predictions of wear profiles and wear characteristics throughout the gear's life cycle.

Although there are numerous studies of the dynamic characteristics of planetary gears and methods for improvement of their efficiency, the challenge of optimization of the safety factor against pitting (Sh) and the safety factor against tooth breakage (Sf) in planetary gear pairs remains significant. Optimization of these safety factors is important for ensuring the long-term reliability and safe operation of planetary gears, especially under the high loads and demanding operational conditions. The aim of this study was to optimize the safety factor from pitting (Sh) and safety factor from tooth breakage (Sf) of planetary gear pairs. Optimization is performed by combining the Analytic Hierarchy Process (AHP) and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) methods to determine the optimal parameter values based on predefined criteria weights.

2 Planetary transmissions

Mechanical transmissions find their widest application in engineering. Mechanical transmissions transfer mechanical energy from the driving to the driven machine, often altering speed, torque and sometimes direction of movement. Planetary gear transmissions belong to a special group of mechanical power transmissions. Initially, their application was focused on solving problems for transport and military vehicles. As they developed, their range of applications expanded continuously. Effective use of space within the planetary gear housing is achieved by the ability to fill the space between the central gears with a larger number of satellites, contributing to reduced loads and selection of smaller gear modules. Due to their compact design, they are two to three times lighter than the conventional transmissions of the same power and gear ratio [1-2]. Some of the main advantages of planetary transmissions include: the high gear ratio with small dimensions, compact construction, high degree of coupling enabling more uniform distribution of loads, capability of transmitting higher torque, ability to achieve various gear ratios, and high efficiency. However, they also exhibit certain drawbacks such as: occurrence of centrifugal forces, sensitivity to changes in center distances, and complex and costly manufacturing processes [1].

Throughout the time, engineers have strived to bring planetary transmissions to perfection, which they have done, because today these transmissions appear in all branches of industry and represent a key segment in most modern technical systems.

3 Methodology

The kinematics and dynamics calculation of the B_{ha}^{b} concept planetary gearbox was performed for the following input parameters: driving machine is EM with 7 kW power and speed of 1200 rpm, the gear ratio of the planetary gearbox is 4 and the transmission efficiency is 0.97. This design of the planetary gearbox has two stages with three satellites each. The calculations of planetary gearbox were conducted based on literature [1], while the calculation of gear pairs was performed using the Inventor software package based on initial data. During the calculation of gear pairs, the safety factor from pitting (Sh) and safety factor from the tooth breakage (Sf) of planetary gear pairs were recorded based on varied factors, along with numerical results used for the optimization of gear pairs in the mentioned planetary gearbox.

The next step in this research was the application of the AHP and TOPSIS methods to achieve optimal values of safety factors for the tooth flanks and tooth bottoms of the gear pairs.

3.1 Analytic hierarchy process

The AHP is a multi-criteria decision-making method used for systematic analysis and decision-making in complex problems, involving multiple alternatives and criteria [14]. It breaks down complex problems into simpler and more understandable elements through the hierarchical decomposition. Developed in the 1980s, the method owes its advancement to the research and scientific work of American scientist Thomas Saaty. It is widely applied in various fields such as engineering, management, economics, medicine and others [15]. A pivotal component of the AHP method is Saaty's scale, also known as the comparison scale, which determines the relative importance between criteria. Saaty's scale (Table 1) consists of 9 levels, each level comprising a numerical value and corresponding description [14-18].

Based on existing research and applications of the AHP method, the following steps are provided [16, 18-19]:

Step 1: Creating a pair-wise comparison matrix using the Saaty's scale. Comparison is made for each criterion relative to another, using a value from the Saaty's scale. A criterion compared to itself always has a value of 1.

Table 1 *Saaty's scale*

Importance	
1	Two criteria are equally important.
3	One criterion is moderately more important than the other.
5	One criterion is significantly more important than the other.
7	One criterion is much more important than the other.
9	One criterion is extremely more important than the other.
2,4,6,8	Intermediate values.

