Mathematical model for determining the irradiated area of the lower absorber surface of the double exposure flat-plate water solar collector

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Abstract:

The term double exposure, flat-plate water solar collector is related to the solar collector which has the ability to absorb solar irradiation from the upper and lower surface of its own absorber. Absorption of solar irradiation, from its lower absorber surface is accomplished using reflecting surface placed below the collector. In comparison with conventional flat-plate solar collector, at analyzed collector, insulation mounted in the lower part of the collector box is replaced by glazing. Because of the exclusion of the insulation, therefore reducing overall collector heat losses, absorber of the same has to be coated with selective coating on both sides. Described collector is analyzed in order to determine the possibilities of improving its efficiency, in comparison with conventional collector, which among other things depends on size of the irradiated area of the lower absorber surface. This paper presents the mathematical model for determining the irradiated area of the lower absorber surface of the mentioned analyzed collector-reflector system for different possible positions and dimensions of the reflector relative to the collector. The model can be used for numerical optimization of the positions and dimensions of the reflective surface (reflector) relative to the collector. The basis and reason for the future conducting of the numerical analysis, relies on the fact that it is possible, using reflector, to increase the collector absorbed solar irradiation, specifically for examined case for 6.52% (10:00 h), 12.53% (12:00 h) and 30.11% (14:00 h).

Keywords:

Double exposure flat-plate collector, mathematical model, absorbed irradiation.

1. Introduction

The need for the increasing usage of the renewable energy sources, specifically in this case, solar energy, requires conducting a different researchs in order to improve the efficiency of the solar systems. The most common systems for absorbing solar energy are flat-plate solar water collectors which by upper surface of its own absorber absorb solar irradiation. This paper points the possible increase of the amount od energy absorbed by the modified collector system called double exposure flat-plate water collector. The term double exposure, flat-plate water solar collector is related to the solar collector which has the ability to absorb solar irradiation from the upper and lower surface of its own absorber. Absorption of solar irradiation, from its lower absorber surface is accomplished using reflecting surface placed below the collector. In comparison with conventional flat-plate solar collector, at analyzed collector, insulation mounted in the lower part of the collector box is replaced by glazing. Because of the exclusion of the insulation, therefore reducing overall collector heat losses, absorber of the same has to be coated with selective coating on both sides. There are several studies[1,2,3] relative to this modified collector-reflective system. Within them optimization of the tilt angle of the collector and reflector, without taking into consideration the impact of the position and dimension of the reflector relative to the collector on the system efficiency, is executed. This paper presents the mathematical model for determining the irradiated area of the lower absorber surface of the mentioned analyzed collector-reflector system, for different possible positions and dimensions of the reflector relative to the collector, which can be lately used for numerical optimization of the positions and dimensions of the reflector relative to the collector.

2. Mathematical model

The analyzed double exposure flat-plate solar water collector consists of selective absorber and single glazing on its upper and lower side. Reflective surface (hereinafter reflector) is placed below the collector in parallel with the same (Fig. 1). Collector-reflector system (hereinafter CRS) is tilted at angle *G* relative to horizontal plane, where the collector is fixed while the reflector can be moved in plane parallel to collector plane. The assumption adopted before conducting the analysis is reffered to to the fact that solar beam is specularly reflected.



Fig. 1. a) Schematic diagram of the analysed CRS with unit vectors of the solar beam and normal of the CRS surface, b) analysed combination of case E1 (in IZGa plane), with condition $L_r < L_k$, $L_r/2 < L_k/2 - v$, $v < L_r/2$, and case B5 (in SJGa plane), with condition $x = (L_k)/2 + w$, $L_r > L_k$, $x < L_r/2 < L_k + w$

The influence of the mutual position of the reflector and collector on absorbed irradiation is determined by the size of the irradiated area of the lower absorber surface (A_{ozr}). In order to evaluate the same all CRS is observed in tree planes: plane perpendicular to the CRS plane (IZGa plane) and with view from the south, plane north-south (SJGa plane) and CRS plane. There are different cases of mutual position of the reflector and the collector in both planes for which two parameters ",w" i ",v", which take into account distance of the reflector's axis from left (right) collector's edge, when looking in plane IZGa, are introduced (Fig. 2). The same cases exist when CRS is projected in SJGa plane, with difference that instead of parameters for reflector and collector lenght L, the parameters for their width W are used. It is adopted that in IZGa plane, parameters w, v, x are designates with index 1 and in SJGa plane with index 2.





Fig. 2. The position of the reflector relative to the collector defined by parameter w(v): a) w = 0 (case A), b) $w < L_r/2$ (B), c) $w = L_r/2$ (C), d) $w > L_r/2$ (D), e) $v < L_k/2$ (E) if) $v = L_k/2$ (F)

On the displayed figures parameters used in the model are designated and reffered to: w - the distance of the reflector axis measured from the edge of the collector to the exterior (outside) of the same, v - the distance of the reflector axis from the edge of the collector measured to the interior (inside) of the same, x – the distance between reflector and collector axis, y - the distance between the reflector and the collector, Lr - the reflector length and L_k - the collector length. For each of mentioned possible position of reflector relative to collector, defined by the parameters w (4 cases) and v (2 cases), conditions which include different dimensions of reflector lenght and width relative to collector lenght and width, are considered. Accuratelly, condition when $L_r < L_k$, $L_r = L_k$, $L_r > L_k$, for the ratio of the reflector and collector lenght is separately considered as the condition when W_r $\langle W_k, W_r = W_k, W_r \rangle \langle W_k$, for ratio of the reflector and collector width. In the relations, beside previously described parameters, lenght parameters as λ , ξ , a_p , p_{dnsGa} , b_{pdnsGa} , p_{eewGa} , b_{peewGa} , as well as angle parameters, which define instantaneous sun position, as projection of the solar altitude angle on IZG α plane, $\beta_{ewG\alpha}$, and on SJG α plane, $\beta_{nsG\alpha}$, as well as projection of solar azimuth angle $\gamma_{G\alpha}$ on CRS plane, are also included. The form of the relations for irradiated area of the lower absorber surface depends on whether and how reflected beams form the same area. Because of that, terms like full shading (POS), full irradiation (POZ) and partial irradiation (shading)(DO), are introduced. Full irradiation is situation when solar beams begin to separate from the collector edge or when shadow which collector casts on reflector, does not affect the irradiated area. Otherwise, there is situation called partial irradiation. The term POS points on situation when beams, which falling over the collector edge in one of the planes, affect the forming of irradiated area if and only if the same in other plane fall beside collector edge and reflect on the collector surface. If beams fall over the collector edge in both planes, in that case there is no generation of the mentioned area. For the various situation combinations there are various relations for the instantaneous irradiated area of the lower absorber surface depending on where the same occur whether in IZGa or SJGa plane:

a)
$$POS_{ewGa}$$
- DO_{nsGa} , POS_{ewGa} - POZ_{nsGa} , POS_{nsGa} - DO_{ewGa} , POS_{nsGa} - POZ_{ewGa} i POZ_{ewGa} - POZ_{nsGa}
 $A_{ozr} = \xi \cdot \lambda$ (1)

b) DO_{ewGa} - DO_{nsGa} $A_{ozr} = \xi \cdot a_p + \lambda \cdot b_{pnsGa}$ (2)

c)
$$POZ_{ewG\alpha}$$
- $DO_{nsG\alpha}$
 $A_{ozr} = \xi \cdot \lambda + \lambda \cdot b_{pnsG\alpha}$ (3)

 $d) DO_{ewGa} - POZ_{nsGa}$ $A_{ozr} = \xi \cdot a_p$ (4)

Within this analysis equations of parameters (eq. 5–114), that are included in relations for determination of the irradiated area of the lower absorber surface, as well as all possible

combinations of mutual position of reflector and collector, when looking in $IZG\alpha$ plane, are displayed below:

Equations for β_{nsGa} , β_{ewGa} , γ_{Ga}

$$\cos \beta_{nsG\alpha} = \frac{\cos G(\cos \alpha \cdot \cos \beta \cdot \cos \gamma + \sin \alpha \cdot \cos \beta \cdot \sin \gamma)}{\sqrt{\left[\cos G(\cos \alpha \cdot \cos \beta \cdot \cos \gamma + \sin \alpha \cdot \cos \beta \cdot \sin \gamma)\right]^2 + \left[\sin G(\cos \alpha \cdot \cos \beta \cdot \cos \gamma + \sin \alpha \cdot \cos \beta \cdot \sin \gamma) - \cos G \cdot \sin \beta\right]^2}}$$
(5)
$$\cos \beta_{ewG\alpha} = \frac{\sin \alpha \cdot \cos \beta \cdot \cos \gamma - \cos \alpha \cdot \cos \beta \cdot \sin \gamma}{\sqrt{\left[\sin \alpha \cdot \cos \beta \cdot \cos \gamma - \cos \alpha \cdot \cos \beta \cdot \sin \gamma\right]^2 + \left[\sin G(\cos \alpha \cdot \cos \beta \cdot \cos \gamma + \sin \alpha \cdot \cos \beta \cdot \sin \gamma) - \cos G \cdot \sin \beta\right]^2}}$$
(6)

$$\cos \beta_{\gamma G \alpha} = \frac{\cos G(\cos \alpha \cdot \cos \beta \cdot \cos \gamma + \sin \alpha \cdot \cos \beta \cdot \sin \gamma)}{\sqrt{\left[\cos G(\cos \alpha \cdot \cos \beta \cdot \cos \gamma + \sin \alpha \cdot \cos \beta \cdot \sin \gamma)\right]^2 + \left[\sin \alpha \cdot \cos \beta \cdot \cos \gamma - \cos \alpha \cdot \cos \beta \cdot \sin \gamma\right]^2}}$$

Equations for $\lambda(\xi)$

$$E1 (E1=E2=E4=E5) \text{ For } 0^{\circ} < \gamma_{Ga} < 180^{\circ}, tg\beta^{*}_{ewGa} = y/(L_r/2+v_l)$$
$$POS_{ewGa}: 90^{\circ} > \beta_{ewGa} > \beta^{*}_{ewGa} \Longrightarrow \lambda = \frac{L_r}{2} + v_1 - \frac{y}{tg\beta_{ewGa}}$$
(8)

E1 For $180^{\circ} < \gamma_{Ga} < 360^{\circ}$, $tg\beta^{*}_{ewGa} = y/(L_r/2 - v_l)$, $tg\beta^{**}_{ewGa} = y/(L_r/2 + v_l)$, $tg\beta^{***}_{ewGa} = y/(L_r/2 + v_l)$, $tg\beta^{**}_{ewGa} = y/(L_r/2 + v$

$$DO_{ewG\alpha}: 90^{\circ} > \beta_{ewG\alpha} \ge \beta_{ewG\alpha}^{*} \Longrightarrow \lambda = \frac{2y}{tg\beta_{ewG\alpha}}, \ \beta_{ewG\alpha}^{*} > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{**} \Longrightarrow \lambda = \frac{L_{r}}{2} - v_{1} + \frac{y}{tg\beta_{ewG\alpha}}$$
(9)

$$POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^{**} \ge \beta_{ewG\alpha} \ge \beta_{ewG\alpha}^{***} \Longrightarrow \lambda = L_r, \ \beta_{ewG\alpha}^{***} > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{****} \Longrightarrow \lambda = L_k - v_1 + \frac{L_r}{2} - \frac{y}{tg\beta_{ewG\alpha}}$$
(10)

E2 For
$$180^{\circ} < \gamma_{G\alpha} < 360^{\circ}$$
, $tg\beta^{*}_{ewG\alpha} = y/L_r$, $tg\beta^{**}_{ewG\alpha} = y/(L_k - v_l - L_r/2)$, $tg\beta^{***}_{ewG\alpha} = y/(L_k - v_l + L_r/2)$
 $DO_{ewG\alpha}$: $90^{\circ} > \beta_{ewG\alpha} > \beta^{*}_{ewG\alpha} \Rightarrow \lambda = \frac{y}{tg\beta_{ewG\alpha}}$ (11)

$$POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^* \ge \beta_{ewG\alpha} \ge \beta_{ewG\alpha}^{**} \Longrightarrow \lambda = L_r, \ \beta_{ewG\alpha}^{**} > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{***} \Longrightarrow \lambda = L_k - v_1 + \frac{L_r}{2} - \frac{y}{tg\beta_{ewG\alpha}}$$
(12)

E3 (E3=E6) For $0^{\circ} < \gamma_{Ga} < 180^{\circ}$, $tg\beta^{*}_{ewGa} = y/(v_1 - Lr/2)$, $tg\beta^{**}_{ewGa} = y/(L_r/2 + v_1)$

$$POS_{ewG\alpha}: 90^{\circ} > \beta_{ewG\alpha} \ge \beta_{ewG\alpha}^{*} \Longrightarrow \lambda = L_{r}, \ \beta_{ewG\alpha}^{*} > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{**} \Longrightarrow \lambda = \frac{L_{r}}{2} + v_{1} - \frac{y}{tg\beta_{ewG\alpha}}$$
(13)

 $E3 \quad For \quad 180^{\circ} < \gamma_{Ga} < 360^{\circ}, \quad tg\beta^{*}_{ewGa} = y/(v_1 - Lr/2), \quad tg\beta^{**}_{ewGa} = y/(L_r/2 + v_1), \quad tg\beta^{***}_{ewGa} = y/(L_k - v_1 - L_r/2), \quad tg\beta^{****}_{ewGa} = y/(L_k - v_1 - L_r/2), \quad tg\beta^{****}_{ewGa} = y/(L_k - v_1 - L_r/2), \quad tg\beta^{***}_{ewGa} = y/(L_k - v_1 - L_r/2), \quad tg\beta^{**}_{ewGa} = y/(L_k - v_$

 $DO_{ewGa}: \ \beta_{ewGa}^* > \beta_{ewGa} > \beta_{ewGa}^{**} \Longrightarrow \lambda = \frac{L_r}{2} - v_1 + \frac{y}{tg\beta_{ewGa}}$ (15)

 $POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^{**} \ge \beta_{ewG\alpha} \ge \beta_{ewG\alpha}^{***} \Longrightarrow \lambda = L_r, \ \beta_{ewG\alpha}^{***} > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{****} \Longrightarrow \lambda = L_k - v_1 + \frac{L_r}{2} - \frac{y}{tg\beta_{ewG\alpha}}$ (16)

E4 (E4=E7) For $180^{\circ} < \gamma_{Ga} < 360^{\circ}$, $tg\beta^{*}_{ewGa} = y/(L_r/2 - v_l)$, $tg\beta^{**}_{ewGa} = y/(L_k - v_l - L_r/2)$, $tg\beta^{***}_{ewGa} = y/(L_k - v_l - L_r/2)$, $tg\beta^{***}_{ewGa} = y/(L_k - v_l - L_r/2)$

$$DO_{ewG\alpha}: 90^{\circ} > \beta_{ewG\alpha} \ge \beta_{ewG\alpha}^{*} \Longrightarrow \lambda = \frac{2y}{tg\beta_{ewG\alpha}}, \ \beta_{ewG\alpha}^{*} > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{**} \Longrightarrow \lambda = \frac{L_{r}}{2} - v_{1} + \frac{y}{tg\beta_{ewG\alpha}}$$
(17)

 $POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^{**} \ge \beta_{ewG\alpha} > \beta_{ewG\alpha}^{***} \Longrightarrow \lambda = L_k - v_1 + \frac{L_r}{2} - \frac{y}{tg\beta_{ewG\alpha}}$ (18)

E5 (E5=E8) For $180^{\circ} < \gamma_{G\alpha} < 360^{\circ}$, $tg\beta^{*}_{ewG\alpha} = y/(L_k - v_l - L_r/2)$, $tg\beta^{**}_{ewG\alpha} = y/(L_k - v_l + L_r/2)$ $DO_{ewG\alpha}$: $90^{\circ} > \beta_{ewG\alpha} > \beta^{*}_{ewG\alpha} \Rightarrow \lambda = \frac{y}{tg\beta_{ewG\alpha}}$ (19)

 $POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^* \ge \beta_{ewG\alpha} > \beta_{ewG\alpha}^{**} \Longrightarrow \lambda = L_k - v_1 + \frac{L_r}{2} - \frac{y}{tg\beta_{ewG\alpha}}$ (20)

E6 (E6=E9) For $180^{\circ} < \gamma_{Ga} < 360^{\circ}$, $tg\beta^{*}_{ewGa} = y/(v_1 - Lr/2)$, $tg\beta^{**}_{ewGa} = y/(L_k - v_1 - L_r/2)$, $tg\beta^{***}_{ewGa} = y/(L_k - v_1 - L_r/2)$, $tg\beta^{**}_{ewGa} = y/(L_k - v_1 - L_r/2)$.

