An Enhanced Method for Quantifying Energy Loss due to Diffusion in Coronary Arteries with Stenosis

Aleksandar Milovanović[1,*], Velibor Isailović[2], Igor Saveljić[1], Nenad Filipović[2]

[1] Institute for Information Technologies, University of Kragujevac, 34000 Kragujevac, Serbia; e-mail: acakg85@hotmail.com , isaveljic@kg.ac.rs
[2] Faculty of Engineering, University of Kragujevac, 34000 Kragujevac, Serbia; e-mail: velibor@kg.ac.rs , fica@kg.ac.rs

* *Corresponding author*

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Abstract: Energy loss in stenotic coronary arteries significantly affects blood flow efficiency and contributes to the development of ischemic heart disease. Various methods are employed to diagnose coronary artery stenosis and assess its functional significance, among which Fractional Flow Reserve (FFR) plays a key role. FFR quantifies the pressure drop caused by a stenosis in a coronary artery. Numerous studies have shown that the pressure drop due to diffusion constitutes the dominant component of total pressure loss in coronary stenosis, highlighting the importance of its reliable quantification. This paper presents an enhanced method for calculating the diffusive pressure drop, validated through the studies reported in [1] and [2].

Keywords: coronary artery stenosis, fractional flow reserve, pressure drop, inertion, diffusion

1. Introduction

Coronary artery disease is the most prevalent among cardiovascular conditions, which remains the leading cause of mortality worldwide [3, 4]. In over 90% of cases, the disease results from atherosclerosis, which impairs the blood supply to the heart muscle, leading to myocardial ischemia and often to myocardial infarction. Timely diagnosis is therefore essential to prevent adverse clinical outcomes. Among the parameters used to assess the functional severity of coronary stenosis, FFR stands out as one of the most reliable. This parameter is based on the fact that stenosis, induces a pressure drop proportional to the reduction in cross-sectional area caused by atherosclerotic narrowing.

The largest portion of the total pressure drop is attributed to diffusion-induced pressure loss [2] which arises primarily due to viscous friction, but it also includes the pressure drop caused by the acceleration of blood flow along the stenotic segment. For these reasons, accurate determination of this pressure loss is of crucial importance. The following section presents an innovative approach to addressing this task.

2. Determination of the pressure drop due to diffusion

It is assumed that at the entrance of the stenosis (Figure 1), the velocity is uniform [5]. Under the influence of the boundary layer, the initial uniform velocity gradually contracts toward the axis of the stenosis, until a fully developed flow is reached. The pressure drop due to diffusion is derived from the continuity equation and the Navier–Stokes equations for axisymmetric flow. After introducing non-dimensional variables:

$$x=\frac{z\nu}{v_0R^2},\quad y=\frac{r}{R},\quad u=\frac{v_z}{v_0},\quad v=v_r\frac{R}{\nu},\quad p=\frac{P}{\rho v_0^2} \tag{1}$$

the corresponding boundary conditions,

$$\begin{aligned} &\text{for } x=0, \quad u=1 && \text{for all } y\\ &\text{for } y=1, \quad u=0 && \text{for all } x\\ &\text{for } y=0, \quad \partial u/\partial y=0 && \text{for all } x \end{aligned} \tag{2}$$

and the cross-sectional integration the above equations take the following form:

$$\int_0^1 yu\,dy=\frac{1}{2} \tag{3}$$

$$\frac{d}{dx}\int_0^1 yu^2\,dy=-\frac{1}{2}\frac{dp}{dx}+\left(\frac{\partial u}{\partial y}\right)_{y=1} \tag{4}$$

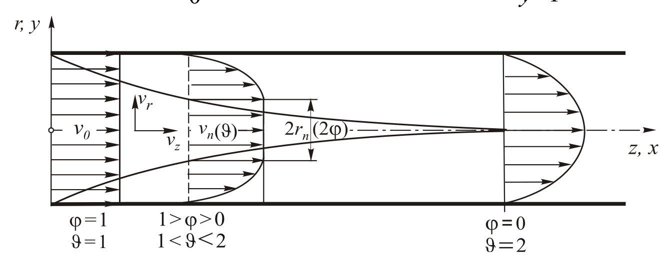

Figure 1. Schematic of key flow parameters

According to Figure 1 axial velocity ϑ has **the following distribution:**

$$\left.\begin{aligned} &u=\vartheta && (0\le y\le\varphi)\\ &u=\vartheta\left[1-\left(\frac{y-\varphi}{1-\varphi}\right)^2\right] && (\varphi\le y\le 1) \end{aligned}\right\} \tag{5}$$

where ϑ and φ are functions of the x-coordinate only, and at x = 0, according to boundary conditions (2), φ =1 and $\vartheta = u = 1$, with ϑ denoting the velocity of the inviscid core.