After the pair-wise comparisons, matrix A is formed where a_{ij} represents the relative preference of criterion A_i over criterion A_j.

$$A = [a_{ij}] = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ \frac{1}{a_{12}} & 1 & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{1}{a_{1n}} & \frac{1}{a_{2n}} & \cdots & 1 \end{bmatrix}, \tag{1}$$

Step 2: Calculating the normalized weight w_j using Equations (2) and (3):

$$w_j = \frac{GM_i}{\sum_{i=1}^{n} GM_i}. \tag{2}$$

where, the value GM_i is calculated as follows:

$$GM_i = \left(\prod_{i=1}^{n} a_{ij}\right)^{\frac{1}{n}} \tag{3}$$

Step 3: Determining the maximum eigenvalue of the comparison matrix $\lambda_{\max}$ and calculating the consistency index CI:

$$CI = \frac{\lambda_{\max} - n}{(n-1)}. \tag{4}$$

Step 4: Calculating the random consistency index RI (Table 2) for the criteria used in the decision-making process. Criteria represent different factors or characteristics used in the decision-making. Each criterion has its importance relative to the others. Criteria are included as input parameters, when used to determine or rank alternatives, while they are assigned output values when used to generate results based on input data.

Step 5: Calculate the consistency ratio CR using Equation (5):

$$CR = \frac{CI}{RI} \tag{5}$$

If the CR value is 0.1 or lower, accept the obtained criteria weights. However, if the calculated CR exceeds 0.1, it indicates unsatisfactory consistency, and it is necessary to repeat the comparison, i.e., recreate the comparison matrix as shown in Step 1.

3.2 TOPSIS method

The TOPSIS is a multi-criteria decision-making method used for ranking and selecting the best alternative, considering multiple criteria. It was developed in 1981 by Hwang and Yoon. The best alternative is the one with the shortest Euclidean distance from the positive ideal solution (PIS) and the greatest Euclidean distance from the negative ideal solution (NIS). This method is highly efficient compared to other metaheuristic methods, due to its numerous characteristics, such as reliability, ease of reaching solutions, procedural transparency, compatibility with other methods, and more. It finds extensive applications across various industries, including the project management, financial analysis, industrial engineering, and marketing for analyzing and selecting marketing strategies and products. The steps used in its implementation are detailed below based on research [14-23].

Step 1: Formation of a decision matrix from all the alternatives and criteria. Assuming that the alternatives are denoted by M and criteria by N, the decision matrix *D* takes the following form:

$$D = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1N} \\ x_{21} & x_{22} & \cdots & x_{2N} \\ \cdot & \cdot & \cdots & \cdot \\ x_{M1} & x_{M2} & \cdots & x_{MN} \end{bmatrix}. \tag{6}$$

Step 2: Formation of the normalized decision matrix. The normalized value r_{ij} can be calculated using:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^{2}}}, \tag{7}$$

where: x_{ij} - performance score of alternative i with respect to criterion j.

Step 3: Definition of criterion weights and formation of the weighted normalized decision matrix. The weighted normalized value v_{ij} can be calculated using:

$$v_{ij} = w \cdot r_{ij}, \tag{8}$$

where: r_{ij} - normalized value,
w_j - criterion weight.

Step 4: Definition of the PIS and NIS. When defining the positive ideal in Equation (9) and negative ideal solution in Equation (10), it is crucial to adhere to the recommendation regarding whether the criteria are considered from a benefit or cost perspective.

$$v_{ij}^{+} = \left\{\left(\sum_{i}^{\max} v_{ij} / j \in J\right); \left(\sum_{i}^{\min} v_{ij} / j \in J'\right) / \atop i = 1,2.\ldots n\right), \tag{9}$$

$$v_{ij}^{-} = \left\{\left(\sum_{i}^{\min} v_{ij} / j \in J\right); \left(\sum_{i}^{\max} v_{ij} / j \in J'\right) / \atop i = 1,2.\ldots n\right), \tag{10}$$

Here, $J = (j = 1, 2, ..., n)/j$ refers to criteria considered from a benefit perspective, while $J' = (j = 1, 2, ..., n)/j$ refers to criteria considered from a cost perspective.

