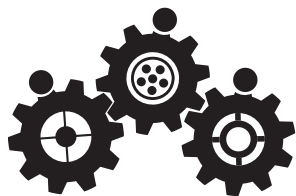




University of Banja Luka
Faculty of Mechanical Engineering



DEMI 2025

**17th International Conference on
Accomplishments in Mechanical
and Industrial Engineering**

PROCEEDINGS



**MAŠINSKI
FAKULTET**
PODGORICA

Banja Luka, 29–30 May 2025

University of Banja Luka
Faculty of Mechanical Engineering

PROCEEDINGS
DEMI 2025

Banja Luka, May 2025

**17TH INTERNATIONAL CONFERENCE ON ACCOMPLISHMENTS
IN MECHANICAL AND INDUSTRIAL ENGINEERING**

DEMI 2025

Supported by:

MINISTRY FOR SCIENTIFIC AND TECHNOLOGICAL DEVELOPMENT
AND HIGHER EDUCATION

Organizer and publisher:

FACULTY OF MECHANICAL ENGINEERING
UNIVERSITY OF BANJA LUKA

Co-organizer:

FACULTY OF MECHANICAL ENGINEERING
UNIVERSITY OF NIŠ, SERBIA

FACULTY OF MECHANICAL ENGINEERING
UNIVERSITY OF PODGORICA, MONTENEGRO

FACULTY OF TECHNICAL SCIENCES, REȘIȚE
BABES-BOLYAI UNIVERSITY OF CLUJ-NAPOCA
ROMANIA

REYKJAVIK UNIVERSITY, ICELAND

Publisher:

Faculty of Mechanical Engineering
University of Banja Luka

Editor of chief:

Assistant Professor Dejan Branković, PhD

Executive editor:

Milivoj Stipanović, BsC

ORGANIZING COMMITTEE

Assist. Prof. Dejan Branković, PhD
Chairman of Organizing Committee
Faculty of Mechanical Engineering, University of Banja Luka

Assoc. Prof. Stevo Borojević, PhD

Prof. Aleksandar Milašinović, PhD

Prof. Zorana Tanasić, PhD

Prof. Petar Gvero, PhD

Assoc. Prof. Branislav Sredanović, PhD

Assoc. Prof. Milovan Kotur, PhD

Assist. Prof. Biljana Vranješ, PhD

Assist. Prof. Jovan Škundrić, PhD

Assist. Prof. Biljana Prochaska, PhD

Assist. Prof. David C. Finger, PhD

Prof. Uroš Karadžić, PhD

Prof. Codruța-Oana HAMAT, PhD

Assoc. Prof. Miloš Kocić, PhD

Assist. Prof. Marko Mančić, PhD

Vukašin Pavlović, Assistant, PhD

Saša Laloš, Assistant, MSc

Milisav Marković, Senior Assistant, MSc

Danijela Kardaš, Senior Assistant, MSc

Milan Pupčević, Senior Assistant, MSc

Zoran Grahovac, Assistant, MSc

Dragan Erceg, Senior Assistant, MSc

Gordana Tošić, Senior Assistant, MSc

Petar Babić, Senior Assistant, MSc

Jelena Marković, Assistant, BSc

Nikola Despenić, Assistant

Vladimir Mitrović, Assistant

Sanja Maglov, MA+

Svetislav Bajić, BA Law

Nedeljka Sladojević Putnik, BA Economics

Milivoj Stipanović, BSc

SCIENTIFIC COMMITTEE

Prof. Stevo Borojević, PhD, Chairman of
Scientific Committee
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Zdravko Milovanović, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Darko Knežević, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Milan Tica, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Gordana Globočki-Lakić, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Petar Gvero, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Aleksandar Milašinović, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Zorana Tanasić, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Izet Bjelonja, PhD
Faculty of Mechanical Engineering
University of Sarajevo

Prof. Milan Zeljković, PhD
Faculty of Technical Sciences
University of Novi Sad

Prof. Slobodan Tabaković, PhD
Faculty of Technical Sciences
University of Novi Sad

Prof. Franci Pušavec, PhD
Faculty of Mechanical Engineering
University of Ljubljana

