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10th International Congress
Motor Vehicles & Motors 2024

ECOLOGY -
VEHICLE AND ROAD SAFETY
- EFFICIENCY

Proceedings

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PREDGOVOR

U oktobru se na Fakultetu inženjerskih nauka Univerziteta u Kragujevcu tradicionalno održava skup istraživača i naučnika koji se bave proučavanjem motornih vozila, motora i drumskog saobraćaja. Od 1979. do 2004. godine održano je trinaest bienalnih MVM simpozijuma koji su 2006. prerasli u Međunarodni kongres MVM. Od tada je održano devet MVM kongresa, a oktobra 2024. godine Fakultet inženjerskih nauka je organizovao deseti međunarodni kongres MVM od 10. do 11. oktobra 2024. godine.

Na deseti kongres Motorna vozila i motori, MVM2024 dostavljen je veliki broj naučnih radova iz Srbije i inostranstva. Kongres tradicionalno podržavaju Ministarstvo za nauku, tehnološki razvoj i inovacije Republike Srbije, Univerzitet u Kragujevcu, Fakultet inženjerskih nauka i međunarodni časopis „Mobility and Vehicle Mechanics".

Tema Kongresa MVM 2024 bila je „Ekologija – Bezbednost vozila i na putevima – Efikasnost". Tokom ovog istraživačkog putovanja, učesnici su puno naučili kroz rad na različitim sekcijama, koje su pokrivale širok spektar tema u vezi sa inženjerstvom u automobilskoj industirji, od fundamentalnih istraživanja do industrijskih primena, naglašavaju interakciju između vozača, vozila i životne sredine i stimulišući naučnu interakcije i saradnju.

Međunarodni naučni odbor u saradnji sa organizacionim odborom izradio je podsticajan naučni program. Program je ponudio preko 54 prezentacije radova, uključujući predavanja po pozivu i radove u sekcijama. Prezentacije na ovom kongresu obuhvatile su aktuelna istraživanja u oblasti motornih vozila i motora sprovedena u 12 zemalja iz celog sveta.

Zadovoljstvo nam je bilo što su nam uvodničari bili profesor Emrulah Hakan Kaleli (sa Tehničkog univerziteta Yıldız, Turska), profesor Ralph Putz (sa Univerziteta Landshut UAS, Nemačka) i profesori Nenad Miljić i Slobodan Popović (sa Univerziteta u Beogradu, Srbija). Izazovi i rešenja u korišćenju vodonika kao goriva za motore sa unutrašnjim sagorevanjem, korišćenje aditiva nanoborne kiseline dodatog u motorno ulje, kao i evropska politika o budućoj mobilnosti na putevima su bile teme uvodnih predavanja.

Sigurni smo da je ovaj program pokrenuo živu diskusiju i podstakao istraživače na nova dostignuća.

10. Kongres MVM 2024. finansijski je podržalo Ministarstvo za nauku, tehnološki razvoj i inovacije Republike Srbije.

Zahvaljujemo se iskusnim i mladim istraživačima koji su prisustvovali i prezentovali svoju stručnost i inovativne ideje na našem kongresu.

Posebnu zahvalnost dugujemo članovima međunarodnog naučnog odbora i svim recenzentima za njihov značajan doprinos visokom nivou kongresa.

Naučni i organizacioni komitet Kongresa MVM2024

FOREWARD

In October, the Faculty of Engineering University of Kragujevac traditionally holds gatherings of researchers and academics who study motor vehicles, engines and road traffic. From 1979 to 2004, thirteen, biennal MVM Symposiums have been held and they grew into an International Congress MVM in 2006. Since then, ninth MVM Congresses have been held, and in October 2024, the Faculty of Engineering organized the tenth International Congress MVM from 10th to 11th October 2024.

A large number of scientific papers from the Serbia and abroad were submitted to the tenth Congress “MVM2024”. Congress is traditionally supported by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia, University of Kragujevac, Faculty of Engineering and the International Journal “Mobility and Vehicle Mechanics”.

The theme of the Congress MVM 2024 was "Ecology - Vehicle and Road Safety - Efficiency". Along this journey we learned from the various sessions, which broadly cover a wide range of topics related to automotive engineering from fundamental research to industrial applications, highlight the interaction between the driver, vehicle and environment and stimulate scientific interactions and collaborations.