$$POS_{ewGa}: 90^{\circ} > \beta_{ewGa} \ge \beta_{ewGa}^{*} \Longrightarrow \lambda = L_{r}$$

$$(21)$$

$$DO_{ewG\alpha}: \ \beta_{ewG\alpha}^* > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{**} \Longrightarrow \lambda = \frac{L_r}{2} - v_1 + \frac{y}{tg\beta_{ewG\alpha}}$$
(22)

$$POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^{**} \ge \beta_{ewG\alpha} > \beta_{ewG\alpha}^{***} \Longrightarrow \lambda = L_k - v_1 + \frac{L_r}{2} - \frac{y}{tg\beta_{ewG\alpha}}$$
(23)

 $E7 (E7 = E8 = E10 = E11a = E11b) \text{ For } 0^{\circ} < \gamma_{Ga} < 180^{\circ}, tg\beta^{*}_{ewGa} = y/(L_k - v_l - L_r/2), tg\beta^{**}_{ewGa} = y/(L_k/2), tg\beta^{**}_{ewGa} = y/(L_r/2 + v_l)$

$$POS_{ewGa}: 90^{\circ} > \beta_{ewGa} \ge \beta_{ewGa}^{*} \Longrightarrow \lambda = \frac{L_{r}}{2} + v_{1} - \frac{y}{tg\beta_{ewGa}}$$
(24)

$$DO_{ewG\alpha}: \beta_{ewG\alpha}^* > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{**} \Longrightarrow \lambda = \frac{L_r}{2} - L_k + v_1 + \frac{y}{tg\beta_{ewG\alpha}}$$
(25)

$$POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^{**} \ge \beta_{ewG\alpha} > \beta_{ewG\alpha}^{***} \Longrightarrow \lambda = \frac{L_r}{2} + v_1 - \frac{y}{tg\beta_{ewG\alpha}}$$
(26)

E9 For $0^{\circ} < \gamma_{Ga} < 180^{\circ}$. $tg\beta^{*}_{ewGa} = y/(v_1 - Lr/2)$, $tg\beta^{**}_{ewGa} = y/(L_k - v_1 - L_r/2)$, $tg\beta^{***}_{ewGa} = y/(L_k/2)$, $tg\beta^{***}_{ewGa} = y/(L_r/2 + v_1)$

 $POS_{ewG\alpha}: 90^{\circ} > \beta_{ewG\alpha} \ge \beta_{ewG\alpha}^{*} \Longrightarrow \lambda = L_{r}, \ \beta_{ewG\alpha}^{*} > \beta_{ewG\alpha} \ge \beta_{ewG\alpha}^{**} \Longrightarrow \lambda = \frac{L_{r}}{2} + v_{1} - \frac{y}{tg\beta_{ewG\alpha}}$ (27)

$$DO_{ewGa}: \beta_{ewGa}^{**} > \beta_{ewGa} > \beta_{ewGa}^{***} \Longrightarrow \lambda = \frac{L_r}{2} - L_k + v_1 + \frac{y}{tg\beta_{ewGa}}$$
(28)

 $POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^{***} \ge \beta_{ewG\alpha} > \beta_{ewG\alpha}^{****} \Longrightarrow \lambda = \frac{L_r}{2} + v_1 - \frac{y}{tg\beta_{ewG\alpha}}$ (29)

E10 (E10=E11a) For $180^{\circ} < \gamma_{Ga} < 360^{\circ}$, $tg\beta^{*}_{ewGa} = y/(L_r/2 - v_l)$, $tg\beta^{**}_{ewGa} = y/(L_k/2)$, $tg\beta^{***}_{ewGa} = y/(L_k/2)$

$$DO_{ewG\alpha}: 90^{\circ} > \beta_{ewG\alpha} \ge \beta_{ewG\alpha}^{*} \Longrightarrow \lambda = \frac{2y}{tg\beta_{ewG\alpha}}, \ \beta_{ewG\alpha}^{*} > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{**} \Longrightarrow \lambda = \frac{L_{r}}{2} - v_{1} + \frac{y}{tg\beta_{ewG\alpha}}$$
(30)

 $POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^{**} \ge \beta_{ewG\alpha} > \beta_{ewG\alpha}^{***} \Longrightarrow \lambda = L_k - v_1 + \frac{L_r}{2} - \frac{y}{tg\beta_{ewG\alpha}}$ (31)

E11b (*E11b=E12=E13=E14=E15*) For $180^{\circ} < \gamma_{Ga} < 360^{\circ}$, $tg\beta^{*}_{ewGa} = y/(L_k/2)$, $tg\beta^{**}_{ewGa} = y/(L_r/2 - v_l)$, $tg\beta^{***}_{ewGa} = y/(L_k-v_l+L_r/2)$

 $DO_{ewG\alpha}: 90^{\circ} > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{*} \Longrightarrow \lambda = \frac{2y}{tg\beta_{ewG\alpha}}$ (32)

 $POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^* \ge \beta_{ewG\alpha} \ge \beta_{ewG\alpha}^{**} \Longrightarrow \lambda = L_k , \ \beta_{ewG\alpha}^{**} > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{***} \Longrightarrow \lambda = L_k - v_1 + \frac{L_r}{2} - \frac{y}{tg\beta_{ewG\alpha}}$ (33)

E12 For
$$0^{\circ} < \gamma_{Ga} < 180^{\circ}, tg\beta^{*}_{ewGa} = y/(L_k/2), tg\beta^{**}_{ewGa} = y/(L_r/2 + v_l)$$

 $DO_{ewGa}: 90^{\circ} > \beta_{ewGa} > \beta^{*}_{ewGa} \Longrightarrow \lambda = \frac{y}{tg\beta_{ewGa}}$
(34)

 $POZ_{ewG\alpha}: \ \beta^*_{ewG\alpha} \ge \beta_{ewG\alpha} > \beta^{**}_{ewG\alpha} \Longrightarrow \lambda = \frac{L_r}{2} + v_1 - \frac{y}{tg\beta_{ewG\alpha}}$ (35)

E13 For $0^{\circ} < \gamma_{G\alpha} < 180^{\circ}, tg\beta^{*}_{ewG\alpha} = y/(L_r/2 + v_l - L_k), tg\beta^{**}_{ewG\alpha} = y/(L_k/2), tg\beta^{***}_{ewG\alpha} = y/(L_r/2 + v_l)$ $DO_{ewG\alpha}: 90^{\circ} > \beta_{ewG\alpha} \ge \beta^{*}_{ewG\alpha} \Longrightarrow \lambda = \frac{2y}{tg\beta_{ewG\alpha}}, \ \beta^{*}_{ewG\alpha} > \beta^{**}_{ewG\alpha} \Longrightarrow \lambda = \frac{L_r}{2} + v_l - L_k + \frac{y}{tg\beta_{ewG\alpha}}$ (36)

$$POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^{**} \ge \beta_{ewG\alpha} > \beta_{ewG\alpha}^{***} \Longrightarrow \lambda = \frac{L_r}{2} + v_1 - \frac{y}{tg\beta_{ewG\alpha}}$$
(37)

E14 For $0^{\circ} < \gamma_{Ga} < 180^{\circ}$, $tg\beta^{*}_{ewGa} = y/(L_r/2 + v_l - L_k)$, $tg\beta^{**}_{ewGa} = y/(L_r/2 + v_l)$ DO_{ewGa} : $90^{\circ} > \beta_{ewGa} > \beta^{*}_{ewGa} \Longrightarrow \lambda = \frac{2y}{tg\beta_{ewGa}}$ (38)

$$POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^* \ge \beta_{ewG\alpha} > \beta_{ewG\alpha}^{**} \Longrightarrow \lambda = \frac{L_r}{2} + v_1 - \frac{y}{tg\beta_{ewG\alpha}}$$
(39)

E15 For $0^{\circ} < \gamma_{Ga} < 180^{\circ}$, $tg\beta^{*}_{ewGa} = y/(L_k/2)$, $tg\beta^{***}_{ewGa} = y/(L_r/2 + v_l - L_k)$, $tg\beta^{****}_{ewGa} = y/(L_r/2 + v_l)$ DO_{ewGa} : $90^{\circ} > \beta_{ewGa} > \beta^{*}_{ewGa} \Rightarrow \lambda = \frac{2y}{tg\beta_{ewGa}}$ (40)

$$POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^* \ge \beta_{ewG\alpha} \ge \beta_{ewG\alpha}^{**} \Longrightarrow \lambda = L_k, \ \beta_{ewG\alpha}^{**} > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{***} \Longrightarrow \lambda = \frac{L_r}{2} + v_1 - \frac{y}{tg\beta_{ewG\alpha}} (41)$$