Substituting equation (5) into equation (3) yields an expression for the velocity of the inviscid core as a function of its non-dimensional radius φ:

$$\vartheta=\frac{6}{3+2\varphi+\varphi^2} \tag{6}$$

while substituting equations (5) and (6) into equation (4) leads to the following relation:

$$\frac{d\varphi}{dx}=\frac{5\left(3+2\varphi+\varphi^2\right)^2}{1+4\varphi+9\varphi^2+4\varphi^3}\left[\frac{1}{1-\varphi}+\frac{dp}{dx}\,\frac{3+2\varphi+\varphi^2}{24}\right] \tag{7}$$

Since both the pressure and the parameter φ depend on the x-coordinate, an additional relation is required in addition to equation (7). It can be derived by applying the equation (4) at the wall of the coronary artery ($y = 1$), where the velocities $u = v = 0$:

$$\frac{dp}{dx}=-\frac{48}{(1-\varphi)(6+\varphi)} \tag{8}$$

By substituting (5) and (6) into (8), the pressure gradient is obtained:

$$\frac{dp}{dx}=-\frac{48}{(1-\varphi)(6+\varphi)} \tag{9}$$

Substituting this expression for the pressure gradient into equation (7) yields:

$$x=\frac{z\pi\mu}{\rho Q}=\int_{\varphi}^{1}\frac{(1-\varphi)(6+\varphi)\left(1+4\varphi+9\varphi^2+4\varphi^3\right)}{5\varphi(3+2\varphi)\left(3+2\varphi+\varphi^2\right)^2}d\varphi \tag{10}$$

Assuming $L= L_{sten}$ and knowing the length of the stenosis, the dimensionless radius φ at the end of the stenosis can be calculated from (10), which is necessary to determine ΔP.

The pressure drop can be calculated using the expression proposed by Huo et al. [5]:

$$\Delta P=\rho\frac{v_0^2}{2}f_x Re\,x \tag{11}$$

where the apparent friction coefficient as a function of the pressure gradient can, taking into account (9), as well as the first and fourth substitutions from (1), be expressed as:

$$f_x=\frac{-2D}{4\rho v_0^2}\frac{dP}{dz}=\frac{16\lambda}{Re}=\frac{96}{Re(1-\varphi)(6+\varphi)} \tag{12}$$

Substituting (10) and (12) into equation (11) yields the pressure drop due to diffusion:

$$\Delta P_{diff}=\rho\frac{v_0^2}{2}\frac{96}{5}\int_{\varphi}^{1}\frac{\left(1+4\varphi+9\varphi^2+4\varphi^3\right)}{\varphi(3+2\varphi)\left(3+2\varphi+\varphi^2\right)^2}d\varphi \tag{13}$$

When $\varphi \geq 0.05$ along the entire length of the stenosis (L_{sten}) - that is the case of a short stenosis - the corresponding pressure drop is calculated using expression (13):

$$\Delta P_{diff}^{\varphi\geq 0,05}=\frac{\rho Q^2}{2A_{sten}^2}\frac{96}{5}\int_{\varphi_0}^{1}\frac{\left(1+4\varphi+9\varphi^2+4\varphi^3\right)}{\varphi(3+2\varphi)\left(3+2\varphi+\varphi^2\right)^2}d\varphi \tag{14}$$

In the case of a long stenosis (Figure 2), fully developed flow (φ = 0.05) is reached at a certain distance from the stenosis entrance (L_{entr}) and continues up to the stenosis exit.

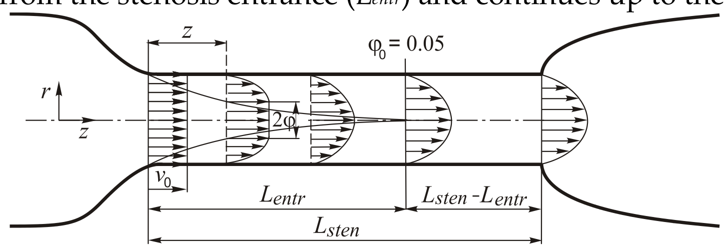

Figure 2. Schematic of long stenosis

The stenosis length is thus divided into two parts: an entrance region of length L_{entr}, and a region of fully developed flow with length L_{sten} - L_{entr}, where L_{entr} is calculated using expression (9), by setting $z = L_{entr}$ and taking $\varphi = 0.05$ as the lower limit of the integral.

$$L_{entr} = \frac{\rho Q}{\pi \mu} \int_{0.05}^{1} \frac{(1-\varphi)(6+\varphi)\left(1+4\varphi+9\varphi^2+4\varphi^3\right)}{5\varphi(3+2\varphi)\left(3+2\varphi+\varphi^2\right)^2} d\varphi \tag{15}$$

In this case, the pressure drop due to acceleration (eq. 14) should be supplemented by the pressure drop due to friction over the length L_{sten} - L_{entr} where the flow is fully developed. (Here, the lower limit of the integral is set to $\varphi_0 = 0.05$:

$$\Delta P_{diff}^{\varphi<0,05} = \frac{\rho Q^2}{2A_{sten}^2} \frac{96}{5} \int_{0.05}^{1} \frac{\left(1+4\varphi+9\varphi^2+4\varphi^3\right)}{\varphi(3+2\varphi)\left(3+2\varphi+\varphi^2\right)^2} d\varphi + \int_{0}^{L_{sten}-L_{entr}} \frac{8\pi\mu Q}{A_{sten}^2} dx \tag{16}$$

3. Conclusions

This paper presents an improved mathematical model for determining the pressure drop due to diffusion in coronary arteries. The proposed model was validated on a sample of 62 stenosis by comparing the calculated results (published in studies [1] and [2]) with clinical data.

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