The International Scientific Committee in collaboration with the Organising Committee built up a stimulating scientific program. The program offered over 54 presentations, including key-note speakers and paper sessions. The presentations to this conference covered current research in motor vehicle and motors conducted in 12 countries from all over the world.

We were pleased to have professor Emrullah Hakan Kaleli (from Yıldız Technical University, Türkiye), professor Ralph Pütz (from Landshut University UAS, Germany) and professors Nenad Miljić and Slobodan Popović (from University of Belgrade, Serbia) as the keynote speakers, addressing Challenges and solutions in using hydrogen as a fuel for internal combustion engines, using nanoboric acid (nBA) additive added in engine oil, as well as European policy on future road mobility.

We are sure this program will trigger lively discussion and will project researchers to new developments.

The 10th Congress MVM 2024 was financially supported by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia.

We would like to thank experienced and young researchers, for attending and bringing their expertise and innovative ideas to our conference.

Special thanks are due to the International Scientific Board Members and all reviewers for their significant contribution in the high level of the conference.

Scientific and Organizational committee of Congress MVM2024

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MVM2024-036

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SHAPE FUNCTION OPTIMIZATION FOR STATIC ANALYSIS OF COMPOSITE MATERIALS USED IN AUTOMOTIVE INDUSTRY

ABSTRACT: This study investigates the details of the static analysis of composite materials, which are increasingly important in the automotive industry due to their exceptional mechanical properties. The main focus is on the comparative analysis between already existing and newly developed shape functions, which aim to accurately describe the deformation behavior of composite structures. Special attention is paid to higher order shear deformation theories that use shape functions for more detailed modeling of the behavior of composite laminate plate. Numerical simulations are carried out using Matlab programming codes to precisely quantify the material responses under different loading conditions. The validation of the results is done through a rigorous comparison with the findings of relevant research. In particular, symmetrical cross-ply and angle-ply laminates are investigated to elucidate their specific mechanical properties and practical applicability in industrial settings. These results are crucial for the development of optimized structural solutions in the automotive industry, providing theoretical insights and experimental confirmation of the performance of composite materials in real operating scenarios.

KEY WORDS: static analysis, higher order shear deformation theories, shape functions, composite materials, numerical simulation, Matlab

INTRODUCTION

Composite laminates represent a key innovation in the modern automotive industry, enabling significant advancements in vehicle performance, efficiency, and safety. These materials, including carbon fiber and glass fibers, offer a range of advantages over traditional materials such as steel and aluminum. The primary benefit of composite laminates lies in their ability to reduce vehicle weight. This reduction in mass contributes to improved fuel

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efficiency and lower emissions of harmful gases, which is crucial in the contemporary context of combating climate change and global warming.

In addition to weight reduction, composites offer high strength and impact resistance, which significantly enhances vehicle safety. These materials absorb energy more effectively during collisions, thereby increasing passenger protection. Furthermore, the ability of composites to be molded into aerodynamic shapes allows for the optimization of vehicle performance and further improves fuel efficiency. Additionally, the corrosion resistance of composite materials contributes to the extended lifespan of components and reduces the need for maintenance.

In modern automotive design, composite laminates enable the creation of complex shapes and aesthetically pleasing finishes, contributing to innovative design solutions and enhancing the visual identity of vehicles. Although the production of composite materials can be energy-intensive, their durability and fuel efficiency over the vehicle's lifespan make a significant contribution to reducing the overall environmental impact.

In the application of composite materials, racing cars, such as Formula 1 vehicles, utilize composite laminates due to their extreme strength and lightweight properties. In mass-produced vehicles, composites are used to manufacture various components, including hoods, bumpers, and interior panels. Additionally, in electric vehicles, weight reduction is crucial for optimizing battery range, making composites a key element in this segment.

This paper presents a static analysis of composite laminates that can be used in the automotive industry, focusing on their behavior under static bending problems. The aim is to provide a comprehensive overview of the innovations these materials offer, as well as to explore future trends and opportunities in this field.

A detailed review of the use of composite materials in the automotive industry is provided in [1]. Olsson and Olsson emphasize that the use of composite materials, such as carbon fiber and glass fibers, enables the development of lighter and more efficient vehicles, thereby reducing fuel consumption and emissions of harmful gases, which is particularly important in the fight against climate change [2]. Zhang and colleagues further highlight the importance of the mechanical properties of these materials, particularly their resistance to dynamic forces, which contributes to vehicle safety [3].