Step 5: Calculation of the distance of each alternative from the PIS D^+ (11) and NIS D^- (12) using the Euclidean distance:

***Table 2** Values of random consistency index depending on the number of criteria*

Number of criteria	1	2	3	4	5	6	7	8	9
Random consistency index RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45

$$D_i^{+} = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^{+})}, \quad (11)$$

$$D_i^{-} = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^{-})}. \quad (12)$$

Step 6: Calculation of the relative closeness to the ideal solution using:

$$C_i^{+} = \frac{D_i^{-}}{(D_i^{+} + D_i^{-})}, \quad (13)$$

where C_i^+ is within the range $0 \le C_i^+ \le 1$.

Step 7: Ranking of alternatives. Based on the values of C_i^+, alternatives are ranked in descending order. The alternative with C_i^+ closest to 1 will be ranked as the 1st.

In this study, the AHP method was selected due to its capability to rank alternative criteria through a hierarchical structure, enabling clear definition and quantification of the weights of various safety factors in the optimization of gear pairs. In contrast, the TOPSIS method is well-suited for selecting the optimal solution through comparison to the ideal solution, thereby ensuring precise ranking of alternatives. This combination provides a systematic and efficient approach to optimization in line with the objectives of this study.

4 Optimization of gear pairs in planetary gear transmission

Before conducting the optimization of gear pairs in the planetary gear transmission, it is necessary to perform the kinematic and dynamic calculations of the planetary gear transmission (Figure 1). The concept of B_{ha}^{b} shown in the Figure 1 consists of a central gear designated as "a", located on the input shaft "I" of the planetary gear transmission, engaged with satellite gear "g". Satellites "g" and "f" are mounted on shaft "II", with satellite "f" engaged with an internally toothed central gear identified as "b". The carrier of a satellite "h" is connected to the output shaft "III".

After the calculation, the number of teeth for the gears of the planetary gear transmission was determined ($z_a = 21, z_g = 29, z_f = 40, z_b = 90$), satisfying all checks, and the number of satellites N = 3 was adopted. To optimize the gear pairs, 18 different alternatives were created using 3 input parameters (tooth width, module and material). Table 3 provides the values of the input parameters.

The aim of optimization is to obtain optimal values for the safety factor from pitting (Sh) and safety factor from the tooth breakage (Sf) for both gear pair. Since optimization is performed for both gear pairs, the criteria considered are Sh_1, Sf_1, Sh_2 and Sf_2. These criteria values are obtained based on the input parameters in the Auto-desk Inventor software package. Table 4 presents various variations of input and output parameters with criterion values.

4.1 Application of AHP and TOPSIS methods

The procedure for implementing these methods is described in the previous section, so here are presented only the results obtained from these methods. Based on the first step of the TOPSIS method, a decision matrix A was formed, followed immediately by the calculation of the normalized decision matrix, as shown in Table 4.

During the optimization, it is very important to correctly determine the criterion weights, considering

