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APPLYING SAVAGE CRITERION TO DECISION MAKING IN PRODUCTION MANAGEMENT

Abstract: *In circumstances of unemployed industrial capacities by current contracts and a lengthy production cycle the production management is mainly grounded on assumptions and predictions. The paper analyzes the possibility of applying the game theory and Savage criterion to production planning under conditions of uncertainty in business operations. Considering the intensity and rate of changes in the state of production system and relevant impact on the production program trends, the aim of the paper is to demonstrate a quantifiable basis for strategic decision making in production management.*
Key words: *strategy, criteria, decision making, production management, uncertainty, prediction*

1. INTRODUCTION

Considerations of production activities, from program orientation over production program, and vice versa, require identification of external and internal determinants acting within the framework of the time axis, from prediction to short-term planning. Production program means making program orientation concrete, from the medium- to short-term viewpoint, with its design being based upon facts more or less susceptible to quantification. Given that the production program is defined by the type and quantity of products, to design it is a complex process with far-reaching outcomes.
The paper presents a quantitative method that summarizes in itself a regression analysis and extrapolation of trends as a starting point for applying the rules and criteria used in the game theory. Considering that relevant decisions about production orientations are made at strategic level, with a high degree of uncertainty, a concept is offered to the top management enabling further quantitative analysis with the aim to make strategic decisions in the domain of production management.

2. PRODUCTION PLANNING UNDER CONDITIONS OF UNCERTAINTY

Business and manufacturing systems belong to a group of organizational, complex, dynamic, stochastic and open systems and as such tend to their own degeneration unless corresponding inputs are continuously provided. This means that entropy has a growing trend in these systems. The systems for manufacturing armament and military equipment (subject of investigations) have a specific position and role. The manufacturing of mentioned products is primarily organized for the needs of the country's defense and security, and for the sales on the world markets and domination in the region and the world. The prices of products, depending on the globalization level of business operations, can but need not be necessarily influenced by the law of supply and demand. Production is governed by special legal provisions and operating a business is determined by the range of products and reliability in the use of products, tight deadlines for delivery, demands for product standard version modification, possibility of supplying specific materials and parts, material, financial, technical-technological and human resources for the system.
As for spatial demand, the globalization of business operations proceeds on the broadest, world scale. Consequently, it is needed to observe how external limitations are acting: scientific-

research resources and achievements, international relations and legal provisions, capital circulation and integration processes at the regional and world levels. Offer on the market, with corresponding corrections imposed by import and inventories, indicates the volume and structure of production in its quality and range of products. In contrast, demand indicates the volume and structure of needs. The ratio of supply to demand, in structure and volume, can be used as an indicator of production orientation.

However, having in mind that the products of armament and military equipment are special-purpose products, global-level investigations of the market are difficult in respect of predicting demands by monitoring and analyzing all elements that are important. In addition, the fact is that production is often organized for the 'unknown customers'. As there are no reliable indicators of the market needs and possibilities of the product placement, and aiming to reduce the risk of failure in production orientation, the behavior of the market has to be monitored in the previous period, whereby future trends are more predictable. Identification of possible alternatives in the market behavior and their ranking, from the viewpoint of adopted criteria, makes possible to establish a quantitative basis for the corresponding simulation processes that are used to seek optimal solutions.

Production planning under conditions of uncertainty, when the distribution of the future states of the system is unknown, can be treated as a process that consists of the following stages:

1. *Problem formulation.* This stage comprises the analysis of the system's production program over a longer period of time in order to define the time horizon for collecting statistical data (experimental area), time horizon for predicting the future states (prediction area) and selection of regression curves to describe alternative states for the production of articles or groups of articles. If the subject of analysis is a group of products or production program, a representative product should be chosen. In the experimental area the product placement should be analyzed, whereby a set of data is provided: $\{(q_1, t_1), (q_2, t_2), \ldots, (q_n, t_n)\}$ for regression analysis. Using the production program analysis of 'Sloboda' Co. Cacak, in the 18 years' period, experimental areas (5 years) and prediction areas (1 year) [2], and types of regression dependencies (first-, second-, third- and fourth-degree polynomials, exponential and geometric regression) were determined [3].