The analysis of the fracture behavior of composite laminates during collisions, as described by Aboudi, Bednarcyk, and Gates, demonstrates that these materials have superior energy absorption capabilities, thereby reducing the risk of passenger injuries [4]. On the other hand, Kumar and Chung focus on the development of advanced lightweight materials that further enhance the energy efficiency of vehicles [5], while Wang, Li, and Jiang provide insights into numerical simulations of composite laminate buckling, which is crucial for ensuring structural stability under load [6].

Additionally, research related to static stress problems in composite laminates often involves shear deformation theories based on shape functions. For instance, studies by M. K. K. Yang and X. F. Zhang provide a detailed analysis of buckling problems using advanced mathematical models that incorporate complex shape functions for more accurate predictions of composite structure behavior under various loads [7]. Furthermore, research in this field frequently employs shear deformation theories, such as those by Reddy and Hinton, to analyze composite laminates with the aim of better assessing their load-bearing capacity and stability [8][9].

BASIC THEORETICAL FOUNDATIONS OF HIGHER-ORDER SHEAR DEFORMATION THEORY (HSDT)

Higher-Order Shear Deformation Theory (HSDT) represents a sophisticated approach for analyzing composite laminates, enabling more accurate modeling of shear deformations within material layers. Unlike classical shear deformation theories, HSDT takes into account multiple orders of shear deformations and rotations, allowing for a more detailed representation of complex deformations in composite structures.

In the undeformed configuration of a laminate plate, an arbitrary point is selected, and a tangent is placed on the elastic line of the mid-surface. A line normal to the tangent is considered. According to classical plate theory, the normal at the selected point remains a straight line and is perpendicular to the tangent at the observed point. According to the First-Order Shear Deformation Theory (FSDT), an additional rotation of the normal occurs due to shear, although the normal still maintains its geometric linearity. In real cases, there is a change in the geometric linearity of the normal, which is defined by the assumed displacement shapes described by Higher-Order Shear Deformation Theories (HSDTs). Even in the case of material linearity, Higher-Order Shear Deformation Theory introduces the concept of geometric nonlinearity. The direction of the initial normal changes according to nonlinear mathematical curves.

HSDT theories are based on shape functions. This theory utilizes shape functions that include linear and quadratic terms with respect to the layer thickness, thereby enhancing the accuracy of simulations. These shape functions are the subject of research by many authors and can be polynomial, trigonometric, exponential, hyperbolic, etc.
The assumed displacement shapes that define geometric nonlinearity are:

$$
\begin{aligned}
u(x,y,z) &= u_0(x,y) - z\frac{\partial w}{\partial x}(x,y) + f(z)\theta_x, \\
v(x,y,z) &= v_0(x,y) - z\frac{\partial w}{\partial y}(x,y) + f(z)\theta_y, \\
w(x,y,z) &= w_0(x,y).
\end{aligned} \tag{1}
$$

where:

u_0, v_0, w_0 - displacements of points on the mid-surface of the laminate,

$\frac{\partial w}{\partial x}$, $\frac{\partial w}{\partial y}$ - angles of rotation of the normal relative to the vertical axis due to bending,

$f(z)$ - shape functions.

Based on the assumed displacement shapes, the components of the strain vector in the domain of linear elasticity are determined. Further, using Hooke's law for anisotropic materials, the components of the stress vector are also found.
If a rectangular plate of dimensions $a \times b \times h$ is considered using the principle of virtual displacements and introducing a sine function as a load in the form of:

$$F_z = q_0 \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$$

The equilibrium equations are obtained:

$$
\begin{aligned}
\delta u_0 &: \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0, \\
\delta v_0 &: \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0, \\
\delta w_0 &: \frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} - F_z = 0, \\
\delta \theta_x &: \frac{\partial P_x}{\partial x} + \frac{\partial P_{xy}}{\partial y} - R_x = 0, \\
\delta \theta_y &: \frac{\partial P_{xy}}{\partial x} + \frac{\partial P_y}{\partial y} - R_y = 0,
\end{aligned} \tag{2}
$$

where the forces and moments are defined as:

$$\left(N_{xx}, N_{yy}, N_{xy}, Q_x, Q_y\right) = \int_{h^-}^{h^+} \left(\sigma_{xx}, \sigma_{yy}, \tau_{xy}, \tau_{xz}, \tau_{yz}\right) dz \quad \left(M_{xx}, M_{yy}, M_{xy}, R_x, R_y\right) = \int_{h^-}^{h^+} \left(\sigma_{xx}, \sigma_{yy}, \tau_{xy}, \tau_{xz}, \tau_{yz}\right) z dz. \tag{3}$$