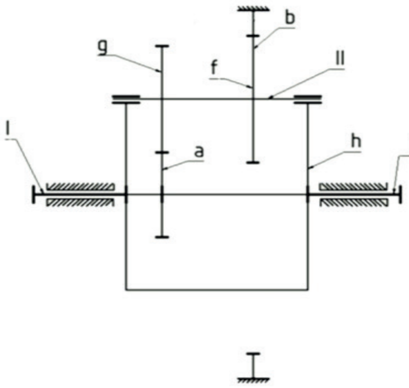


Figure 1 *Concept B_{ha}^{b} of the planetary gear transmission*

Table 3 *Input parameters*

Parameter	Mark	Unit	Level I	Level II	Level III
Width of gear pair	b	mm	20	22	24
Module of gear pair	m	mm	2.25	2.5	2.75
Material of gear pair	M	/	1 (16MnCr5)	2 (34CrNiMo6)	/

that they have a key role in ranking the alternatives. Following this, the determination of criterion weights, using the AHP method, and formation of the weight-normalized decision matrix, were performed according to the steps described in the previous section. Criteria are assigned to output parameters. Criteria weights have been determined, with Sh_1 and Sh_2 each accounting for 33.3%, and Sf_1 and Sf_2 each accounting for 16.7%. Based on the calculated consistency ratio CR, which is 0, it can be concluded that the obtained criteria weights

Table 4 *Combinations of input parameters with the decision matrix*

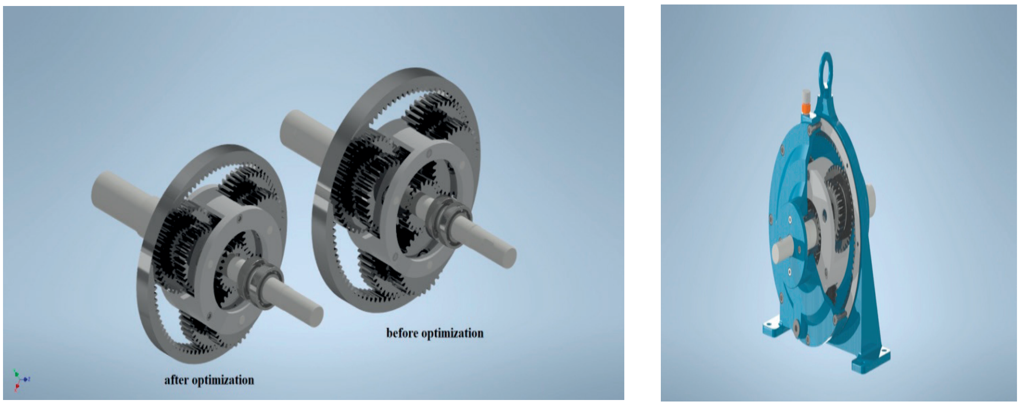
Alter.	Parameters			Decision matrix				Normalized decision matrix			
	b	m	M	Sh_1	Sf_1	Sh_2	Sf_2	Sh_1	Sf_1	Sh_2	Sf_2
1	20	2.25	1	1.117	4.229	2.673	3.969	0.217	0.180	0.220	0.186
2	22	2.25	1	1.150	4.529	2.741	4.212	0.223	0.193	0.382	0.198
3	24	2.25	1	1.178	4.798	2.802	4.443	0.229	0.205	0.391	0.209
4	20	2.5	1	1.226	5.034	2.898	4.596	0.238	0.215	0.404	0.216
5	22	2.5	1	1.258	5.349	2.970	4.875	0.244	0.228	0.414	0.229
6	24	2.5	1	1.290	5.668	3.036	5.140	0.250	0.242	0.423	0.241
7	20	2.75	1	1.349	5.824	3.116	5.244	0.262	0.248	0.435	0.246
8	22	2.75	1	1.363	6.210	3.192	5.559	0.264	0.265	0.445	0.261
9	24	2.75	1	1.397	6.578	3.262	5.858	0.271	0.281	0.455	0.275
10	20	2.25	2	1.034	4.410	2.475	4.139	0.201	0.188	0.345	0.194
11	22	2.25	2	1.065	4.723	2.538	4.393	0.207	0.201	0.354	0.206
12	24	2.25	2	1.091	5.004	2.595	4.633	0.212	0.213	0.362	0.218
13	20	2.5	2	1.134	5.250	2.684	4.793	0.220	0.224	0.374	0.225
14	22	2.5	2	1.165	5.578	2.751	5.084	0.226	0.238	0.384	0.239
15	24	2.5	2	1.194	5.911	2.812	5.360	0.232	0.252	0.392	0.252
16	20	2.75	2	1.227	6.073	2.886	5.469	0.238	0.259	0.403	0.257
17	22	2.75	2	1.262	6.476	2.957	5.797	0.245	0.276	0.412	0.272
18	24	2.75	2	1.294	6.860	3.022	6.109	0.251	0.293	0.421	0.287

