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AN APPROACH FOR BUSINESS-MANUFACTURING SYSTEMS MODELING IN ORDER TO APPLY OPTIMIZATION PROCESSES

Abstract: *The optimal production program is based on the assumption that there are quantitative measures of comparison with other admissible solutions, and is often a compromise between the desired goals and constraints which act as a condition for achieving extreme solutions. The importance of optimization of the production is significant because it is the basis for all other plans. The purpose of this paper is to propose an approach for modeling the business-production system in order to determine the optimal production program. The paper deals with the applied simulation and optimization processes by which the mentioned system can lead to optimum-required state. A mathematical model was defined using the linear programming method and the results were presented using the Mathematica software package.*

Keywords: *Optimization, Production program, Software package Mathematica*

1. Introduction

Business-manufacturing systems (BMS) belong to a group of organizational, complex, dynamic, stochastic, and open systems, and as such, tend towards their own degeneration if various approaches to the application of optimization processes are not continuously implemented.

The question of the production program is of strategic importance for the business, survival, and development of BMS. It is natural for managers to aim for an expected-desired state that is also optimal. However, desires are always hindered by constraints that must always be taken into account when applying optimization methods and techniques. In market business conditions, the production program represents the result of the optimal use of production potentials while respecting real market constraints. If the BMS has "excess" capacity, it means that it is not optimally utilizing its capacity through specific contracts, and it is necessary to determine the reason for this through appropriate expert teams. If there is a "shortage" of capacity, it means that the demand is greater than the production capacity. Then, it is necessary to identify "bottlenecks" in production and remove them through more efficient organization or investment, thus creating conditions for fulfilling contractual obligations.

The researchers aim to define a mathematical model that enables finding optimal solutions at different levels of management and from different aspects. According to Joppen et al. (2019), the biggest challenge is optimizing the overarching production planning and the lack of an overall optimum. The following methods are most commonly applied for production planning: LP (linear programming), NLP (nonlinear programming), and MILP (mixed integer

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linear programming) (Kallrath et al., 2005). Numerous studies deal with this issue, and a small part of it is presented in this paper.

Joppen et al. (2019) present a practical framework for optimizing production management processes, which represents a holistic approach to optimizing production planning and control processes. Their framework describes how conflicting objectives can be systematically analyzed and how a reasonable operational status can be derived. The authors address issues related to medium and short-term production planning, including defining goal analysis, initial state analysis, and optimization.

Trachenko et al. (2021) developed an economic-mathematical model for the rational formation of a business process management system in the engineering services sector, taking into account the time factor. The results of using the optimization model for the management of business processes such as design, electro-maintenance, production of electrical equipment, commissioning and adjustment, and consulting were presented.

Hervert-Escobar & López-Pérez (2020) propose an optimization model that maximizes the fulfilment of requirements, taking into account typical constraints from production planning formulations, as well as production constraints in real-life scenarios such as limited product changeovers and minimum machine run lengths. The authors utilized a mixed-integer programming model.

Various models can be found in the literature, such as: a model (two linear programming models) for maximizing Net Present Value (Petridis et al., 2020); a model for minimizing the aggregate production costs (Golari et al., 2017; Naeem et al., 2013); a model for optimizing the input technical effectivenes and efficiency of production process (Donini & Barbiroli, 1997); a mixed integer linear programming model for optimizing the placement of composite parts in an autoclave (Dios et al., 2017); a mixed integer linear programming model for integrating production, inventory, distribution, and routing decisions in a single framework (Miranda et al, 2018); a model for finding the optimum production rate in factories that use seasonally produced raw materials (Kioulafas & Kapralos, 1980); a fuzzy multi-objective linear programming model that formulates the problem of mold production and outsourcing decisions (Wang et al., 2013); a cost model for evaluating the cost and benefit analysis of implementing Industry 4.0 elements in the production plant (Alami & ElMaraghy, 2021).