*

B1 (**B1=B2=B3=B4**) For
$$0^{\circ} < \gamma_{G\alpha} < 180^{\circ}$$
, $tg\beta^{*}_{ewG\alpha} = y/(L_r/2 - w_l)$
 $POS_{ewG\alpha}: 90^{\circ} > \beta_{ewG\alpha} > \beta^{*}_{ewG\alpha} \Rightarrow \lambda = \frac{L_r}{2} - w_1 - \frac{y}{tg\beta_{ewG\alpha}}$
(42)

For $180^{\circ} < \gamma_{Ga} < 360^{\circ}$, $tg\beta^{*}_{ewGa} = y/(L_r/2 - w_l)$, $tg\beta^{**}_{ewGa} = y/(L_r/2 + w_l)$, $tg\beta^{***}_{ewGa} = y/(L_k + w_l - L_r/2)$, $tg\beta^{****}_{ewGa} = y/(L_k + w_l + L_r/2)$

$$DO_{ewGa}: 90^{\circ} > \beta_{ewGa} > \beta_{ewGa}^{*} \Longrightarrow \lambda = \frac{2y}{tg\beta_{ewGa}}$$
(43)

 $POZ_{ewGa}: \ \beta_{ewGa}^* \ge \beta_{ewGa} > \beta_{ewGa}^{**} \Longrightarrow \lambda = \frac{L_r}{2} - w_1 + \frac{y}{tg\beta_{ewGa}}$ (44)

$$POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^{**} \ge \beta_{ewG\alpha} \ge \beta_{ewG\alpha}^{***} \Longrightarrow \lambda = L_r, \ \beta_{ewG\alpha}^{***} > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{****} \Longrightarrow \lambda = L_k + w_1 + \frac{L_r}{2} - \frac{y}{tg\beta_{ewG\alpha}}$$
(45)

B2 For $180^{\circ} < \gamma_{Ga} < 360^{\circ}, tg\beta^{*}_{ewGa} = y/(L_{r}/2 - w_{l}), tg\beta^{***}_{ewGa} = y/(L_{r}/2 + w_{l}), tg\beta^{****}_{ewGa} = y/(L_{k} + w_{l} + L_{r}/2)$ $DO_{ewGa}: 90^{\circ} > \beta_{ewGa} > \beta^{*}_{ewGa} \Rightarrow \lambda = \frac{2y}{tg\beta_{ewGa}}$ (46)

 $POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^* \ge \beta_{ewG\alpha} > \beta_{ewG\alpha}^{**} \Longrightarrow \lambda = \frac{L_r}{2} - w_1 + \frac{y}{tg\beta_{ewG\alpha}}$ (47)

$$POZ_{ewG\alpha}: \ \beta_{ewG\alpha} = \beta_{ewG\alpha}^{**} \Longrightarrow \lambda = L_r, \ \beta_{ewG\alpha}^{**} > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{***} \Longrightarrow \lambda = L_k + w_1 + \frac{L_r}{2} - \frac{y}{tg\beta_{ewG\alpha}}$$
(48)

B3 For $180^{\circ} < \gamma_{Ga} < 360^{\circ}$, $tg\beta^{*}_{ewGa} = y/(L_r/2 - w_l)$, $tg\beta^{**}_{ewGa} = y/(L_k + w_l - L_r/2)$, $tg\beta^{***}_{ewGa} = y/(L_r/2 + w_l)$, $tg\beta^{***}_{ewGa} = y/(L_k + w_l + L_r/2)$

$$DO_{ewG\alpha}: 90^{\circ} > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{*} \Longrightarrow \lambda = \frac{2y}{tg\beta_{ewG\alpha}}$$
(49)

 $POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^* \ge \beta_{ewG\alpha} > \beta_{ewG\alpha}^{**} \Longrightarrow \lambda = \frac{L_r}{2} - w_1 + \frac{y}{tg\beta_{ewG\alpha}}$ (50)

 $POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^{**} \ge \beta_{ewG\alpha} \ge \beta_{ewG\alpha}^{***} \Longrightarrow \lambda = L_k, \ \beta_{ewG\alpha}^{***} > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{****} \Longrightarrow \lambda = L_k + w_1 + \frac{L_r}{2} - \frac{y}{tg\beta_{ewG\alpha}}$ (51)

B4 (**B4=B5=B6=B7=B8=B9**) For $180^{\circ} < \gamma_{Ga} < 360^{\circ}$, $tg\beta^{*}_{ewGa} = y/(L_r/2 - w_l)$, $tg\beta^{**}_{ewGa} = y/(L_r/2 + w_l)$, $tg\beta^{**}_{ewGa} = y/(L_r/2 + w_l)$, $tg\beta^{**}_{ewGa} = y/(L_r/2 + w_l)$, $tg\beta^{**}_{ewGa} = y/(L_r/2 - w_l)$

$$DO_{ewGa}: 90^{\circ} > \beta_{ewGa} > \beta_{ewGa}^{*} \Longrightarrow \lambda = \frac{2y}{tg\beta_{ewGa}}$$
(52)

$$POZ_{ewGa}: \ \beta_{ewGa}^{*} \ge \beta_{ewGa} \ge \beta_{ewGa}^{**}, \Rightarrow \lambda = L_{k}, \ \beta_{ewGa}^{**} > \beta_{ewGa} > \beta_{ewGa}^{***} \Rightarrow \lambda = L_{k} + w_{1} + \frac{L_{r}}{2} - \frac{y}{tg\beta_{ewGa}}$$
(53)

$$B5 \ For \ 0^{\circ} < \gamma_{Ga} < 180^{\circ}, \ tg\beta^{*}_{ewGa} = y/(L_{k} + w_{l} - L_{r}/2), \ tg\beta^{**}_{ewGa} = y/(L_{k}/2), \ tg\beta^{***}_{ewGa} = y/(L_{r}/2 - w_{l})$$

$$POS_{ewGa}: \ 90^{\circ} > \beta_{ewGa} \ge \beta_{ewGa}^{*} \Rightarrow \lambda = \frac{L_{r}}{2} - w_{1} - \frac{y}{tg\beta_{ewGa}}$$
(54)

$$DO_{ewGa}: \ \beta_{ewGa}^* > \beta_{ewGa} > \beta_{ewGa}^{**} \Longrightarrow \lambda = \frac{L_r}{2} - L_k - w_1 + \frac{y}{tg\beta_{ewGa}}$$
(55)

$$POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^{**} \ge \beta_{ewG\alpha} > \beta_{ewG\alpha}^{***} \Longrightarrow \lambda = \frac{L_r}{2} - w_1 - \frac{y}{tg\beta_{ewG\alpha}}$$
(56)

B6 For $0^{\circ} < \gamma_{G\alpha} < 180^{\circ}$, $tg\beta^{*}_{ewG\alpha} = y/(L_k/2)$, $tg\beta^{**}_{ewG\alpha} = y/(L_r/2-w_l)$ $DO_{ewG\alpha}$: $90^{\circ} > \beta_{ewG\alpha} > \beta^{*}_{ewG\alpha} \Longrightarrow \lambda = \frac{y}{tg\beta_{ewG\alpha}}$ (57)

$$POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^* \ge \beta_{ewG\alpha} > \beta_{ewG\alpha}^{**} \Longrightarrow \lambda = \frac{L_r}{2} - w_1 - \frac{y}{tg\beta_{ewG\alpha}}$$
(58)

B7 For
$$0^{\circ} < \gamma_{G\alpha} < 180^{\circ}$$
, $tg\beta^{*}_{ewG\alpha} = y/(L_r/2 - L_k - w_l)$, $tg\beta^{**}_{ewG\alpha} = y/(L_k/2)$, $tg\beta^{***}_{ewG\alpha} = y/(L_r/2 - w_l)$
 $DO_{ewG\alpha}$: $90^{\circ} > \beta_{ewG\alpha} \ge \beta^{*}_{ewG\alpha} \Longrightarrow \lambda = \frac{2y}{tg\beta_{ewG\alpha}}, \beta^{*}_{ewG\alpha} > \beta^{**}_{ewG\alpha} \Longrightarrow \lambda = \frac{L_r}{2} - L_k - w_l + \frac{y}{tg\beta_{ewG\alpha}}$ (59)
 $POZ_{ewG\alpha}$: $\beta^{**}_{ewG\alpha} \ge \beta_{ewG\alpha} > \beta^{***}_{ewG\alpha} \Longrightarrow \lambda = \frac{L_r}{2} - w_l - \frac{y}{tg\beta_{ewG\alpha}}$ (60)