Prof. Mića Vukić, PhD
Faculty of Mechanical Engineering
University of Niš

Prof. dr Milenko Sekulić
Faculty of Technical Sciences
University of Novi Sad

Prof. Mileta Janjić, PhD
Faculty of Mechanical Engineering
University of Montenegro

Assist. Prof. Davorin Kramar, PhD
University of Ljubljana
Slovenia

Prof. Simo Jokanović, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. dr Platon Sovilj
Faculty of Technical Sciences
University of Novi Sad

Prof. dr Gordana Stefanović
Faculty of Mechanical Engineering
University of Niš

Prof. dr Miladin Stefanović
Faculty of Engineering Sciences
University of Kragujevac

Prof. Strain Posavljak, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Vid Jovišević, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Živko Babić, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Milan Banić, PhD
Faculty of Mechanical Engineering
University of Niš

Prof. Branko Blanuša, PhD
Faculty of Electrical Engineering
University of Banja Luka

Prof. Tihomir Latinović, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Sead Pašić, PhD
Faculty of Mechanical Engineering
“Džemal Bijedić” University in Mostar

Prof. Borut Kosec, PhD
Faculty of Natural Sciences and
Engineering University of Ljubljana

Senior Scient. Eng. Milica Grahovac, PhD
Lawrence Berkeley National
Laboratory
USA

Prof. Doina Frunzaverde, PhD
Faculty of Engineering Resita
Babeş-Bolyai University

Prof. Mihajlo Stojčić, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Assoc. Prof. Siniša Bikić, PhD
Faculty of Technical Sciences
University of Novi Sad

Assist. Prof. Esad Tombarević, PhD
Faculty of Mechanical Engineering
University of Montenegro

Prof. Valentina Golubović-Bugarski, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Snežana Petković, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Bojan Knežević, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Goran Janjić, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Mladen Todić, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Milovan Kotur, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Dragoslav Dobraš, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Đorđe Čiča, PhD,
Faculty of Mechanical Engineering,
University of Banja Luka

Prof. Branislav Sredanović, PhD,
Faculty of Mechanical Engineering,
University of Banja Luka

Assist. Prof. Dejan Branković, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Slobodan Lubura, PhD
Faculty of Electrical Engineering
University of East Sarajevo

Prof. Sanda Midžić – Kurtagić, PhD
Faculty of Mechanical Engineering
University of Sarajevo

Assoc. Prof. Srđan Vasković, PhD
Faculty of Mechanical Engineering
University of East Sarajevo

Prof. Vladimir Stojanović, PhD
Faculty of Mechanical Engineering
University of Niš

Prof. Milan Radovanović, PhD
Faculty of Mechanical Engineering
University of Belgrade

Prof. Dragoslava Stojiljković, PhD
Faculty of Mechanical Engineering
University of Belgrade

Prof. Nebojša Manić, PhD
Faculty of Mechanical Engineering
University of Belgrade

Prof. Milan Lečić, PhD
Faculty of Mechanical Engineering
University of Belgrade

Prof. Neven Duić, PhD
Faculty of Mechanical Engineering
and Naval Architecture
University of Zagreb

Assist. Prof. Srđan Savić, PhD
Faculty of Technical Sciences
University of Novi Sad

Prof. Dejan Mitrović, PhD
Faculty of Mechanical Engineering
University of Niš

Prof. Goran Janevski, PhD
Faculty of Mechanical Engineering
University of Niš

Prof. Uroš Karadžić, PhD
Faculty of Mechanical Engineering
University of Montenegro

Prof. Deaconu Sorin, PhD
University Politehnica Timisoara
Romania

Prof. dr Vinko Babić, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Goran Petrović, PhD
Faculty of Mechanical Engineering
University of Niš

Prof. Radoje Vujadinović, PhD
Faculty of Mechanical Engineering
University of Montenegro

Assist. Prof. David C. Finger, PhD
School of Science and Engineering
University of Reykjavik, Iceland

Assoc. Prof. Edin Berberović, PhD
Polytechnical Faculty
University of Zenica

Prof. Živojin Stamenković, PhD
Faculty of Mechanical Engineering
University of Niš