NUMERICAL RESULTS

The mathematical model for the application of HSDT is based on solving differential equations that include these shape functions. These equations are used to calculate stresses and deformations in each layer of the laminate.
The use of HSDT within the finite element method allows for detailed numerical simulation of composite laminates [10]. Shape functions are applied to precisely model stresses and deformations within each element, contributing to improved analysis and optimization of composite structures. This methodology is particularly significant for analyzing thicker laminates and complex loading conditions, where classical theories do not provide sufficient accuracy.
In the development of a new shape function, it is simpler to apply analytical methods to solve differential equations, which has been done in this paper.
In the context of macromechanical analysis of the static behavior of composite laminates, problems of bending of cross-ply laminate plates have been considered. Contemporary research, such as that referenced in papers [11], [12], [13], has significantly contributed to the understanding of this issue. Specifically, a square laminate plate under sinusoidal loading has been examined using various deformation theories based on shape functions. In this study, MATLAB programs have been developed that combine symbolic and numerical methods, where the number of introduced variables depends on the number of independent variables defined by the assumed displacement shapes.

The boundary conditions along the edges of a simply supported rectangular plate based on [13] are:

$$v_0 = w_0 = \theta_y = N_x = M_x = P_x = 0, \text{ at the edges where } x = 0 \text{ and } x = a,$$
$$u_0 = w_0 = \theta_x = N_y = M_y = P_b = 0, \text{ at the edges where } y = 0 \text{ and } y = b. \tag{4}$$

Considering the previously defined boundary conditions based on [13], it is possible to assume the Navier-type solution in the form of:

$$u_0(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b},$$
$$v_0(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b},\ \ \theta_x(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} T_{xmn} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b}, \tag{5}$$
$$w_0(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b},\ \theta_y(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} T_{ymn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b}.$$

where $U_{mn}, V_{mn}, W_{mn}, T_{xmn}, T_{ymn}$ arbitrary parameters that need to be determined.

To enable comparison of the results with reference results from the literature, a square laminate plate of dimensions a×aa×a, or, in a general case, a rectangular plate of type a×ba×b, composed of layers of material with the following values for engineering constants was considered:

$$E_1/E_2 = 25,\ G_{12}/E_2 = \ = 0.5,\ G_{13}/E_2 = \ = 0.5,\ G_{23}/E_2 = 0.2,\ \nu_{12} = \nu_{13} = \nu_{23} = 0.25$$

The numerical procedure for obtaining results was carried out in MATLAB, combining symbolic and numerical programming. In the presentation of numerous values, normalization was performed:

$$\bar{w} = \frac{100E_2h^3}{q_0a^4} w\left(\frac{a}{2},\frac{b}{2},0\right); \quad \bar{\sigma}_{xx} = \frac{h^2}{qb^2}\sigma_{xx}\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right)$$
$$\bar{\sigma}_{yy} = \frac{h^2}{q_0b^2}\sigma_{yy}\left(\frac{a}{2},\frac{b}{2},\frac{h}{4}\right); \quad \bar{\sigma}_{xy} = \frac{h^2}{q_0b^2}\sigma_{xy}\left(0,0,\frac{h}{2}\right)$$
$$\bar{\sigma}_{yz} = \frac{h}{q_0b}\sigma_{yz}\left(\frac{a}{2},0,0\right); \quad \bar{\sigma}_{xz} = \frac{h}{q_0b}\sigma_{xz}\left(0,\frac{b}{2},0\right)$$

For the analysis of the bending problem, this paper proposes the shape function:

$$f(z) = z\left(\cosh\left(\frac{z}{h}\right) - 1.388\right) \tag{7}$$

In the development of the new shape function, consideration was given to the fundamental assumptions that the shape functions must satisfy, namely that the function be odd with respect to the zz-coordinate and that it meets the zero-stress boundary conditions.

The main advantage of the newly introduced shape function is its simple form, and using this shape function eliminates the need for numerical integration, which significantly reduces calculation time.