Table 5 *Determination of PIS, NIS, their distances, relative closeness and rank*

Alter.	Sh_1	Sf_1	Sh_2	Sf_2	D^+	D^-	C_i	Rank
1	0.372	0.706	0.073	0.031	0.028	0.456	0.943	1
2	0.383	0.756	0.127	0.033	0.083	0.399	0.827	3
3	0.392	0.801	0.130	0.035	0.121	0.353	0.745	5
4	0.408	0.841	0.135	0.036	0.161	0.311	0.659	7
5	0.419	0.893	0.138	0.038	0.212	0.257	0.549	9
6	0.430	0.947	0.141	0.040	0.264	0.203	0.434	11
7	0.449	0.973	0.145	0.041	0.295	0.174	0.371	13
8	0.454	1.037	0.148	0.044	0.357	0.109	0.235	15
9	0.465	1.099	0.151	0.046	0.418	0.047	0.101	17
10	0.344	0.736	0.115	0.032	0.052	0.429	0.893	2
11	0.354	0.789	0.118	0.034	0.094	0.375	0.799	4
12	0.363	0.836	0.121	0.036	0.139	0.328	0.702	6
13	0.378	0.877	0.125	0.038	0.181	0.284	0.611	8
14	0.388	0.931	0.128	0.040	0.236	0.229	0.493	10
15	0.398	0.987	0.131	0.042	0.292	0.174	0.373	12
16	0.409	1.014	0.134	0.043	0.321	0.144	0.310	14
17	0.420	1.081	0.137	0.045	0.388	0.080	0.170	16
18	0.431	1.146	0.140	0.048	0.453	0.036	0.074	18
PIS	0.344	0.706	0.073	0.031				
NIS	0.465	1.146	0.151	0.048				

***Table 6** Comparative overview of both gear pairs mass before and after the optimization*

	Before optimization		After optimization - alternative 1	
	gear pair *a-g*	gear pair *b-f*	gear pair *a-g*	gear pair *b-f*
Width of gear pair (mm)	25	25	20	20
Module of gear pair(mm)	2.5	2.5	2.25	2.25
Mass (kg)	1.065	3.641	0.794	2.359



(a) (b)

***Figure 2** Planetary gearbox: (**a**) gear pairs before and after optimization; (**b**) model*

are acceptable. Using the templates for forming the weight matrix of decision-making, the determination of the positive ideal solution (PIS) and negative ideal solution (NIS) is approached, with their values shown in Table 5. Table 5, also presents the distances of each alternative from the positive ideal solution D^+ and negative ideal solution D^- using the Euclidean distance, as well as the values of relative proximity to the ideal solution C_i. Finally, the last column displays the ranking of alternatives based on C_i.

As shown in Table 5, by ranking the values of C_i, it is concluded that the optimal parameter combination is alternative 1, where the parameter values are as follows: gear width 20 mm, module 2.25 mm, and material 1 (steel 16MnCr5). On the other hand, the least favorable parameter combination is alternative 18, where the parameter values are: width 24, module 2.75, and material 2 (34CrNiMo6). According to the chosen optimal version, the safety factor values Sh_1 and Sf_1 for the first gear pair are 1.117 and 4.229, respectively, while for the second gear pair Sh_2 and Sf_2 are 2.673 and 3.969, respectively.