Prof. Miloš Simonović, PhD
Faculty of Mechanical Engineering
University of Niš

Prof. Predrag Živković, PhD
Faculty of Mechanical Engineering
University of Niš

Prof. Hasan Smajić, PhD
University of Applied Sciences Keln
Germany

Prof. Mirjana Laković, PhD
Faculty of Mechanical Engineering
University of Niš

Prof. Jezdimir Knežević, PhD
CEO at MIRCE Academy
United Kingdom

Prof. Radivoje Mitrović, PhD
Faculty of Mechanical Engineering
University of Belgrade

Prof. Vlastimir Nikolić, PhD
Faculty of Mechanical Engineering
University of Niš

Prof. Nenad T. Pavlović, PhD
Faculty of Mechanical Engineering
University of Niš

Prof. Codruța-Oana HAMAT, PhD
Faculty of Technical Sciences,
REȘIȘTE
Romania

Prof. Igor Vušanović, PhD
Faculty of Mechanical Engineering
University of Montenegro

Prof. Gelu Ovidiu Tirian, PhD
University Politehnica Timisoara
Romania

Prof. Dejan Lukić, PhD
Faculty of Technical Sciences
University of Novi Sad

Prof. Saša Živanović, PhD
Faculty of Mechanical Engineering
University of Belgrade

Prof. Mijodrag Milošević, PhD
Faculty of Technical Sciences
University of Novi Sad

Prof. Vojislav Novaković, PhD
NTNU, Norway

Prof. Milan Rackov, PhD
Faculty of Technical Sciences
University of Novi Sad

Prof. Mladomir Milutinović, PhD
Faculty of Technical Science
University of Novi Sad

Prof. Dragan Milčić, PhD
Faculty of Mechanical Engineering
University of Niš

Prof. Radoslav Tomović, PhD
Faculty of Mechanical Engineering
University of Montenegro

Prof. Janko Jovanović, PhD
Faculty of Mechanical Engineering
University of Montenegro

Prof. Nebojša Radić, PhD
Faculty of Mechanical Engineering
University of East Sarajevo

Prof. dr Tomaž Berlec
Faculty of Mechanical Engineering
University of Ljubljana

Prof. dr Janez Kušar
Faculty of Mechanical Engineering
University of Ljubljana

Prof. Miodrag Hadžistević, PhD
Faculty of Technical Sciences
University of Novi Sad

Prof. Branko Štrbac, PhD
Faculty of Technical Sciences
University of Novi Sad

Prof. Peđa Milosavljević, PhD
Faculty of Mechanical Engineering
University of Niš

Prof. Jelena Šaković Jovanović, PhD
Faculty of Mechanical Engineering
University of Montenegro

Prof. Milija Krajišnik, PhD
Faculty of Mechanical Engineering
University of East Sarajevo

Prof. Ilija Ćosić, Emeritus
Faculty of Technical Sciences
University of Novi Sad

Prof. Mirko Soković, PhD

University of Ljubljana
Slovenia

Prof. Miroslav Bobrek, PhD
Faculty of Mechanical Engineering
University of Banja Luka

Prof. Igor Budak, PhD
Faculty of Technical Sciences
University of Novi Sad

Prof. Žarko Ćojbašić, PhD
Faculty of Mechanical Engineering
University of Niš

Assist. Prof. Ivan Grujić, PhD
Faculty of Engineering Sciences
University of Kragujevac

Assist. Prof. Nadica Stojanović, PhD
Faculty of Engineering Sciences
University of Kragujevac

Prof. Vladimir Jovanović, PhD
Faculty of Mechanical Engineering
University of Belgrade

Prof. Saša Randjelović, PhD
Faculty of Mechanical Engineering
University of Niš

Prof. Dušica Pešević, PhD
Faculty of Natural Sciences and
Mathematics University of Banja Luka



Banja Luka
29–30 May 2025

DEMI 2025
**17th International Conference on
Accomplishments in Mechanical and
Industrial Engineering**
www.demi.mf.unibl.org



Influence of Kerr's elastic foundation on free vibration of functionally graded plate