2. *Creating mathematical models.* A five-year dynamics of products' placement was used to define stochastic dependencies of a representative product as a function of time, and vice versa. The selected approximation curves (1)-(6) define the likely alternative states of the system in the experimental area (5 years) and by trend extrapolation in the prediction area too (one year). The game theory, i.e. the theory of statistical solutions represents a mathematical basis for the formation of a game matrix of the alternatives-states of the system at the interval comprising both intervals (Tabs. 1 and 2).

$$q_1 = a_0 + a_1 \cdot t \quad (t_1 = b_0 + b_1 \cdot q) \tag{1}$$

$$q_2 = a_0 + a_1 \cdot t + a_2 \cdot t^2 \quad (t_2 = b_0 + b_1 \cdot q + b_2 \cdot q^2) \tag{2}$$

$$q_3 = a_0 + a_1 \cdot t + a_2 \cdot t^2 + a_3 \cdot t^3 \quad (t_3 = b_0 + b_1 \cdot q + b_2 \cdot q^2 + b_3 \cdot q^3) \tag{3}$$

$$q_4 = a_0 + a_1 \cdot t + a_2 \cdot t^2 + a_3 \cdot t^3 + a_4 \cdot t^4 \quad (t_4 = b_0 + b_1 \cdot q + b_2 \cdot q^2 + b_3 \cdot q^3 + b_4 \cdot q^4) \tag{4}$$

$$q_5 = a \cdot t^b \quad (t_5 = a \cdot q^b) \tag{5}$$

$$q_6 = a \cdot b^t = a \cdot e^{kt} \; (k = \ln b) \quad (t_6 = a \cdot b^q = a \cdot e^{kq}) \tag{6}$$

Uncertain situations are of zero-ingredient-of-conflict because the states are the outcomes of the alternatives (regression curve) chosen by a decision-maker. In order to overcome the above statement, apart from regressions $q_i = f_i(t)$, the regressions $t_i = f_i(q)$ were introduced. The integrated model contains regressions $q_i = f_i(t)$ and multi-functions $p_i = f_i(t)$ that are inverse to functions $t_i = f_i(q)$. Six types of alternatives' curves $t_i(q)$ can define, to the maximum, twelve multi-functions $p_i = f_i(t)$.

Table 1. A general game model $q_i – S_j$ (zero-ingredient-of-conflict situations)

THE STATES OF THE SYSTEM / ALTERNATIVES	S_j , $j = \overline{1,n}$									
	EXPERIMENTAL AREA					PREDICTION AREA				
$q_i = f_i(t)$, $i = \overline{1,m}$	S_1	S_2	S_3	...	S_j	S_{j+1}	S_{j+2}	S_{j+3}	...	S_n
$q_1 = f_1(t)$	a_{11}	a_{12}	a_{13}	...	a_{1j}	a_{1j+1}	a_{1j+2}	a_{1j+3}	...	a_{1n}
$q_2 = f_2(t)$	a_{21}	a_{22}	a_{23}	...	a_{2j}	a_{2j+1}	a_{2j+2}	a_{2j+3}	...	a_{2n}
.	.	.	.	...	.	.	.	.	...	.
$q_i = f_i(t)$	a_{i1}	a_{i2}	a_{i3}	...	a_{ij}	a_{ij+1}	a_{ij+2}	a_{ij+3}	...	a_{in}
.	.	.	.	...	.	.	.	.	...	.
$q_m = f_m(t)$	a_{m1}	a_{m2}	a_{m3}	...	a_{mj}	a_{mj+1}	a_{mj+2}	a_{mj+3}	...	a_{mn}

Table 2. A general game model $t_i – S_j$ (zero-ingredient-of-conflict situations)

THE STATES OF THE SYSTEM / ALTERNATIVES	S_j , $j = \overline{1,n}$									
	EXPERIMENTAL AREA					PREDICTION AREA				
$t_i = f_i(q)$, $i = \overline{1,k}$	S_1	S_2	S_3	...	S_j	S_{j+1}	S_{j+2}	S_{j+3}	...	S_n
$t_1 = f_1(q)$	b_{11}	b_{12}	b_{13}	...	b_{1j}	b_{1j+1}	b_{1j+2}	b_{1j+3}	...	b_{1n}
$t_2 = f_2(q)$	b_{21}	b_{22}	b_{23}	...	b_{2j}	b_{2j+1}	b_{2j+2}	b_{2j+3}	...	b_{2n}
.	.	.	.	...	.	.	.	.	...	.
$t_i = f_i(q)$	b_{i1}	b_{i2}	b_{i3}	...	b_{ij}	b_{ij+1}	b_{ij+2}	b_{ij+3}	...	b_{in}
.	.	.	.	...	.	.	.	.	...	.
$t_k = f_k(q)$	b_{m1}	b_{m2}	b_{m3}	...	b_{mj}	b_{mj+1}	b_{mj+2}	b_{mj+3}	...	b_{mn}

3. *Choice of the optimal alternative (alternatives).* In the choice of the optimal alternative a number of methods and criteria can be used, from extremely pessimistic to extremely optimistic. The argumentation for decision making is most dependent of the manager's methods and style of work. In ranking the alternatives, the Savage criterion will be applied.
4. *Qualitative analysis of the solutions obtained.*
5. *Creation of plans for the adopted volume of production.*
6. *Preparation and organization of production*.
7. Analysis of the outcomes of decisions made.

