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BUCKLING ANALYSIS OF LAMINATED COMPOSITE PLATES

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Abstract: This paper deals with laminated plates subjected to uniaxial compression. The effects of lamination scheme and material properties on the critical loads are considered, as well as variations of boundary conditions and direction of the acting loads. The buckling analysis was conducted using the finite elements method implemented in the Simcenter Femap software package. The obtained results were compared with the already available ones.

Keywords: Buckling, laminated plates, composite materials.

1. INTRODUCTION

With the advancement of materials and production technologies in recent years, laminated composites have increased their application in advanced, modern engineering structures. Laminated composites made of advanced fibre-reinforced materials are considered as the primary materials for future generations of aircraft and marine structures. Their indispensable application will certainly be reflected in both, biomedicine and electronics, and they are also already widely used in construction. Laminated composite shells/panels exhibit complexity due to nonlinearity and anisotropy. In practical problems, such structures are often subjected to different types of in-plane loads, which requires knowledge of their stability behaviour. Considering the above, the assessment of the buckling load is very important.

Given that in practical situations laminated structures are exposed to different loads, with anisotropy and nonlinearity, they have become the subject of interest of many researchers.

The authors of paper [1] were dedicated to finite elements through linear analysis, while monograph [2] focuses on non-linear analysis. The researchers of paper [3] dealt with buckling of laminated square plates and plates made of functionally graded materials. They presented calculations in the case of plates subjected to uniaxial compression. In the article [4], laminated composite plates were analysed using higher-order shear deformation theory to predict deflections and stresses, while in [5], a higher-order shear deformation theory is used to analyse laminated anisotropic composite plates for deflections, stresses, natural frequencies and buckling loads. The subject of the paper [6] is the buckling of rectangular layered plates. The effects of plate aspect ratio, lamination scheme, number of layers and material properties on the critical loads are studied. In [7] was analysed buckling of composite plates using shear deformable finite elements. [8] in addition to buckling, also analysed vibration of laminated composite plates using various plate theories. The attention of the authors [9] is also directed towards the buckling of laminated plates, but with a variation of the boundary conditions. In addition to buckling, many authors also dealt with bending of composite plates as in article [10]. Reference [11] shows the uniaxial and biaxial buckling of rectangular plates through the application of the new theory of trigonometric shear and normal deformation. The widely known Mindlin theory is presented in the paper [12], while a much earlier research and theory that precedes the above is presented in the paper [13]. The stability of multi-layered plates was investigated in [14]. In paper [15], the authors dealt with the buckling of unsymmetrically laminated plates, emphasizing the importance of theories that include shear deformations. Fibre-reinforced plastic composites are used in many light constructions, which additionally motivated the authors to deal with this topic. With the help of another researcher, the authors of the previously published article analysed the buckling of orthotropic laminate plates under uniaxial compression using third-order shear deformation theory [16]. The work produced good results in comparison with the results available so far. The great advantage of this approach is its applicability regardless of the plate thickness, and the speed of calculation. [17] considered orthotropic laminate plates under the influence of uniaxial loading. A method for determining the buckling load was developed within Reddy's third-order shear deformation theory. The plate is simply-supported with additional rotational restraints at the unloaded edges. The approach is simple, computationally very efficient and shows good agreement with available solutions. The authors of the paper [18] analysed the rectangular Mindlin plates consisting of laminated composites with symmetrical layers, or with isotropic materials. Axial compression was also considered here, and the verification of the results was done by comparing with exact solutions and/or solutions obtained with high-fidelity finite elements. [19] presents a highly efficient approximate computer model that offers a valuable tool for the preliminary design of lightweight structures that use advanced materials and that can be deformed by shearing. In paper [20] an approach that is effective regardless of plate thickness is presented, and paper [21] considers the buckling and post-buckling behaviour of a layered composite shell in a combination of temperature loading and applied mechanical loads. The presented numerical results are based on higher-order shear deformation theory.

2. BUCKLING OF LAMINATED PLATE

In this section, analyses of a square layered plate $(a \times a)$ with the following material characteristics were carried out:

$$E_1 / E_2 = 20(30,40),\ G_{12} = G_{13} = 0.6\,E_2,\ G_{23} = \\ = 0.5E_2,\ \nu_{12} = \nu_{13} = 0.25$$

(E – Young's modulus, G – shear modulus, ν – Poisson's ratio).

The geometric characteristics of the plate with equal thickness of all layers are adopted as:

$$a / h = 10,\ a = 0.1\ \mathrm{m}\ i\ h = 0.01\,\mathrm{m}.$$

Figure 1 shows the laminate square plate with N layers.