G. Bogdanović^a, D. Čukanović^b, A. Radaković^c, P. Knežević^b

^aUniversity of Kragujevac, Faculty of Engineering, Sestre Janjić 6, 34000 Kragujevac, Serbia

^bUniversity of Priština in Kosovska Mitrovica, Faculty of Technical Sciences, Knjaza Miloša 7, 38220 Kosovska Mitrovica, Serbia

^cInstitute for Information Technologies Kragujevac, Jovana Cvijića bb, 34000 Kragujevac, Serbia

Abstract The introduction of the paper presents an overview of composite materials. Emphasis is placed on their continuing importance as a key area of research and their significant impact on various industrial applications. Special attention is paid to functionally graded materials (FGM), which are classified as modern composites. The primary objective of this study is to develop analytical formulations and solutions for the analysis of free vibrations of functionally graded plates supported on Kerr elastic foundation. The kinematic displacement-strain relations are represented using a higher-order shear deformation theory based on shape functions. The material properties of FG plates are assumed to vary through the thickness following a power law distribution. The applied shape functions ensure that the boundary conditions are satisfied without stresses at the plate surfaces, eliminating the need for shear correction factors. Various numerical examples are given to highlight the effects of different values of the power law index, aspect ratio and aspect-to-thickness ratio. The results are compared with those from the quasi-3D theory and the conclusions suggest that the proposed theory is both accurate and effective in predicting the vibration behavior of functionally graded plates.

Keywords functionally graded materials, higher-order shear deformation theory, shape functions, Kerr's elastic foundations, free vibration

1. INTRODUCTION

Over the past four decades, composite materials have experienced a significant transformation, establishing themselves as essential materials in modern engineering components. Among them, fiber-reinforced composites have gained particular attention due to their outstanding stiffness-to-weight and strength-to-weight ratios, which have facilitated their extensive application across a wide range of industries.

A key advantage of laminated fiber-reinforced composites lies in the flexibility they offer to designers, not only in shaping and dimensioning structural elements but also in optimizing the material properties. These laminates demonstrate excellent mechanical performance. However, their principal limitation is the tendency toward delamination, which occurs when fibers detach from the matrix under the influence of external loads, moisture, or elevated temperatures.

In this context, a new class of composite materials known as functionally graded materials (FGM) has gained prominence due to their enhanced mechanical properties. A significant number of studies in the literature [1, 2] focus on the processing and modelling of these materials to make them suitable for use in

Corresponding author

PhD, Dragan Čukanović, Associate professor,
dragan.cukanovic@pr.ac.rs

University of Priština, Faculty of Technical Sciences
Knjaza Miloša 7
38220 Kosovska Mitrovica, Serbia

various industrial sectors. These materials are frequently categorized as advanced engineering materials due to their capability to maintain consistent performance under demanding operating conditions. The primary structural advantage of FGM over conventional laminated composites is their enhanced resistance to failure at material interfaces. This issue is effectively mitigated with FGM by the gradual variation of material composition across the structure.

Initial applications of FGM were observed in thermal barrier coatings for aerospace components and in fusion reactors within the nuclear energy sector. Subsequently, due to their tailored gradation of constituent properties, FG materials have since found use in other areas such as military technology, medicine, dentistry, the automotive industry.

The rapid advancement of engineering technologies has prompted extensive research into the mechanical behaviour of FGM, particularly under varying loading conditions. Comprehensive analyses of their dynamic and structural responses are essential for the effective design of FGM components intended for industrial applications. Models of functionally graded plates on elastic foundations are widely employed as reliable tools for addressing engineering problems in various disciplines [3-6].

2. THEORETICAL FORMULATIONS

2.1 Description of model

A functionally graded (FG) rectangular plate with length a , width b , and thickness h is considered within a Cartesian coordinate system (x, y, z) , as illustrated in Fig. 1.

The FG plate is assumed to rest on a Kerr-type elastic foundation. This foundation model is characterized by three stiffness parameters: k_s representing the shear layer stiffness, k_u and k_l denoting the stiffness coefficients of the upper and lower elastic layers, respectively.