B8 For $0^{\circ} < \gamma_{G\alpha} < 180^{\circ}$, $tg\beta^{*}_{ewG\alpha} = y/(L_{r}/2 - L_{k} - w_{l})$, $tg\beta^{**}_{ewG\alpha} = y/(L_{r}/2 - w_{l})$ $DO_{ewG\alpha}$: $90^{\circ} > \beta_{ewG\alpha} > \beta^{*}_{ewG\alpha} \Longrightarrow \lambda = \frac{2y}{tg\beta_{ewG\alpha}}$ (61)

$$POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^* \ge \beta_{ewG\alpha} > \beta_{ewG\alpha}^{**} \Longrightarrow \lambda = \frac{L_r}{2} - w_1 - \frac{y}{tg\beta_{ewG\alpha}}$$
(62)

B9 For $0^{\circ} < \gamma_{G\alpha} < 180^{\circ}$, $tg\beta^{*}_{ewG\alpha} = y/(L_k/2)$, $tg\beta^{**}_{ewG\alpha} = y/(L_r/2 - L_k - w_l)$, $tg\beta^{***}_{ewG\alpha} = y/(L_r/2 - w_l)$ $DO_{ewG\alpha}$: $90^{\circ} > \beta_{ewG\alpha} > \beta^{*}_{ewG\alpha} \Rightarrow \lambda = \frac{2y}{tg\beta_{ewG\alpha}}$ (63)

 $POZ_{ewG\alpha}: \beta_{ewG\alpha}^* \ge \beta_{ewG\alpha} \ge \beta_{ewG\alpha}^{**} \Longrightarrow \lambda = L_k, \beta_{ewG\alpha}^{**} > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{***} \Longrightarrow \lambda = \frac{L_r}{2} - w_1 - \frac{y}{tg\beta_{ewG\alpha}}$ (64)

$$DI DI=D2=D3 For 0^{*} < \gamma_{Ga} < 180^{\circ} NOTHING$$

$$D1 For 180^{\circ} < \gamma_{Ga} < 360^{\circ}, tg\beta^{*}_{ewGa}=y/w_{l}, tg\beta^{***}_{ewGa}=y/L_{r}+w_{l}, tg\beta^{****}_{ewGa}=y/L_{k}+L_{r}+w_{l}$$

$$POZ_{ewGa}: \beta^{***}_{ewGa} > \beta^{****}_{ewGa} \Rightarrow \lambda = \frac{y}{tg\beta_{ewGa}} - w_{l}, \beta^{**}_{ewGa} \ge \beta^{****}_{ewGa} \Rightarrow \lambda = L_{r} \qquad (65)$$

$$POZ_{ewGa}: \beta^{***}_{ewGa} > \beta^{****}_{ewGa} \Rightarrow \lambda = L_{r} + L_{k} + w_{l} - \frac{y}{tg\beta_{ewGa}} \qquad (66)$$

$$D2 For 180^{\circ} < \gamma_{Ga} < 360^{\circ}, tg\beta^{*}_{ewGa}=y/W_{l}, tg\beta^{***}_{ewGa}=y/L_{r}+w_{l}, tg\beta^{****}_{ewGa}=y/L_{k}+L_{r}+w_{l}$$

$$POZ_{ewGa}: \beta^{***}_{ewGa} > \beta^{****}_{ewGa} \Rightarrow \lambda = L_{r} + L_{k} + w_{l} - \frac{y}{tg\beta_{ewGa}} \qquad (66)$$

$$D2 For 180^{\circ} < \gamma_{Ga} < 360^{\circ}, tg\beta^{*}_{ewGa}=y/w_{l}, tg\beta^{***}_{ewGa}=y/L_{r}+w_{l}, tg\beta^{****}_{ewGa}=y/L_{k}+L_{r}+w_{l}$$

$$POZ_{ewGa}: \beta^{**}_{ewGa} > \beta^{***}_{ewGa} \Rightarrow \lambda = L_{r} + L_{k} + w_{l} - \frac{y}{tg\beta_{ewGa}} \Rightarrow \lambda = L_{r} \qquad (67)$$

$$POZ_{ewGa}: \beta^{**}_{ewGa} > \beta_{ewGa} > \beta^{***}_{ewGa} \Rightarrow \lambda = L_{r} + L_{k} + w_{l} - \frac{y}{tg\beta_{ewGa}} \Rightarrow \lambda = L_{r} \qquad (68)$$

$$D3 For 180^{\circ} < \gamma_{Ga} < 360^{\circ}, tg\beta^{*}_{ewGa}=y/w_{l}, tg\beta^{**}_{ewGa}=y/L_{k}+w_{l}, tg\beta^{***}_{ewGa}=y/L_{k}+w_{l}, tg\beta^{***}_{ewGa}=y/L_{r}+w_{l}, tg\beta^{***}_{ewGa}=y/L_{r}+w_{l}+w_{l}$$

$$POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^* > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{**} \Rightarrow \lambda = \frac{y}{tg\beta_{ewG\alpha}} - w_1, \ \beta_{ewG\alpha}^{**} \ge \beta_{ewG\alpha} \ge \beta_{ewG\alpha}^{***} \Rightarrow \lambda = L_k$$
(69)

$$POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^{***} > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{****} \Longrightarrow \lambda = L_r + L_k + w_1 - \frac{y}{tg\beta_{ewG\alpha}}$$
(70)

$$C1 C1 = C2 = C3 For 0 < \gamma_{G\alpha} < 180 \text{ NOTHING}$$

$$C1 \text{ For } 180^{\circ} < \gamma_{G\alpha} < 360^{\circ}, tg\beta^{*}_{ewG\alpha} = y/L_r, tg\beta^{**}_{ewG\alpha} = y/L_k, tg\beta^{***}_{ewG\alpha} = y/L_k + L_r$$

$$POZ_{ewG\alpha}: 90^{\circ} > \beta_{ewG\alpha} > \beta^{*}_{ewG\alpha} \Rightarrow \lambda = \frac{y}{tg\beta_{ewG\alpha}}, \beta^{*}_{ewG\alpha} \ge \beta_{ewG\alpha} \ge \beta^{**}_{ewG\alpha} \Rightarrow \lambda = L_r \quad (71)$$

$$POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^{**} > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{***} \Longrightarrow \lambda = L_r + L_k - \frac{y}{tg\beta_{ewG\alpha}}$$
(72)

$$C2 \text{ For } 180^{\circ} < \gamma_{G\alpha} < 360^{\circ}, \ tg\beta^{*}_{ewG\alpha} = y/L_r, \ tg\beta^{**}_{ewG\alpha} = y/L_k + L_r$$
$$POZ_{ewG\alpha}: 90^{\circ} > \beta_{ewG\alpha} > \beta^{*}_{ewG\alpha} \Longrightarrow \lambda = \frac{y}{tg\beta_{ewG\alpha}}, \ \beta_{ewG\alpha} = \beta^{*}_{ewG\alpha} \Longrightarrow \lambda = L_r \quad (73)$$

$$POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^* > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{**} \Longrightarrow \lambda = L_r + L_k - \frac{y}{tg\beta_{ewG\alpha}}$$
(74)

C3 For
$$180^{\circ} < \gamma_{Ga} < 360^{\circ}$$
, $tg\beta^{*}_{ewGa} = y/L_k$, $tg\beta^{**}_{ewGa} = y/L_r$, $tg\beta^{***}_{ewGa} = y/L_k + L_r$

$$POZ_{ewG\alpha}: 90^{\circ} > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{*} \Longrightarrow \lambda = \frac{y}{tg\beta_{ewG\alpha}}, \beta_{ewG\alpha}^{*} \ge \beta_{ewG\alpha} \ge \beta_{ewG\alpha}^{**} \Longrightarrow \lambda = L_{k}$$
(75)

$$POZ_{ewG\alpha}: \ \beta_{ewG\alpha}^{**} > \beta_{ewG\alpha} > \beta_{ewG\alpha}^{***} \Longrightarrow \lambda = L_r + L_k - \frac{y}{tg\beta_{ewG\alpha}}$$
(76)

Equations for λ for $0^{\circ} < \gamma_{G\alpha} < 180^{\circ}$ and $180^{\circ} < \gamma_{G\alpha} < 360^{\circ}$ are indentical to equations for ξ for $270^{\circ} < \gamma_{G\alpha} < 90^{\circ}$ and $90^{\circ} < \gamma_{G\alpha} < 270^{\circ}$ respectively (SJG α plane).