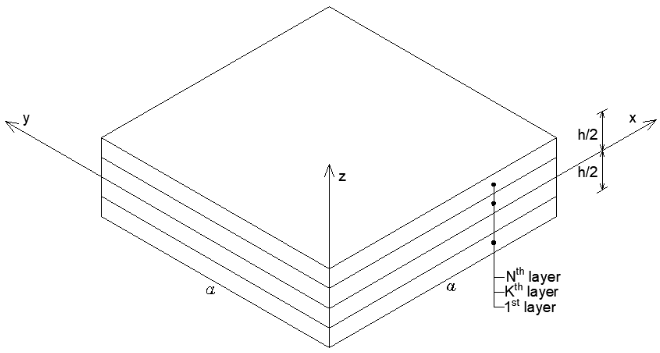

Figure 1. *Square layered plate*

Boundary conditions for the initially analysed model presenting simply-supported plates (SSSS) are defined in Table 1.

Table 1. *Boundary conditions, SSSS laminated square plate [3]*

Boundary conditions	Face $y = 0$	Face $y = a$	Face $x = 0$	Face $x = a$
SSSS	$w = 0,\ \theta_y = 0$	$v = w = 0,\ \theta_y = 0$	$w = 0,\ \theta_x = 0$	$u = w = 0,\ \theta_x = 0$

2.1 Ratio variation (E_1/E_2)

Table 1 shows the results of the normalized critical load of simply-supported plates (SSSS) of the aforementioned geometric characteristics, but with different E_1/E_2 ratios. The analysis was carried out for plates with a symmetrical arrangement of layers (3 and 5 layers) within the realized thickness of the plate. Different types of lamination, as well as the normalized load, are shown in Table 2. Pre-processing and post-processing of FEA models were performed in the Simcentar Femap software package with a defined mesh size 10×10. Structural analysis was conducted using NX Nastran software, which operates based on the finite element method.

Table 2. *Normalized critical load of a simply-supported laminated plate*

Lamination scheme	Source	E_1 / E_2		
		20	30	40
(0° / 90° / 0°)	Present	14.862	18.811	21.999
	[10]	15.215	20.428	24.977
	[11]	15.003	19.002	22.330
	[14]	15.019	19.304	22.880
	HSDT	15.300	19.675	23.339
	FSDT	14.985	19.027	22.315
	CPT	19.712	27.936	36.160
(0° / 90° / 0° / 90° / 0°)	Present	15.678	20.345	24.293
	[10]	16.234	21.435	25.976
	[11]	15.828	20.643	24.756
	[14]	15.653	20.466	24.593
	HSDT	15.783	20.578	24.676
	FSDT	15.736	20.485	24.547
	CPT	19.712	27.936	36.160

HSDT – Higher-order shear deformation theory (Reddy, 1984), FSDT – First order shear deformation theory (Mindlin, 1951), CPT – The classical plate theory (Kirchhoff, 1850)

Normalization was performed according to the following expression:

$$\bar{N}_{cr} = \frac{N_0\, a^2}{E_2\, h^3} \tag{1}$$

Where N_0 represents in-plane load that, in the case of previous analyses (results in Table 2), acts in the direction of the x-axis (Figure 2).

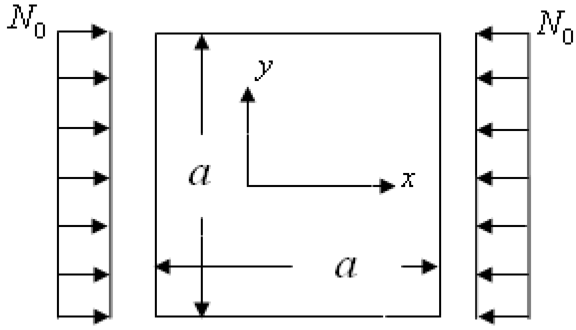

Figure 2. *Square plate subjected to uniaxial compression*

The presented results are in good agreement with the already available data in the mentioned articles. In addition to the good agreement of the results, it is noticeable that the highest load can be presented in the case of $E_1/E_2 = 40$, and in the following, only the plates with this defined ratio will be additionally processed.

2.2 Square plates with different layer arrangements and different boundary conditions

As it was said at the end of the previous subsection, the analysis will be directed towards plates with the ratio $E_1/E_2 = 40$. The normalized critical load is shown in Table 3 in the case of a simply-supported plate (SSSS), while in Table 4 is presented in the case of all edges clamped (CCCC). The boundary conditions for the CCCC plate are shown in Table 5.