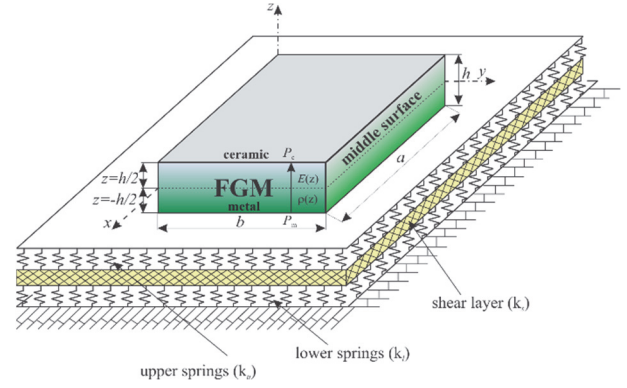


Fig. 1. The geometry of rectangular FG plate resting on Kerr's elastic foundation

2.2 Material properties

FG plates are composed of two primary phases: a ceramic phase and a metallic phase. These materials are typically manufactured by gradually varying the volume fraction of the constituent phases along the thickness direction. In a typical configuration, the ceramic and metal phases are distributed such that one is concentrated at the top surface and the other at the bottom surface of the plate. This continuous gradation through the thickness results in corresponding variations in the mechanical properties of the plate, enhancing its structural performance and resistance to thermal or mechanical loads.

The material properties, including Young's modulus and density, vary continuously along the thickness direction of the FG plate according to a power-law distribution. These properties are estimated using the rule of mixtures, which provides an effective approximation of the overall behaviour based on the volume fractions of the constituent materials as:

$$\begin{aligned} E(z) &= E_m + E_{cm} \left(\frac{1}{2} + \frac{z}{h} \right)^p, \quad E_{cm} = E_c - E_m, \\ \rho(z) &= \rho_m + \rho_{cm} \left(\frac{1}{2} + \frac{z}{h} \right)^p, \quad \rho_{cm} = \rho_c - \rho_m, \\ V_f &= \left(\frac{1}{2} + \frac{z}{h} \right)^p. \end{aligned} \quad (1)$$

where E_c and E_m represent the Young's module of the ceramic and metal constituents, respectively. V_f denotes the volume fraction of the constituents, while p is the volume fraction exponent-parameter that governs the gradation

profile and significantly influences the mechanical response of the FG plate. Poisson's ratio ν is typically assumed to remain constant throughout the thickness.

2.3 Displacement fields

According to the higher-order shear deformation theory (HSDT) based on shape functions the displacement field are expressed can be written as:

$$\begin{aligned} u &= u_0(x, y, t) - zw_{0,x} + f(z)\theta_x, \\ v &= v_0(x, y, t) - zw_{0,y} + f(z)\theta_y, \\ w &= w_0(x, y), \end{aligned} \quad (2)$$

where $f(z)$ is the representative shape function that denotes the distribution of the transverse shear strains and stresses through the thickness of FG plates. Here, the shape function developed by the authors of this paper is used [7]:

$$f(z) = z \left(\cosh\left(\frac{z}{h}\right) - 1,388 \right). \quad (3)$$

The previously mentioned assumed displacement field at any point within the FG plate serves as the foundation for establishing the strain-displacement relationships. Using a generalized Hooke's law, the components of unit loads are obtained.

2.4 Equations of motion

In order to get equations of motion, it is necessary to define the strain energy in the following form:

$$U_s = \int_{-h/2}^{h/2} \int_A \left(\sigma_{xx} \varepsilon_{xx} + \sigma_{yy} \varepsilon_{yy} + \sigma_{zz} \varepsilon_{zz} + \tau_{xy} \gamma_{xy} + \tau_{xz} \gamma_{xz} + \tau_{yz} \gamma_{yz} \right) dAdz, \quad (4)$$

then additional strain energy caused by the Kerr's elastic foundation as:

$$U_e = \frac{1}{2} \int_A \left\{ \frac{k_l k_u}{k_l + k_u} w^2 + \frac{k_s k_u}{k_l + k_u} \left[\left(\frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] \right\} dA, \quad (5)$$

as well as kinetics energy in form:

$$K = \frac{1}{2} \int_{-h/2}^{h/2} \int_A \rho(z) \left[\left(\frac{\partial u}{\partial t} \right)^2 + \left(\frac{\partial v}{\partial t} \right)^2 + \left(\frac{\partial w}{\partial t} \right)^2 \right] dAdz. \quad (6)$$

where is $\rho(z)$ variable according to (1).