In the case when $\gamma_{G\alpha}=0^{\circ}$, 360° $(180^{\circ}) \rightarrow \beta_{ewG\alpha}=90^{\circ}$, and $\gamma_{G\alpha}=90^{\circ}$ $(270^{\circ}) \rightarrow \beta_{nsG\alpha}=90^{\circ}$, irradiated area is determined as $A_{ozr} = \xi \cdot \lambda_0$, $A_{ozr} = \xi_0 \cdot \lambda$ respectively, where equations for ξ and λ remain same while equations for λ_0 and ξ_0 differs from case to case (E1=E2=E4=E5=E7=E8=E10=E11a=E11b=E12 - $\lambda_0 = L_r/2+v_1$, $\xi_0 = W_r/2+v_2$, E3=E6=E9 - $\lambda_0 = L_r$, $\xi_0 = W_r$, E13=E14=E15 - $\lambda_0 = L_k$, $\xi_0 = W_k$, B1=B2=B3=B4=B5=B6 - $\lambda_0 = L_r/2-w_1$, $\xi_0 = W_r/2-w_2$, B7=B8=B9 - $\lambda_0 = L_k$, $\xi_0 = W_k$).

Equations for $p_{nsG\alpha}$

 $E1 (E1=E4) For 90^{\circ} < \gamma_{Ga} < 270^{\circ}, tg\beta_{nsGa}^{*}=y/W_{r}/2-v_{2}, tg\beta_{nsGa}^{**}=y/W_{r}/2+v_{2}$ $DO_{nsGa} : 90^{\circ} > \beta_{nsGa} \ge \beta_{nsGa}^{*} \Longrightarrow p_{nsGa} = \frac{\xi}{2}, \beta_{nsGa}^{*} > \beta_{nsGa} > \beta_{nsGa}^{**} \Longrightarrow p_{nsGa} = \xi - \frac{W_{r}}{2} + v_{2} \quad (77)$ $E1 (E1=E2=E3=E4=E5=E6) For 270^{\circ} < \gamma_{Ga} < 90^{\circ} NOTHING$ $E2 (E2=E5=E8) For 90^{\circ} < \gamma_{Ga} < 270^{\circ} \rightarrow p_{nsGa} = \xi$ $E3 (E3=E6=E9) For 90^{\circ} < \gamma_{Ga} < 270^{\circ} \rightarrow p_{nsGa} = v_{2} - W_{r}/2 + \xi$ $E7 (E7=E10=E11a) For 90^{\circ} < \gamma_{Ga} < 270^{\circ}, tg\beta_{nsGa}^{*} > \beta_{nsGa} > \beta_{nsGa}^{**} \Rightarrow p_{nsGa} = \xi - \frac{W_{r}}{2} + v_{2} \quad (78)$ $E7 (E7=E8=E9=E10=E11a) For 270^{\circ} < \gamma_{Ga} < 270^{\circ} \rightarrow p_{nsGa} = \xi - \frac{W_{r}}{2} + v_{2} \quad (78)$ $E7 (E7=E8=E9=E10=E11a=E11b) For 270^{\circ} < \gamma_{Ga} < 270^{\circ} \rightarrow p_{nsGa} = \xi/2$ $E11b (E11b=E12=E13=E14=E15) For 90^{\circ} < \gamma_{Ga} < 270^{\circ} \rightarrow p_{nsGa} = \xi/2$ $E12 For 270^{\circ} < \gamma_{Ga} < 90^{\circ} \rightarrow p_{nsGa} = \xi$ $E13 For 270^{\circ} < \gamma_{Ga} < 90^{\circ} \Rightarrow p_{nsGa} = \xi$ $E13 For 270^{\circ} < \gamma_{Ga} < 90^{\circ} \Rightarrow p_{nsGa} = \xi$ $E13 For 270^{\circ} < \gamma_{Ga} < 90^{\circ} \Rightarrow p_{nsGa} = \xi$ $E13 For 270^{\circ} < \gamma_{Ga} < 90^{\circ} \Rightarrow p_{nsGa} = \xi$ $E13 For 270^{\circ} < \gamma_{Ga} < 90^{\circ} \Rightarrow p_{nsGa} = \xi$ $E13 For 270^{\circ} < \gamma_{Ga} < 90^{\circ} \Rightarrow p_{nsGa} = \xi$ $E13 For 270^{\circ} < \gamma_{Ga} < 90^{\circ} \Rightarrow p_{nsGa} = \xi$ $E13 For 270^{\circ} < \beta_{nsGa} <math>\ge \beta_{nsGa}^{*} \Rightarrow p_{nsGa} = \frac{\xi}{2}, \beta_{nsGa}^{*} > \beta_{nsGa} > \beta_{nsGa}^{**} \Rightarrow p_{nsGa} = \xi - \frac{W_{r}}{2} - v_{2} + W_{k} \quad (79)$

$$E14 (E14=E15) \text{ For } 270^{\circ} < \gamma_{Ga} < 90^{\circ} \rightarrow p_{nsGa} = \xi/2$$

$$B1 (B1=B2=B3=B4=B5=B6=B7=B8=B9) \text{ For } 90^{\circ} < \gamma_{Ga} < 270^{\circ} \rightarrow p_{nsGa} = \xi/2$$

$$B1 (B1=B2=B3=B4) \text{ For } 270^{\circ} < \gamma_{Ga} < 90^{\circ} \text{ NOTHING}$$

$$B5 \text{ For } 270^{\circ} < \gamma_{Ga} < 90^{\circ} \rightarrow p_{nsGa} = W_k + w_2 - W_r/2 + \xi$$

$$B6 \text{ For } 270^{\circ} < \gamma_{Ga} < 90^{\circ} \rightarrow p_{nsGa} = \xi$$

$$B7 \text{ For } 270^{\circ} < \gamma_{Ga} < 90^{\circ}, tg\beta^*_{nsGa} = y/(W_r/2 - W_k - w_2), tg\beta^{**}_{nsGa} = y/W_k/2$$

$$DO_{nsGa}: 90^{\circ} > \beta_{nsGa} \ge \beta^*_{nsGa} \Rightarrow p_{nsGa} = \frac{\xi}{2}, \beta^*_{nsGa} > \beta_{nsGa} \Rightarrow \beta^{**}_{nsGa} \Rightarrow p_{nsGa} = \xi - \frac{W_r}{2} + w_2 + W_k \quad (80)$$

$$B8 (B8=B9) \text{ For } 270^{\circ} < \gamma_{Ga} < 90^{\circ} \rightarrow p_{nsGa} = \xi/2$$

Equations for $p_{ewG\alpha}$ are almost indentical to equations for $p_{nsG\alpha}$, with difference that in that case parameters are defined for IZG α plane and for angle interval of $\gamma_{G\alpha}$, $180^{\circ} < \gamma_{G\alpha} < 360^{\circ}$ i $0^{\circ} < \gamma_{G\alpha} < 180^{\circ}$.

