



# A pseudo-rigid-body approach for dynamic analysis of planar compliant mechanisms

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## ABSTRACT

A new approach to the determination of the natural frequencies of planar flexure-hinge mechanisms with lumped compliance and small deformations is presented. Variable thickness flexure hinges are considered. The approach is based on a new pseudo-rigid body model (PRBM) in which a flexure-hinge mass is discretized into three particles. The positions and masses of the particles are determined using the Routh method for representing rigid bodies by equipomental systems of particles. The flexure hinges can be considered in the frame of either Euler–Bernoulli or Timoshenko beam theory, where both lateral and axial deformations of the flexure hinges are considered. Using the proposed PRBM, the rigid multibody model of the entire compliant mechanism is obtained. A straightforward procedure is presented to form both linearized differential equations of motion and a corresponding frequency equation of the mechanism based on the Lagrange equations of the second kind. The procedure does not require the calculation of internal forces. In several numerical examples, the validity, accuracy, and generality of the approach are demonstrated by comparison with results obtained by the finite element analysis and other approaches published in the literature. The numerical examples confirm that the approach can be used to accurately determine the frequencies corresponding to the higher vibration modes of compliant mechanisms.

## 1. Introduction

Compliant mechanisms are widely used in various branches of engineering due to a number of advantages they provide compared to classical mechanisms (see, e.g., [1–5]). Namely, in compliant mechanisms, the role of classic joints (revolute joints, prismatic joints, etc.) is performed by flexure hinges. In this way, unwanted effects such as joint friction, wear, and backlash in joints, which regularly occur during the movement of classic mechanisms, were avoided. Kinetostatics and dynamics of compliant mechanisms represent a very actual and attractive field of scientific research for scientists and engineers. A very detailed overview of the results and current problems in this field can be found in [6]. For this reason, in this introductory part, we will only give a narrower overview of those references that are related to dynamic analysis of planar flexure-based compliant mechanisms with small deformations and lumped compliance. The main characteristic of compliant mechanisms with lumped compliance is that they are presented in the form of multibody systems composed of rigid bodies with arbitrary geometry interconnected by flexure hinges where only sources of compliance are concentrated in flexure hinges (see, e.g., [7–9]). Here, in accordance with the theory of small vibration of continuous systems (see, e.g., [10]), the term small deformations means that the

slope of a flexure hinge neutral axis is much less than unity and that the axial and transverse displacements of points at the neutral axis are much less than the flexure hinge length.

In papers [11–14], the dynamics of compliant mechanisms are considered by using various approaches under the assumption that flexure hinges are massless. The transfer matrix method was used in [11], whereas the Newton–Euler equations of motion were applied in [12,13] in studying the dynamics of spatial and planar compliant mechanisms, respectively. In [14], the pseudo-rigid-body model (PRBM) for quasi-static analysis of compliant mechanisms shown in [15] was used for the determination of vibration frequencies of the RRR compliant micro-motion stage. On the other hand, in papers [16–36], the mass of flexure hinges is taken into account. So in [16–24] the transfer matrix method was used, the dynamic stiffness method was applied in [25–29], and, finally, in [30] the multi-compliant-body matrix method was presented. The finite element method (FEM) has also been used in various problems of the dynamics of compliant mechanisms, which can be seen, for example, in [31–33]. In today's research, in addition to experimental tests, the FEM has been established as a standard procedure for verifying the accuracy of various computational techniques for

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