Using the Hamilton's principles, equations of motion become:

$$\begin{aligned} \delta u_0: N_{xx,x} + N_{xy,y} &= I_1 \ddot{u}_0 - I_2 \ddot{w}_{0,x} + I_4 \ddot{\theta}_x, \\ \delta v_0: N_{yy,y} + N_{yx,x} &= I_1 \ddot{v}_0 - I_2 \ddot{w}_{0,y} + I_4 \ddot{\theta}_y, \\ \delta w_0: M_{xx,xx} + 2M_{xy,xy} + M_{yy,yy} &- \frac{k_l k_u}{k_l + k_u} w_0 + \frac{k_s k_u}{k_l + k_u} (w_{0,xx} + w_{0,yy}) \\ &= I_1 \ddot{w}_0 + I_2 (\ddot{u}_{0,x} + \ddot{v}_{0,y}) - I_3 (\ddot{w}_{0,xx} + \ddot{w}_{0,yy}) + I_5 (\ddot{\theta}_{x,x} + \ddot{\theta}_{y,y}), \\ \delta \theta_x: P_{xx,x} + P_{xy,y} - R_x &= I_4 \ddot{u}_0 - I_5 \ddot{w}_{0,x} + I_6 \ddot{\theta}_x, \\ \delta \theta_y: P_{xy,x} + P_{yy,y} - R_y &= I_4 \ddot{v}_0 - I_5 \ddot{w}_{0,y} + I_6 \ddot{\theta}_y. \end{aligned} \quad (7)$$

where are the mass moments of inertias defined in form:

$$\begin{aligned} I_1 &= \int_{-h/2}^{h/2} \rho(z) dz, & I_2 &= \int_{-h/2}^{h/2} \rho(z) z dz, \\ I_3 &= \int_{-h/2}^{h/2} \rho(z) f(z) dz, & I_4 &= \int_{-h/2}^{h/2} \rho(z) z^2 dz, \\ I_5 &= \int_{-h/2}^{h/2} \rho(z) z f(z) dz, & I_6 &= \int_{-h/2}^{h/2} \rho(z) (f(z))^2 dz. \end{aligned} \quad (8)$$

2.5 Analytical Navier's procedure

To derive analytical solutions of equations of motion (7), assumed displacement functions and boundary conditions corresponding to a simply supported plate are employed, in accordance with Navier's solution technique given as follows:

$$\begin{aligned} u_0(x, y, t) &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} U_{mn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{i\omega t}, \\ v_0(x, y, t) &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} V_{mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{i\omega t}, \\ w_0(x, y, t) &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{i\omega t}, \\ \theta_x(x, y, t) &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} T_{xmn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{i\omega t}, \\ \theta_y(x, y, t) &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} T_{ymn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{i\omega t}. \end{aligned} \quad (9)$$

By substituting (9) into (7) the eigenvalue equation can be obtained:

$$[\mathbf{K} - \omega^2 \mathbf{I}] \bar{\mathbf{U}} = \mathbf{0}, \quad (10)$$

The smallest eigenvalue obtained from the solution of equation (10) corresponds to the fundamental natural frequency of the FG plate.

3. NUMERICAL EXAMPLES AND RESULTS

To validate the derived theoretical formulations, a series of numerical simulations were conducted and compared with results obtained using other shear deformation theory. The analysis was carried out on an FG plate composed of metal and ceramic phases, whose mechanical properties are listed in Table 1.