Authors often justify or even accept the second-ranked alternative alongside the top-ranked alternative as the optimal solution in lot of scientific papers. This occurs in specific cases where the top-ranked alternative, due to various factors, cannot be selected as the most favorable solution. In this study, based on the results, it is observed that alternative 10 is ranked 2, and in the cases where alternative 1, for some reason cannot be accepted, alternative 10 would be considered as the optimal solution.

4.2 Analysis and discussion of results

The optimization of gear pairs achieved satisfactory results concerning the mass and overall dimensions, primarily of the gear pairs and subsequently of the entire planetary gearbox. Reducing the mass significantly impacts the inertia of the gearbox, which contributes to increased efficiency by reducing the energy required for the gearbox's operation and movement. Considering that planetary gearboxes are more expensive than other power transmissions, it is crucial to minimize production costs as much as possible. By reducing the mass and, consequently, the overall dimensions the consumption of materials and the time required for manufacturing are directly impacted, dictating the final costs of the planetary gearbox.

Besides the manufacturing costs and achieving greater efficiency, the power transmissions may encounter issues during handling and assembly. The reduction in mass positively affects easier handling, transport and assembly. Table 6 provides a comparative overview of data related to the width, module, mass and overall dimensions of gear pairs a-g and b-f before and after the optimization.

Given that the values of the width and module for both gear pairs are smaller after optimization, there has been a reduction in the mass and dimensions of the tip diameters. Consequently, during the design process, the housing, shafts, and the carrier of the satellites would have smaller dimensions and mass, which would further contribute to savings in materials and production resources. After applying the optimization methods to both gear pairs and obtaining the necessary results, modeling and assembly of the planetary gearbox elements into a single unit were performed. The dimensions of the carrier were adjusted according to the values obtained from the optimization, while the dimensions of the shafts remained the same. Figure 2a illustrates the difference in the gear pair a-g and b-f before and after optimization, while Figure 2b shows the model of a planetary gearbox.

The application of optimization methods on gear pairs is increasing, many researchers use optimization methods in their studies and scientific papers, striving to achieve the best possible results in this field. Singh et al. [22] optimized the surface temperature, wear rate and efficiency of polymer gears produced by injection molding using a combination of AHP-TOPSIS methods. For the optimization, 27 different combinations were created. The input parameters were material (ABS, HDPE and POM), rotational speed (600, 900 and 1200 min^{-1}) and torque (0.8, 1.2 and 1.6 Nm). The output values, or criteria, included the surface temperature, wear rate and efficiency. The results showed that the best performance of polymer gears is achieved with POM material, at 900 min^{-1} and 0.8 Nm. Nasr et al. [8], applied the Genetic Algorithm to optimize a planetary gearbox. The aim of the optimization was to reduce the weight and increase the gear ratio, while maintaining the safety factor within permissible limits. The input parameters included module, number of teeth, width, internal diameter of the central gear and satellite gears, and external diameter of the internally toothed gear. Various material types were studied, including the low alloy steel, stainless steel, aluminum and PET plastic. The results showed that the greatest weight savings in the planetary gearbox were achieved using the PET plastic gears. Miladinovic et al. [24] focused on the optimization of a Ravigneaux planetary gearbox. The goal was to reduce the weight, while considering the safety factor values. The input parameters considered were three different types of materials, three module values and three gear width values. The Taguchi and ANN methods were used for optimization and it was concluded that the module had the most significant impact on the safety factor, while the gear width had the least impact. Miladinovic et al. [25] applied the Taguchi method in their research to select optimal parameters for the driving gear in a planetary gearbox. For optimization, an L18 matrix was created with input parameters including module, material and gear width. The ANOVA analysis was used to determine the influence of parameters on the safety factor. The results showed that the module had the most significant influence on the safety factor against tooth flank failure at 80.953%, while the material had the least influence at 0.615%. The width had an influence of 18.392% on the safety factor. The Taguchi method yielded the optimal combination of parameters, where the module value was 2.25 mm, width 27 mm and material 16MnCr5. Gupta et al. [26] focused on selecting the optimal material for the production of gears in a planetary gearbox. The eight different materials were considered, with mechanical properties (tensile strength, yield strength, hardness) and costs being the criteria for optimization. In their study, next to the TOPSIS method, the COPRAS was, also applied. Comparing the results, they concluded that the optimal material for gear production is EN30A, with EN24 being the next best option.