This paper aims to propose a different approach for modeling BMS in order to determine the optimal production program from the perspective of maximum utilization of available workforce capacity, within a one-year time interval, and to strive to describe the production system, as a hierarchical system with multiple levels, by a mathematical model. A mathematical model will be defined for the selected business-production system, using the method of linear programming, and the results will be presented by applying the Mathematica software package.

2. Defining the Mathematical Model

We can explore the optimal production program at the level of the BMS as a whole or at the level of its subsystems. If we model the BMS as a system that cannot be further broken down, then we determine the optimal production program at one level. The corresponding parameters in the mathematical model are usually grouped at the product level and constraints at the level of the system as a whole. However, optimizing the production program of a BMS by considering it as a complex system, consisting of multiple subsystems (possible horizontal and vertical decomposition), is a much more complex task. The question arises whether the optimum at the system level is also the optimum at the subsystem level, i.e., whether there is a unity of goals between subsystems and the system as a whole.

When studying and modeling hierarchical systems, it is necessary to define organizational levels, description levels, and decision-making levels. Constraints in the model are closely related to organizational levels, management criteria for decision-making levels, and corresponding parameters in the mathematical model can be grouped at the level of products, parts, or technological variants of production. The long-term aspect of the production program is usually based on assumptions and predictions, where qualitative elements prevail. Short-term production program planning is based on facts, with quantifiable elements prevailing. Depending on the adopted concept of programming orientation, from prediction to short-term planning, various methods and techniques can be used.

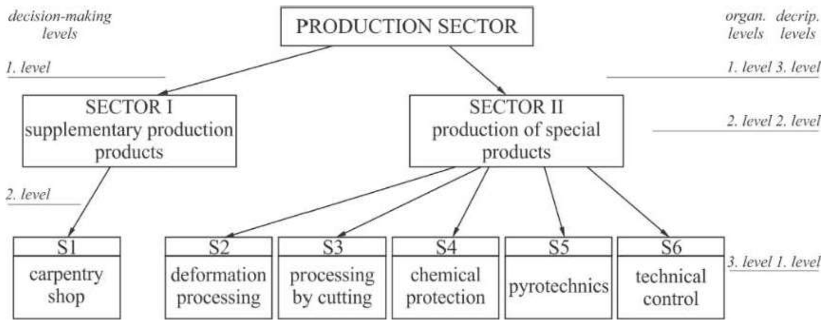

Figure 1. Hierarchical representation of the selected BMS Defining the coefficients in the mathematical model

As the goal of this paper is to determine the optimal production program from the perspective of maximizing the use of available workforce capacity and to describe the production system using a mathematical model, the most suitable method is linear programming (LP). The advantage of using this method is that there is a well-developed procedure (method) for solving such problems. However, it should also be noted that the application of LP is based on the assumption that relevant variables are deterministic, which does not correspond to reality, as they are stochastic variables. In this context, it is necessary to thoroughly investigate all parameters that are included in the model and perform a sensitivity analysis so that the obtained solution has practical value. The application of the LP method involves defining a mathematical model consisting of an objective function or management criterion and a set of constraints. Figure 1 shows a system model in the form of a graph, with a hierarchical arrangement of subsystems and connections between levels. Vertical decomposition is performed with three organizational levels, three levels of description, and two levels of decision-making. The mathematical model for production optimization in the matrix form is presented by equations (1) – (4). Relation (1) represents the objective function that needs to be maximized, taking into account the optimal solution from the perspective of maximum utilization of available production human resources. Constraints (2) define the available production human resource capacities (B), while constraints (3) define the minimum (D) and maximum (G) market demands**.** To apply software tools, it is necessary to represent the mathematical model in an expanded form.

$$max\ F(x) = C \cdot X \quad (1)$$

$$A \cdot X \leq B \quad (2)$$

$$D \leq X \leq G \quad (3)$$

$$X \geq 0 \quad (4)$$

The mathematical model enables finding optimal solutions at the system level as a whole (first decision-making level, third description level), with constraints on the available human resources defined at the third organizational level (level of work units). The production plan includes 41 products (Xi). To define the coefficients (elements of matrices C, A, B, D, and G) in the mathematical model, it is necessary to determine: total manufacturing time at the product level (c_j); manufacturing times for current products by work units (a_{ij}); available potential of production human resources (b_i), in accordance with Relation (5); market absorption capacity (d_k and g_k). If market constraints are not included in the mathematical model, it is assumed that everything produced can be sold.