Table 1. Material properties of FGM constituents

Material	Material properties		
	Elasticity modulus E [GPa]	Poisson's ratio ν	Density ρ [kg/m ³]
Aluminum (Al)	$E_m=70$	$\nu=0.3$	$\rho_m=2702$
Alumina (Al_2O_3)	$E_c=380$	$\nu=0.3$	$\rho_c=3800$

This section presents the results of free vibration analysis for various FG plates with different aspect ratios (a/h), resting on Kerr-type elastic foundations. The influence of the foundation parameters k_u and k_s , as well as the volume fraction exponent p , is investigated. In order to facilitate the presentation and comparison of the natural frequencies for rectangular FG plates, the computed frequencies are normalized according to the following expression:

$$\tilde{\omega} = \omega h \sqrt{\rho_m / E_m}. \quad (11)$$

Table 2 illustrates the influence of the shear layer stiffness and the stiffness of the upper elastic layer on the dynamic behaviour of the FG plate, which is supported by a Kerr-type foundation. In these examples, the stiffness of the lower elastic lower is fixed at $k_l=100$. It is noted that the present results in Table 2 agree very well with results based on quasi-3D theory given in [8]. The obtained results indicate that non-dimensional fundamental frequency increases with the increase of k_u and k_s .

Table 2. Non-dimensional fundamental frequencies of the FG plates resting on Kerr foundation ($k_l=100$)

k_u	k_s	a/h	Theory	$\tilde{\omega}$			
				FGM plate			
				$p=0.5$	$p=1$	$p=2$	$p=5$
100	0	20	[8]	0.0253	0.0231	0.0212	0.0202
			Present study	0.0251	0.0227	0.0208	0.0198
		10	[8]	0.0988	0.0903	0.0830	0.0783
			Present study	0.0981	0.0890	0.0813	0.0769
		20	[8]	0.0329	0.0316	0.0308	0.0305
			Present study	0.0328	0.0314	0.0305	0.0303
100	100	20	[8]	0.1294	0.1245	0.1212	0.1201
			Present study	0.1290	0.1237	0.1201	0.1192
		10	[8]	0.0351	0.0341	0.0335	0.0334
			Present study	0.0350	0.0339	0.0331	0.0330
		20	[8]	0.1385	0.1345	0.1320	0.1316
			Present study	0.1382	0.1338	0.1310	0.1309

The non-dimensional fundamental frequency decreases with the increase of power law index p . This effect can be interpreted by the fact that the FG plates become harder when the volume fraction of ceramic increases.

It is important to note that when $p=0$, the plate is fully homogeneous and composed entirely of ceramic material. In contrast, in the theoretical limit as $p \rightarrow \infty$, the plate also becomes homogeneous, but consists solely of metal. From a practical standpoint, the material can be considered effectively homogeneous for values of p greater than 10. Consequently, for $0 < p < 10$, the plate exhibits the behaviour of a functionally graded material.

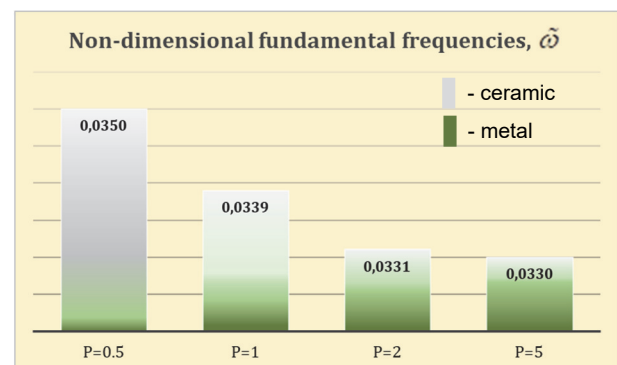


Fig. 2. Non-dimensional fundamental frequencies of the FG plates resting on Kerr foundation ($k_l=100$, $k_s=100$, $k_u=200$)

4. CONCLUSION

In this study, an analytical model based on a higher-order shear deformation theory was developed to analysis the free vibration behavior of functionally graded plates resting on a Kerr-type elastic foundation. The formulation incorporates shape functions that satisfy free boundary conditions on the plate surfaces, thereby eliminating the need for shear correction factors. Material properties of the FGM plates were assumed to vary continuously through the thickness according to a power-law distribution.

The numerical results demonstrate the significant influence of the power-law index, aspect ratio, and thickness-to-length ratio on the dynamic response of the plates. Comparisons with results obtained from quasi-3D theory confirm the accuracy and computational efficiency of the proposed model.

Overall, the presented approach provides a reliable and effective tool for the vibration analysis of functionally graded structures, offering valuable insights for the design and optimization of advanced composite components in engineering applications.