In the following tables, a comparison is made between the results obtained using the newly introduced shape function and those presented in reference papers from the relevant scientific field.

Table 1. Dimensionless values of stresses and displacements under sinusoidal bending force for a symmetric cross-ply laminate $\left[0^0/90^0/90^0/0^0\right]$ за fixed relations $E_1/E_2 = 25$ and $a/h = 4$.

Source	$\bar{w}$	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{yy}$	$\bar{\sigma}_{xy}$	$\bar{\sigma}_{yz}$	$\bar{\sigma}_{xz}$
Present	1.8981	0.6636	0.6328	0.0401	0.2383	0.2052
Reddy [24]	1.8939	0.6806	0.6463	0.0450	0.2390	0.2109
Mantari et al. [34]	1.8940	0.6640	0.6310	0.0440	0.2390	0.2060
Singh and Singh [32]	1.9088	0.7204	0.6370	0.0473	0.2800	0.2318
Ferreira et al. [110]	1.9091	0.6429	0.6265	0.0443	-	0.2173
Wang and Shi [111]	1.9073	0.7361	0.6994	0.0435	-	0.2110

In Tables 1 and 2, the results of this study are compared with relevant literature sources, including the works of Reddy [14], Mantari et al. [15], Singh and Singh [16], Ferreira et al. [17], and Wang and Shi [18].

The values of the central point displacement ($\bar{w}$) indicate minimal differences between the results of this study and the literature, with deviations ranging from 0.07% compared to Reddy's work to 3.14% compared to Wang and Shi.

Normal stresses in the xx-direction ($\bar{\sigma}_{xx}$) show stability, with differences ranging from -0.05% compared to Mantari et al. to +2.86% compared to Wang and Shi. For normal stresses in the yy-direction ($\bar{\sigma}_{yy}$), the differences are also minimal, except in the case of Wang and Shi, where the deviation is +4.71%.

Regarding the values of in-plane shear stresses ($\bar{\sigma}_{xy}$) only noticeable deviation is compared to the results of Wang and Shi, amounting to +2.96%.

In the case of shear stresses $\bar{\sigma}_{xz}$ i $\bar{\sigma}_{yz}$, the results are generally consistent with the reference values, except in the case of Singh and Singh [32], where significant deviations were observed in $\bar{\sigma}_{yz}$ (+22.12%) and $\bar{\sigma}_{xz}$ (+18.18%).

Table 2. Dimensionless values of stresses and displacements under sinusoidal bending force for a symmetric cross-ply laminate for fixed relations $E_1/E_2 = 25$ and $a/h = 10$.

Source	$\bar{w}$	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{yy}$	$\bar{\sigma}_{xy}$	$\bar{\sigma}_{yz}$	$\bar{\sigma}_{xz}$
Present	0.7144	0.5453	0.3894	0.0270	0.1523	0.2638
Reddy [24]	0.7149	0.5589	0.3974	0.0273	0.1530	0.2697
Mantari et al. [34]	0.7150	0.5450	0.3880	0.0270	0.1530	0.2640
Singh and Singh [32]	0.7224	0.5608	0.3880	0.0278	0.1860	0.3118
Ferreira et al. [110]	0.7303	0.5487	0.3966	0.0273	-	0.2993
Wang and Shi [111]	0.7368	0.5609	0.4077	0.0274	-	0.3002

The analysis shows that the results are largely consistent with the reference values, with deviations mostly within acceptable limits for engineering analyses. These small differences confirm the consistency of the numerical methods applied in various studies and can be considered negligible in the context of engineering analyses. Although the differences are minor, they may reflect variations in approaches to modeling boundary conditions or material properties, but they remain within acceptable limits for such analyses. This suggests that the methodology used in this study is generally in line with the literature, with potential specific parameters or assumptions in individual studies that might cause larger discrepancies. Deviations may also result from different theoretical approaches or numerical methods and require further investigation to clarify the causes of the differences.

Table 3. The normalized displacement and stresses of a simply supported rectangular $\left[0^0/90^0/0^0\right]$ laminated plate $b/a = 3$ under a sinusoidal load for fixed relations $E_1/E_2 = 25$ i $a/h = 4$.