Different types of lamination schemes were considered [3]:

Symmetric cross-ply SYCP1	$[0° / 90° / 90° / 0°]$
Symmetric cross-ply SYCP2	$[0° / 90° / 0° / 90° / 0°]$
Antisymmetric cross-ply ASCP	$[0° / 90° / 0° / 90°]$
Symmetric angle-ply SYAP	$[45° / –45° / –45° / 45°]$
Antisymmetric cross-ply (ASAP)	$[30° / –30° / 30° / –30°]$
Unsymmetric cross-ply	$[0° / 15° / 30° / 45°]$

Table 3. *Normalized critical load of a simply-supported laminated plate*

	Symmetric		Antisymmetric		Unsymmetric
	SYCP1	SYAP	ASCP	ASAP	UNSYM
$\overline{N}_{cr}$	23.18(*)	27.49(*)	21.98(*)	31.29(*)	15.80(*)
	23.34 [5]	24.99 [7]	22.58 [8]	31.60 [6]	-
	23.31 [3]	25.74 [3]	22.66 [3]	31.31 [3]	14.64 [3]

(*) Present

Table 4. *Normalized critical load of clamped plate*

	Symmetric		Antisymmetric		Unsymmetric
	SYCP2	SYAP	ASCP	ASAP	UNSYM
$\overline{N}_{cr}$	41.46(*)	30.93(*)	31.07(*)	39.66(*)	23.21(*)
	41.30 [6]	31.08 [5]	-	39.90 [6]	-
	42.58 [3]	30.38 [3]	37.10 [3]	40.15 [3]	24.46 [3]

(*) Present

Table 5. *Boundary conditions, CCCC laminate square plate [3]*

Boundary conditions	Face $y = 0$	Face $y = a$	Face $x = 0$	*Face* $x = a$
CCCC	$w = 0,\ \theta_x = \theta_y = 0$	$v = 0,\ w = 0,\ \theta_x = 0,\ \theta_y = 0$	$w = 0,\ \theta_x = \theta_y = 0$	$u = w = 0,\ \theta_x = \theta_y = 0$

Table 6. *Normalized critical load*

	ASAP				SYCP2			
	SSSS		CCCC		SSSS		CCCC	
	x-axis	y-axis	x-axis	y-axis	x-axis	y-axis	x-axis	y-axis
$\overline{N}_{cr}$	31.29(*)	22.02(*)	39.66(*)	24.53(*)	24.29(*)	19.96(*)	41.46(*)	25.664 (*)
	31.60 [6]	21.60 [6]	39.90 [6]	24.60 [6]	24.40 [6]	22.50 [6]	41.30 [6]	32.30 [6]

(*) Present

Further analysis was conducted for the ASAP case and for the SYCP2 case.

2.3 Changing the direction of the load

For the ASAP and SYCP2 cases, an analysis was carried out in terms of load action in the y direction as well. Normalized critical values are shown in Table 6.

3. CONCLUSIONS

The results shown in the tables reflect a relatively good agreement with the results obtained in the mentioned papers.

Initially, the change in ratio (E_1/E_2) was analysed. The variants of the ratio were 20, 30 and 40. The solutions, in the form of normalized critical load that acting in the direction of the x-axis, indicate that at the highest analysed ratio $E_1/E_2 = 40$, the plate achieves the best bearing capacity.

Different boundary conditions, as well as different orientations of the layers, were then further analysed. The normalized values of the critical load are shown in Tables 3 and 4. The lowest normalized critical load was obtained in the case of non-symmetrical distribution of UNSYM scheme for both, SSSS and CCCC. In the cases of the SSSS plate, the highest critical load appeared in the case of the ASAP scheme, with a normalized load of 31.29. In the case of the CCCC plate, the highest critical load appeared in the case of the SYCP2 scheme, with a normalized load of 41.46. For these reasons, further analysis was performed on such lamination schemes.

Table 6 shows the normalized critical load in the case of simply-supported and clamped plates at ASAP and SYCP2 orientations. From the comparisons shown, it can be concluded that in all cases the bearing capacity is better if the load acts in the direction of the *x*-axis. For simply-supported plates, according to the results, the better choice of lamination scheme would be ASAP, while in the case of a clamped plate, the better choice would be SYCP2.

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ANALIZA IZVIJANJA SLOJEVITIH KOMPOZITNIH PLOČA

Sažetak: Ovaj rad se bavi laminatnim pločama podvrgnutim jednoosnoj kompresiji. Razmatraju se efekti sheme laminacije i svojstava materijala na kritična opterećenja, kao i varijacije graničnih uslova i pravca djelovanja opterećenja. Analiza izvijanja sprovedena je primjenom metoda konačnih elemenata kroz softverski paket Simcenter Femap. Izvršeno je poređenje dobijenih rezultata sa već dostupnim rezultatima.

Ključne riječi: izvijanje, laminatne ploče, kompozitni materijali.

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