Equations for \mathbf{a}_{p} (DO_{ewGa}-DO_{nsGa}) E1 (E1=E4=E7=E10) For $180^{\circ} < \gamma_{Ga} < 270^{\circ}$ (for $270^{\circ} < \gamma_{Ga} < 360^{\circ} \rightarrow tg(360-\gamma_{Ga})$), $tg(\gamma_{Ga}-180^{\circ})^{*} = (L_{r}/2-v_{I})/p_{nsGa}$, $tg(\gamma_{Ga}-180^{\circ})^{**} = (L_{k}-v_{I}-Lr/2)/p_{nsGa}$ $tg(\gamma_{Ga}-180^{\circ})^{*} \le tg(\gamma_{Ga}-180^{\circ}) \le tg(\gamma_{Ga}-180^{\circ})^{**} \Rightarrow a_{p} = L_{r}$ (81)

$$tg(\gamma_{G\alpha} - 180^{\circ}) < tg(\gamma_{G\alpha} - 180^{\circ})^* \Longrightarrow a_p = \frac{L_r}{2} + v_1 + tg(\gamma_{G\alpha} - 180^{\circ}) \cdot p_{nsG\alpha}$$
(82)

$$tg(\gamma_{G\alpha} - 180^{\circ}) > tg(\gamma_{G\alpha} - 180^{\circ})^{**} \Longrightarrow a_p = L_k - v_1 + \frac{L_r}{2} - tg(\gamma_{G\alpha} - 180^{\circ}) \cdot p_{nsG\alpha}$$
(83)

E1 (*E1=E2=E3=E4=E5=E6*) For $0^{\circ} < \gamma_{Ga} < 180^{\circ}$ *NOTHING* (for $90^{\circ} < \gamma_{Ga} < 180^{\circ} \rightarrow tg(180^{\circ} - \gamma_{Ga})$)

E2 (*E2=E3=E5=E6=E8=E9*) For $180^{\circ} < \gamma_{Ga} < 270^{\circ}$ (for $270^{\circ} < \gamma_{Ga} < 360^{\circ} \rightarrow tg(360-\gamma_{Ga})$), $tg(\gamma_{Ga}-180^{\circ})^{*} = 0$, $tg(\gamma_{Ga}-180^{\circ})^{**} = (L_k - v_l - Lr/2)/p_{nsGa}$

$$tg(\gamma_{G\alpha} - 180^{\circ})^* \le tg(\gamma_{G\alpha} - 180^{\circ}) \le tg(\gamma_{G\alpha} - 180^{\circ})^{**} \Rightarrow a_p = L_r$$
(84)

$$tg(\gamma_{G\alpha} - 180^{\circ}) > tg(\gamma_{G\alpha} - 180^{\circ})^{**} \Rightarrow a_p = L_k - v_1 + \frac{L_r}{2} - tg(\gamma_{G\alpha} - 180^{\circ}) \cdot p_{nsG\alpha}$$
(85)

$$E7 (E7 = E8 = E10 = E11a = E11b = E12) For 0^{\circ} < \gamma_{Ga} < 90^{\circ} (for 90^{\circ} < \gamma_{Ga} < 180^{\circ} \rightarrow tg(180^{\circ} - \gamma_{Ga}))$$

$$tg(\gamma_{G\alpha}) \ge tg(\gamma_{G\alpha})^*, tg(\gamma_{G\alpha})^* = 0 \Longrightarrow a_p = \frac{L_r}{2} + v_1 - tg(\gamma_{G\alpha}) \cdot p_{nsG\alpha}$$
(86)

E9 For $0^{\circ} < \gamma_{Ga} < 90^{\circ}$ (for $90^{\circ} < \gamma_{Ga} < 180^{\circ} \rightarrow tg(180^{\circ} - \gamma_{Ga})$), $tg(\gamma_{Ga} - 180^{\circ})^{*} = 0$, $tg(\gamma_{Ga} - 180^{\circ})^{**} = (v_1 - Lr/2)/p_{nsGa}$

$$tg(\gamma_{G\alpha})^{*} \leq tg(\gamma_{G\alpha}) \leq tg(\gamma_{G\alpha})^{**} \Rightarrow a_{p} = L_{r}$$

$$tg(\gamma_{G\alpha}) > tg(\gamma_{G\alpha})^{**} \Rightarrow a_{p} = v_{1} + \frac{L_{r}}{2} - tg(\gamma_{G\alpha}) \cdot p_{nsG\alpha}$$
(88)

E11a (**E11a=E11b**) For $180^{\circ} < \gamma_{Ga} < 270^{\circ}$ (for $270^{\circ} < \gamma_{Ga} < 360^{\circ} \rightarrow tg(360-\gamma_{Ga})$), $tg(\gamma_{Ga}-180^{\circ})^{*} = (L_{k} - v_{1}-Lr/2)/p_{nsGa}$, $tg(\gamma_{Ga}-180^{\circ})^{**} = (L_{r}/2-v_{1})/p_{nsGa}$

$$tg(\gamma_{G\alpha} - 180^{\circ})^* \le tg(\gamma_{G\alpha} - 180^{\circ}) \le tg(\gamma_{G\alpha} - 180^{\circ})^{**} \Longrightarrow a_p = L_k$$
(89)

$$tg(\gamma_{G\alpha} - 180^{\circ}) < tg(\gamma_{G\alpha} - 180^{\circ})^* \Longrightarrow a_p = \frac{L_r}{2} + v_1 + tg(\gamma_{G\alpha} - 180^{\circ}) \cdot p_{nsG\alpha}$$
(90)

$$tg(\gamma_{G\alpha} - 180^{\circ}) > tg(\gamma_{G\alpha} - 180^{\circ})^{**} \Longrightarrow a_{p} = L_{k} - v_{1} + \frac{L_{r}}{2} - tg(\gamma_{G\alpha} - 180^{\circ}) \cdot p_{nsG\alpha}$$
(91)

E12 (*E12=E13=E14=E15*) For $180^{\circ} < \gamma_{Ga} < 270^{\circ}$ (for $270^{\circ} < \gamma_{Ga} < 360^{\circ} \rightarrow tg(360-\gamma_{Ga})$), $tg(\gamma_{Ga}-180^{\circ})^* = 0$, $tg(\gamma_{Ga}-180^{\circ})^{**} = (L_r/2-v_l)/p_{nsGa}$

$$g(y_{Ga} - 180^{\circ})' \le tg(y_{Ga} - 180^{\circ}) = g(y_{Ga} - 180^{\circ})' \Rightarrow a_{p} = L_{k} \qquad (92)$$

$$tg(y_{Ga} - 180^{\circ}) > tg(y_{Ga} - 180^{\circ})' \Rightarrow a_{p} = L_{k} - v_{1} + \frac{L_{r}}{2} - tg(y_{Ga} - 180^{\circ}) \cdot p_{mGa} \qquad (93)$$

$$E13 (E13 = E14 = E15) For 0' < y_{Ga} < 90' (for 90' < y_{Ga} < 180' \Rightarrow tg(180'' - y_{Ga})), tg(y_{Ga} - 180'')'' = 0, tg(y_{Ga} - 180'')'' = (L_{r}^{2} + v_{1} - L_{r}^{2}) + L_{r}^{2} - tg(y_{Ga}) > tg(y_{Ga}) = tg(y_{Ga}) > tg(y_{Ga}) = a_{p} = L_{k} \qquad (94)$$

$$tg(y_{Ga}) > tg(y_{Ga}) = tg(y_{Ga} - 180^{\circ}) = d_{p} = L_{r} \qquad (95)$$

$$B1 (B1 = B2) For 180'' = y_{Ga} < 270'' (for 270'' < y_{Ga} < 360'' \to tg(360 - y_{Ga})), tg(y_{Ga''}) = tg(y_{Ga'} - 180^{\circ}) = d_{p} = L_{r} \qquad (96)$$

$$tg(y_{Ga} - 180^{\circ}) < tg(y_{Ga} - 180^{\circ}) = d_{p} = \frac{L_{r}}{2} - w_{1} + tg(y_{Ga} - 180^{\circ}) \cdot p_{mGa} \qquad (97)$$

$$g(y_{Ga} - 180^{\circ}) > tg(y_{Ga} - 180^{\circ}) = d_{p} = L_{k} + w_{1} + \frac{L_{r}}{2} - tg(y_{Ga} - 180^{\circ}) \cdot p_{mGa} \qquad (98)$$

$$B1 (B1 = B2 = B3 = B4) For 0' < y_{Ga} < 180'' NOTHING (for 90'' < y_{Ga} < 180'' \Rightarrow tg(180'' - y_{Ga}))$$

$$g(y_{Ga} - 180^{\circ}) > tg(y_{Ga} - 180^{\circ}) = d_{p} = t_{r}^{2} - w_{1} + tg(y_{Ga} - 180^{\circ}) \cdot p_{mGa} \qquad (98)$$

$$B1 (B1 = B2 = B3 = B4) For 0'' < y_{Ga} < 180'' NOTHING (for 20'' < y_{Ga} < 360'' \Rightarrow tg(360 - y_{Ga}))$$

$$g(y_{Ga} - 180^{\circ}) > tg(y_{Ga} - 180^{\circ}) = d_{p} = t_{r}^{2} - w_{1} + tg(y_{Ga} - 180^{\circ}) \cdot p_{mGa} \qquad (100)$$

$$tg(y_{Ga} - 180^{\circ}) < tg(y_{Ga} - 180^{\circ}) = d_{p} = t_{r}^{2} - w_{1} + tg(y_{Ga} - 180^{\circ}) \cdot p_{mGa} \qquad (100)$$

$$tg(y_{Ga} - 180^{\circ}) > tg(y_{Ga} - 180^{\circ}) = d_{p} = L_{r} = tg(y_{Ga} - 180^{\circ}) \cdot p_{mGa} \qquad (101)$$

$$B5 (B5 = B6) For 0' < y_{Ga} < 90'' (for 90'' < y_{Ga} < 180' \to tg(180'' - y_{Ga})), tg(360 - y_{Ga})), tg(y_{Ga''} - 180^{\circ}) = d_{p} = t_{r} = t$$