Acknowledgement

The authors would like to thank the Ministry of Science, Technological Development and Innovation of the Republic of Serbia for funding the scientific research work, contract no. 451-03-65/2024-03/200155, realized by the Faculty of Technical Sciences in Kosovska Mitrovica, University of Pristina.

REFERENCES

- [1] Boggarapu, V. et al. (2021). State of the art in functionally graded materials. *Composite Structures*, vol. 262, 113596. DOI: [10.1016/j.compstruct.2021.113596](https://doi.org/10.1016/j.compstruct.2021.113596)
- [2] Saleh, B. et al. (2020). 30 Years of functionally graded materials: An overview of manufacturing methods, Applications and Future Challenges. *Composites Part B: Engineering*, vol. 201, 108376. DOI: [10.1016/j.compositesb.2020.108376](https://doi.org/10.1016/j.compositesb.2020.108376)
- [3] Hoang, V. N. V., Thanh, P. T. (2023). Influences of arbitrary-distributed Kerr foundation on free vibration and nonlinear transient response of functionally graded plate in thermal environment. *Thin-Walled Structures*, vol. 188, 110802. DOI: [10.1016/j.tws.2023.110802](https://doi.org/10.1016/j.tws.2023.110802)
- [4] Shahsavari, D., Shahsavari, M., Li, L., Karami, B. (2018). A novel quasi-3D hyperbolic theory for free vibration of FG plates with porosities resting on Winkler/Pasternak/Kerr foundation, *Aerospace Science and Technology*, vol. 72, p. 134-149. DOI: [10.1016/j.ast.2017.11.004](https://doi.org/10.1016/j.ast.2017.11.004)
- [5] Kenanda, M., A. et al. (2025). Thermo-mechanical free vibration analysis of porous solar FGM plates resting on Kerr's foundation using a new logarithmic-hyperbolic shear deformation theory, *Archive of Applied Mechanics*, vol. 95, no. 91. DOI: [10.1007/s00419-025-02779-7](https://doi.org/10.1007/s00419-025-02779-7)
- [6] Jankowski, P. (2020). Effect of Kerr foundation and in-plane forces on free vibration of FGM nanobeams with diverse distribution of porosity, *Acta mechanica et automatica*, vol. 14 no. 3. DOI: [10.2478/ama-2020-0020](https://doi.org/10.2478/ama-2020-0020)
- [7] Čukanović, D., Radaković, A., Bogdanović, G., Milanović, M., Redžović, H., Dragović, D. (2018). New Shape Function for the Bending Analysis of Functionally Graded Plate. *Materials*, vol. 11, no. 12, 2381. DOI: [10.3390/ma11122381](https://doi.org/10.3390/ma11122381)
- [8] Mengzhen, L., Soares, C., G., Yan, R. (2021). Free vibration analysis of FGM plates on Winkler/Pasternak/Kerr foundation by using a simple quasi-3D HSDT, *Composite Structures*, vol. 264, 113643. DOI: [10.1016/j.compstruct.2021.113643](https://doi.org/10.1016/j.compstruct.2021.113643)

CIP - Каталогизација у публикацији
Народна и универзитетска библиотека
Републике Српске, Бања Лука

621.3(082)(0.034.4)
621(082)(0.034.4)

**INTERNATIONAL Conference on Accomplishments in
Mechanical and Industrial Engineering (17 ; 2025 ; Banja
Luka)**

Proceedings [Електронски извор] : DEMI 2025 / [17th
international conference on accomplishments in mechanical and
industrial engineering, Banja Luka, May 2025] ; editor in chief
Dejan Branković]. - El. zbornik. - Banja Luka : Faculty of
Mechanical Engineering, 2025 ([Banja Luka : Mako print]). - 1
електронски оптички диск (CD-ROM) : слика ; 12 cm

Систематски захтјеви: нису наведени. - Насл. са насл. екрана. -
Ел. публикација у ПДФ формату опсега 673 стр. - Тираж 150. -
Библиографија уз сваки рад.

ISBN 978-99976-11-08-6

COBISS.RS-ID 142611457