Source	$\bar{w}$	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{yy}$	$\bar{\sigma}_{xy}$	$\bar{\sigma}_{yz}$	$\bar{\sigma}_{xz}$
Present	2.6378	1.0314	0.1028	0.0263	0.0347	0.2714
Reddy [24]	2.6411	1.0356	0.1028	0.0263	0.0348	0.2724
Mantari et al. [49]	2.6841	1.1180	0.1030	0.0274	0.0360	0.3020
Karama et al. [109]	2.6838	1.0974	0.1038	0.0272	0.0360	0.2982
Touratier [31]	2.6660	1.0670	0.1030	0.0268	0.0355	0.2850

Tables 3 and 4 present the results of this study compared with relevant literature sources, including the works of Reddy [14], Mantari et al. [19], Karama et al. [20], and Touratier [11].

Differences in the central point displacement ($\bar{w}$) between the results of this study and those from the literature are very small, with deviations ranging from 0.13% compared to Reddy to +1.76% compared to Mantari et al. Similarly, compared to Karama et al., the difference is 0.11%, while deviations compared to Mantari et al. are slightly larger, up to 2.15%.

For normal stresses in the xx-direction ($\bar{\sigma}_{xx}$) variations range from 0.41% (compared to Reddy) to 7.73% (compared to Mantari et al.), while deviations compared to Karama et al. are smaller, ranging from -0.11% to 2.34%. For normal stresses in the yy-direction ($\bar{\sigma}_{yy}$) the results are nearly identical, with minimal deviations that are practically negligible. For the shear stress ($\bar{\sigma}_{xz}$ and $\bar{\sigma}_{yz}$), the results showed small variations, with differences reaching up to +4.18% compared to Mantari et al. Larger differences are observed in the case of shear stresses $\bar{\sigma}_{xz}$ and $\bar{\sigma}_{yz}$, with deviations reaching up to 11.26% for $\bar{\sigma}_{yz}$ compared to Mantari et al.

These small deviations suggest a high level of precision in the numerical models used in this study. As with the previous two tables, the differences may be attributed to specific numerical approaches used in various studies, highlighting the need for further research to better understand the causes of these variations.

Table 4. The normalized displacement and stresses of a simply supported rectangular laminated plate under a sinusoidal load for fixed relations $E_1/E_2 = 25$ and $a/h = 10$.

Source	$\bar{w}$	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{yy}$	$\bar{\sigma}_{xy}$	$\bar{\sigma}_{yz}$	$\bar{\sigma}_{xz}$
Present	0.8613	0.6916	0.0398	0.0115	0.0169	0.2857
Reddy [24]	0.8622	0.6924	0.0398	0.0115	0.0170	0.2859
Mantari et al. [49]	0.8800	0.7080	0.0400	0.0118	0.0180	0.3260
Karama et al. [109]	0.8768	0.7043	0.0403	0.0117	0.0175	0.3194
Touratier [31]	0.8700	0.6980	0.0401	0.0116	0.0172	0.3020

CONCLUSION

This study provides a comprehensive analysis of composite laminates using the advanced Higher-Order Shear Deformation Theory (HSDT) to evaluate the static behavior of these materials. The proposed results have been compared with previous research, allowing for an assessment of the accuracy of the numerical methods used and the identification of potential areas for improvement.
The general conclusions are:

1. High precision: The results obtained in this study demonstrate a high level of agreement with the literature values, with deviations generally within acceptable limits. This consistency confirms the accuracy of the numerical methods used and the proposed shape functions, providing reliable data for the analysis of stresses and deformations in composite structures.
2. Analysis of deviations: The largest deviations were observed in shear stresses. Stresses in the yz direction showed minor deviations compared to literature values, while stresses in the xz direction exhibited more significant deviations. These deviations suggest potential variations in methodologies or models used in the analysis, indicating a need for further standardization of methods in future research.
3. Consistency and reliability: Most of the results from this study are consistent and reliable, highlighting the effectiveness of the proposed shape function. Minor deviations in stresses along the x and z directions, though within acceptable limits, suggest that a more detailed analysis could contribute to the improvement of the models and methods.

Further research should focus on:

- Detailed Analysis of Differences: Investigating the discrepancies arising from the use of various shape functions.
- Standardization of Methodologies: It is recommended to standardize methodologies in the analysis of composite laminates to reduce variations between different studies and enhance result reliability.
- Development of New Models: Developing new shape functions and improving numerical techniques can contribute to greater accuracy and efficiency in the analysis of composite structures.

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