Table 1. Overview of the necessary data for the calculation of the coefficients *bi*

Si	z_i	p_{ni}	η_{ri}	b_i
S1	85	1,10	0,75	135166
S2	182	1,22	0,75	320987
S3	235	1,19	0,74	398879
S4	38	1,19	0,73	63628
S5	222	1,18	0,72	363549
S6	71	1,25	0,71	121457
Dr = 257 (work./yr), *Cs* = 7,5 (h/shift)				

Table 1 shows an overview of available workforce capacities per work unit (*Si*) i.e. the coefficients *bi* in the mathematical model are defined.

$$b_i = z_i \cdot \eta_{r_i} \cdot D_r \cdot C_s \cdot p_{n_i} \quad (5)$$

Where: D_r - total number of workdays (workdays/τ), C_s - projected effective working hours per shift (h/shift), z_i - total number of production workers (workers/τ), p_{ni} - average norm-hour execution for the observed organizational unit and time period, η_{ri} - productive human resources utilization level.

Table 2. Input data for defining the mathematical model

Xi	d_k	g_k	c_i
1	*2*	*3*	*4*
X1	5000	18000	0,51203
X2	500	5000	0,28000
X3	5000	17000	0,35000
X4	7000	7000	0,51203
X5	200	2200	0,40789
X6	9000	20000	1,75969
X7	10000	34000	1,45700
X8	12000	67000	1,45700
X9	39900	40000	2,898
X10	21300	21500	2,99500
X11	16800	17000	2,36516
X12	50000	115500	2,27764
X13	300	8500	0,43236
X14	700	28500	2,41000
X15	1000	56700	2,29300
X16	100	1200	2,20300
X17	800	3500	2,04563
X18	900	5500	3,22500
X19	4500	15500	2,79258
X20	35400	35600	0,05568
X21	35400	35600	0,02655
X22	3000	17800	1,60858
X23	500	5500	0,29800
X24	1600	11000	0,30100
X25	500	6000	0,59100
X26	700	3100	0,91900
X27	30	300	1,69500
X28	3000	17900	0,65600
X29	20000	75000	0,61955
X30	200	5000	1,25400
X31	70	2500	0,77100
X32	70	2500	0,64300
X33	70	2500	0,73820
X34	120	1250	9,48600
X35	700	6500	17,51300
X36	3928	9000	16,65867
X37	600	5500	9,85520
X38	30	650	10,32450
X39	300	850	12,01230
X40	90	1000	5,42150
X41	30	350	3,50000

By analyzing the trend of the achieved production over a period of five years, respecting the average values, minimum and maximum constraints were determined (Table 2, columns 2 and 3). In Table 2, column 4 shows the elements of matrix C, Relation (1).

Due to limited space the coefficients a_{ij} in the mathematical model are not shown in the paper.

The mathematical model for optimizing the production program from the aspect of maximizing the use of available workforce capacities by subsystems *Si* while respecting market constraints has the following form, Relations (6-8):

Objective:

$$maxF(x) = 0{,}51203x_1 + 0{,}28x_2 + 0{,}35x_3 + \cdots + 2{,}203x_{16} + \cdots + 5{,}4215x_{40} + 3{,}5x_{41} \quad (6)$$