Equations for $\mathbf{b}_{pnsG\alpha}$ (DO_{ewG\alpha}-DO_{nsGa}) E1 (E1-E15) For 90° < $\gamma_{G\alpha}$ < 270°

$$DO_{nsGa}: b_{pnsGa} = \frac{W_r}{2} + v_2 - p_{nsGa}, \ 2p_{nsGa} + b_{pnsGa} > W_k \Longrightarrow b_{pnsGa} = W_k - 2p_{nsGa}$$
(107)
E1 (E1-E6) For 270° < $\gamma_{Ga} < 90°$ **NOTHING**
E7 (E7-E15) For 270° < $\gamma_{Ga} < 90° \rightarrow b_{pnsGa} = W_k - 2p_{nsGa}$
B1 (B1-B9) For 90° < $\gamma_{Ga} < 270°$

 $DO_{nsG\alpha}: b_{pnsG\alpha} = \frac{W_r}{2} - W_2 - p_{nsG\alpha}, \ 2p_{nsG\alpha} + b_{pnsG\alpha} > W_k \Longrightarrow b_{pnsG\alpha} = W_k - 2p_{nsG\alpha}$ (108) **B1 (B1-B4)** For $270^{\circ} < \gamma_{Ga} < 90^{\circ}$ **NOTHING B5 (B5-B9)** For $270^{\circ} < \gamma_{Ga} < 90^{\circ} \rightarrow b_{pnsGa} = W_k - 2p_{nsGa}$ Equations for a_p , $b_{pewG\alpha}$ (DO_{ewG\alpha}-POZ_{nsGa}) **E1 (E1-E5)** For $180^{\circ} < \gamma_{Ga} < 360^{\circ}$ $DO_{ewG\alpha}$: $b_{pewG\alpha} = \frac{L_r}{2} + v_1 - p_{ewG\alpha}$, $b_{pewG\alpha} + 2p_{ewG\alpha} > L_k \Longrightarrow b_{pewG\alpha} = L_k - 2p_{ewG\alpha}$ (109) DO_{ewGa} : $a_p = b_{pewGa} + \lambda$, $b_{pewGa} + 2p_{ewGa} > L_k \Rightarrow a_p = L_k - 2p_{ewGa} + \lambda$ (110)**E1 (E1-E6)** For $0^{\circ} < \gamma_{Ga} < 180^{\circ}$ **NOTHING E7 (E7-E15)** For $0^{\circ} < \gamma_{Ga} < 180^{\circ}$ DO_{ewGa} : $b_{pewGa} = L_k - 2p_{ewGa}$, $a_p = L_k - 2p_{ewGa} + \lambda$ (111)**B1 (B1-B9)** For $180^{\circ} < \gamma_{Ga} < 360^{\circ}$ $DO_{ewG\alpha}: b_{pewG\alpha} = \frac{L_r}{2} - w_1 - p_{ewG\alpha}, \ 2p_{ewG\alpha} + b_{pewG\alpha} > L_k \Longrightarrow b_{pewG\alpha} = L_k - 2p_{ewG\alpha}$ (112) $DO_{ewG\alpha}: a_p = b_{pewG\alpha} + \lambda, b_{pewG\alpha} + 2p_{ewG\alpha} > L_k \Longrightarrow a_p = L_k - 2p_{ewG\alpha} + \lambda$ (113)**B1 (B1-B4)** For $0^{\circ} < \gamma_{Ga} < 180^{\circ}$ **NOTHING B5 (B5-B9)** For $0^{\circ} < \gamma_{Ga} < 180^{\circ}$ $DO_{ewG\alpha}$: $b_{pewG\alpha} = L_k - 2p_{ewG\alpha}$, $a_p = L_k - 2p_{ewG\alpha} + \lambda$ (114)

3. Results and discussion

The values of the angles G, β_{ewGa} , β_{nsGa} , γ_{Ga} and γ , β determine the combination of the defined situations and therefore the method for determining the irradiated area. For date 20.06., $G = 35^{\circ}$, $\rho = 0.85$, location latitude and longitude 45°, 20°, orientation of the tilted CRS surface of 180°, S/Z = 80% and CRS dimensions of $L_k = 0.6$ m, $W_k = 0.4$ m, $L_r = 0.487$ m, $W_r = 0.618$ m, y = 0.249 m, $v_1 = 0.034$ m, $w_2 = 0.051$ m, the values for relation parameters for irradiated area of the lower absorber surface are calculated, from previously described equations and [6], and shown by table (Table 1):

| | 06:00h | 08:00h | 10:00h | 12:00h | 14:00h | 16:00h | 18:00h |
|-----------------------------|--------|--------|--------|--------|--------|--------|--------|
| i (°) | 81.51 | 54.04 | 27.39 | 14.29 | 35.99 | 63.21 | 90 |
| (τα) _{dir,ref} (-) | 0.10 | 0.74 | 0.83 | 0.84 | 0.81 | 0.62 | 0 |
| (τα) _{dif} (-) | 0.30 | 0.72 | 0.74 | 0.74 | 0.73 | 0.68 | 0 |
| β(°) | 19.75 | 40.81 | 60.39 | 68.09 | 54.38 | 33.75 | 12.99 |
| γ (°) | - | 96.86 | 128.30 | 192.37 | 244.62 | - | - |
| β_{ewGa} (°) | - | 38.00 | 66.41 | 85.28 | 56.96 | - | - |
| β_{nsGa} (°) | - | 62.88 | 74.40 | 76.45 | 72.07 | - | - |
| γ _{Gα} (°) | - | 111.81 | 122.59 | 198.90 | 243.55 | - | - |

Table 1. The calculated parameters i, $(\tau \alpha)_{dir,ref}$, $(\tau \alpha)_{dif}$, β , γ , β_{ewGa} , β_{nsGa} , γ_{Ga} for 20.06., 06 - 18 h

From calculated values of the parameters for these hours it can be concluded that there are only combinations of the situations $DO_{ewG\alpha}$ - $DO_{nsG\alpha}$ and $POS_{ewG\alpha}$ - $DO_{nsG\alpha}$ for angles $\beta = 54.38^{\circ}$, 68.09° and $\beta = 60.39^{\circ}$ respectively. As for the other possible combinations concerns, the same for angles $\beta = 19.75^{\circ}$, 33.75° and 12.99° were not considered because the same angles are less then angle *G*,

which is why $A_{ozr} = 0$. For $\beta = 40.81^{\circ} A_{ozr} = 0$, because no reflected beams hit the lower absorber surface.



Fig. 3. The comparative diagram of the absorbed irradiation (W) of the double exposure collector and collector without reflector

The values of angle $\beta = 54.38^{\circ}$, 60.39° and 68.09° , for which the irradiated area of the lower absorber surface is generated, only contribute to increasing of the absorbed solar irradiation, for 11.43 W (at 10:00 h), 26.44 W (at 12:00 h) and 61.79 W (at 14:00 h), which can be seen from the Fig. 3 that shows the comparative values of the calculated absorbed irradiation for double exposure collector and collector without reflector, using equations from [6]. The major contribution of the increasing of the absorbed solar irradiation is associated for $\beta = 54.38^{\circ}$ at 14:00 h. The values of the irradiated area, obtained for mentioned angles are 62% (54.38°), 20% (68.09°) and 10% (60.39°) of the lower (upper) absorber surface which is 0.24 m².

The described mathematical model allows determining of the instantaneous irradiated area for double exposure flat-plate CRS, for its arbitrary tilt and arbitrary position and dimensions of the reflector relative to collector. Thus, the developed model will be used for numerical determination of the optimal dimensions as well as optimal positions of the reflector relative to collector. The basis and reason for future conducting of that analysis relies on the fact that by reflector it is possible to increase the absorbed solar irradiation of the CRS, as a one of possibilities for increasing its efficiency, specifically in this case for 6.52% (10:00 h), 12.53% (12:00 h) and 30.11% (14:00 h).

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