Based on the previous research, it is planned to include the following elements in the future optimization process, when designing the planetary gear transmission, namely: housing, shafts, satellite carrier and the selection of suitable bearings. Special attention will be paid to the reduction of dimensions, which would result in additional weight reduction without compromising the reliability of the planetary gear transmission. This weight reduction contributes to savings in material and other resources required to manufacture the planetary gears. Therefore, it is recommended that the mentioned elements be included in the optimization process, which would enable a comprehensive analysis of the entire transmission with a special emphasis on dimensions. Such an approach can further improve the efficiency, reduce the production costs and contribute to development of modern, economical mechanical systems, thus ensuring the long-term sustainability and reliability of those transmissions.

5 Conclusions

Throughout the history, mechanical transmissions have played a significant role in various industrial and transportation systems. Their historical development reflects continuous improvements and the discovery of new, innovative solutions aimed at enhancing performance, efficiency and reliability. The application of planetary gear transmissions is diverse, encompassing a wide range of industries such as automotive, aerospace, mechanical and others.

Optimization methods play a crucial role in achieving the desired performance of planetary gear transmissions. Scientists continuously research and strive not only to discover new methods but to improve and refine existing ones to adapt them to specific problems, as well.

Based on the conducted optimization, the optimal variant of parameters was determined to achieve the optimal values of the safety factor from pitting (Sh) and the safety factor from tooth breakage (Sf) for both gear pairs. The key findings from this study indicate that the optimal parameters for the gear pairs are: the width of 20 mm, module of 2.25 mm and material 16MnCr5. The optimal combination of parameters is specified by alternative 1, where the parameter values are as follows: gear pair width of 20 mm, gear pair module of 2.25 mm, and gear pair material of

16MnCr5. The optimization of gear pairs results in approximately a 20% reduction in the weight of the planetary transmission, which subsequently reduces the transmission's inertia, contributing to increased efficiency in terms of reduced energy required for starting and operating the planetary transmission. Given that the planetary gear transmissions are significantly more expensive than other power transmissions, it is essential to contribute as much as possible to reducing the production costs. This optimization not only improves efficiency, but contributes to lowering the operating costs and extending the lifespan, as well. Therefore, during the design of other elements of the optimized transmission, such as the housing, shafts, satellite carrier, and the selection of appropriate bearings, the dimensions would be smaller, the weight reduced, leading to savings in material and other resources needed for production of the transmission. Reduction of the transmission weight directly contributes to the reduction of production costs, which is crucial in a competitive environment. More efficient, lighter and longer-lasting transmissions not only improve operational performance, but also enable companies to remain competitive on the market. This approach can significantly impact the reduction of wear, weight, inertia, vibrations, and noise, while also increasing the durability and robustness of these elements.

This also marks the direction for the future research, optimizing other elements of the transmission, where innovative materials, particularly hybrid composite materials, can be considered for the production of gear pairs. Additionally, the recommendation would be to use other optimization methods to verify the results obtained by this method. Further research should focus on investigating the effect of different materials and optimization methods on performance and reliability of the transmission. Research of efficiency, based on determination of the optimal types of materials, as well as the application of some new materials, can significantly change the dynamics of the transmission, which directly affects reliability. In addition to the types of materials, optimization can include various parameters related to the technology of manufacturing the transmission elements, with the aim of obtaining the highest quality parts, while reducing the costs and the production time.

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Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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