Production human resource constraint $j = \overline{1{,}6}$:

$$\begin{gathered}0{,}017409x_1 + 0{,}00896x_2 + 0{,}021x_3 + \cdots + 0{,}173488x_{40} + 0{,}112x_{41} \le 135166\\ 0{,}087045x_1 + 0{,}0448x_2 + 0{,}0805x_3 + \cdots + 0{,}742745x_{40} + 0{,}4795x_{41} \le 320987\\ 0{,}095238x_1 + 0{,}0532x_2 + 0{,}06545x_3 + \cdots + 1{,}116829x_{40} + 0{,}721x_{41} \le 398879\\ 0{,}06554x_1 + 0{,}0434x_2 + 0{,}06755x_3 + \cdots + 0{,}623472x_{40} + 0{,}4025x_{41} \le 63628\\ 0{,}186891x_1 + 0{,}10444x_2 + 0{,}07x_3 + \cdots + 2{,}087278x_{40} + 1{,}3475x_{41} \le 363549\\ 0{,}059907x_1 + 0{,}0252x_2 + 0{,}0455x_3 + \cdots + 0{,}677688x_{40} + 0{,}4375x_{41} \le 121457 \quad (7)\end{gathered}$$

Market constraints $j = \overline{7{,}88}$:

$$\begin{gathered}x_1 \le 18000;\ x_2 \le 5000;\ x_3 \le 17000; \ldots\ x_{40} \le 1000; x_{41} \le 350;\\ x_1 \ge 5000;\ x_2 \ge 500;\ x_3 \ge 5000; \ldots\ x_{40} \ge 90; x_{41} \ge 30; \quad (8)\end{gathered}$$

3. Results and Discussion

To secure optimal solutions, the use of various software tools can be employed (Grenci, 2022; Cecílio & dos Santos, 2018). In this paper, a program written in the Mathematica software package is used. The program is designed to provide not only optimal solutions at the output (Relation (9)) but also a capacity analysis (Table 3) that indicates the presence of unrealistic constraints and unused capacities. The optimal production program engages 525236 h/year, which represents 37,42 % of the available human resources potential of 1403666 h/year.

Degree of engagement of the workforce at the level S*i*: S1 - 14,84%, S2 - 24,83%, S3 - 27,29%, S4 - 100%, S5 - 53,51%, S6 - 48,11%.

Table 3. Part of the results of capacity analysis

Constraint	Planned Capacity	Unused Capacity
135166	20065,10	115100,90
320987	79704,90	241282,10
398879	108847,00	290032,00
63628	63627,30	0,70
363549	194548,00	169001,00
121457	58438,60	63018,40
18000	5000	13000
5000	500	4500
⋮	⋮	⋮
120	120	0
700	700	0
3928	3928	0
600	600	0
30	30	0
300	300	0
90	90	0
30	30	0

$$\{525236, \{x1 \to 5000, x2 \to 500, x3 \to 5000, x4 \to 7000, x5 \to 200, x6 \to 9000, x7 \to 10000, x8 \to 12000, x9 \to 40000, x10 \to 21500, x11 \to 17000, x12 \to 50000, x13 \to 300, x14 \to 700, x15 \to 1000, x16 \to 100, x17 \to 800, x18 \to 900, x19 \to 4500, x20 \to 35600, x21 \to 35600, x22 \to 3000, x23 \to 500, x24 \to 1600, x25 \to 500, x26 \to 700, x27 \to 30, x28 \to 3000, x29 \to 20000, x30 \to 200, x31 \to 70, x32 \to 70, x33 \to 70, x34 \to 120, x35 \to 700, x36 \to 3929, x37 \to 600, x38 \to 30, x39 \to 300, x40 \to 90, x41 \to 30\}\} \quad (9)$$

4. Conclusion

By applying simulation and optimization processes, it can be concluded that it is necessary to redistribute production human resources in order to increase the utilization rate of the workforce with the optimal production program. The optimal production program engages 525236 h/year, which represents 37,42 % of the available human resources potential of 1403666 h/year. Since the established model has "surpluses" of capacity, it is necessary to redistribute the production of human resources in order to increase the degree of engagement of the available workforce.

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