3. SAVAGE CRITERION

In the choice of optimal strategy the criterion advises risk-orientation. The choice is the production strategy *(i)* for which the level of risk is, in the worst of conditions, minimum. The application of Savage criterion requires the choice of maximum value A_j in accordance with relation (7), for each state of the system in the future (S_j).

$$A_j = \max_j \left\{ \left\{ a_{ij},\ i = \overline{1,m} \right\},\ j = \overline{1,n} \right\} \tag{7}$$

So, if a given (optimal) state happens to occur in the future, and a decision-maker chooses the best alternative, then there would be no regret for the opportunity loss, i.e. regret equals zero (k_{ij} = 0). However, if a decision-maker chooses any other alternative, then there would be regret for the opportunity loss. The elements of a regret matrix will be calculated using relation (8). In this regard, the initial matrix of P game effects (a_{ij}), relation (9), is transformed into a regret matrix (risk) R, relation (10).

$$a_{ij} \Rightarrow k_{ij} \left| \left\{ \left\{ \left\{ A_j - a_{ij} \right\}, i = \overline{1,m} \right\}, j = \overline{1,n} \right\} \right. \tag{8}$$

$$P = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \tag{9}$$

$$R = \begin{bmatrix} k_{11} & k_{12} & \dots & k_{1n} \\ k_{21} & k_{22} & \dots & k_{2n} \\ \dots & \dots & \dots & \dots \\ k_{m1} & k_{m2} & \dots & k_{mn} \end{bmatrix} \tag{10}$$

The k_{ij} elements of regret matrix (loss) R are derived from the elements of a game matrix (a_{ij}) in such a way that for each state of the system – column (j) the maximum value of A_j is sought, taking the value of zero, at a corresponding position, in a risk matrix. Other elements of a regret matrix k_{ij} are obtained by subtracting the elements of the column (A_j), in accordance with relation (8), from the maximum value in the observed column (A_j).
The Savage criterion (K_l) is essentially the ***min-max*** principle applied under conditions of uncertainty. In other words, it is needed to define maximum 'regrets' for each opportunity loss and then to choose the alternative that has a minimum 'regret', relation (11).

$$K_1 = \min_i \left\{ \max_j \left\{ \left\{ k_{ij},\ j = \overline{1,n} \right\}, i = \overline{1,m} \right\}, i = \overline{1,m} \right\} \tag{11}$$

4. MATHEMATICAL MODEL FOR DECISION MAKING

In order to predict market demands for anti-aircraft ammunition, investigations were carried out on the dynamics of production of 32 products manufactured in more than one variant for different weapon systems. A 5-year production dynamics is expressed in 10^3 pieces/year of a chosen representative product [1] described by relation (12):

$$Q = \{\{1, 234.3\}, \{2, 566.4\}, \{3, 256.3\}, \{4, 222.5\}, \{5, 350.6\}\} \tag{12}$$

Depending on the choice of dependent and independent variable, regression curves can be defined by relations (13) – (24).

$$q_1(t) = 359{,}41 - 11{,}13 \cdot t \tag{13}$$

$$q_2(t) = 293{,}56 + 45{,}312857 \cdot t - 9{,}407143 \cdot t^2 \tag{14}$$

$$q_3(t) = -832{,}18 + 1626{,}709524 \cdot t - 612{,}482143 \cdot t^2 + 67{,}0083 \cdot t^3 \tag{15}$$

$$q_4(t) = -2691{,}4 + 5131{,}19167 \cdot t - 2745{,}9125 \cdot t^2 + 583{,}4583 \cdot t^3 - 43{,}0375 \cdot t^4 \tag{16}$$

$$q_5(t) = 301{,}737045466876 \cdot t^{0{,}0123331712} \tag{17}$$

$$q_6(t) = 317{,}3009762 \cdot 0{,}9872533173^t \tag{18}$$

$$t_1(q) = 3{,}44049 - 0{,}001351 \cdot q \tag{19}$$

$$t_2(q) = -8{,}512162 + 0{,}068391 \cdot q - 0{,}0000878495 \cdot q^2 \tag{20}$$

$$t_3(q) = 65{,}29945 - 0{,}5947887 \cdot q + 0{,}0017808 \cdot q^2 - 1{,}6383958 \cdot 10^{-6} \cdot q^3 \tag{21}$$

$$t_4(q) = 2890{,}026 - 37{,}345485 \cdot q + 0{,}17505 \cdot q^2 - 0{,}00035 \cdot q^3 + 2{,}49656 \cdot 10^{-7} \cdot q^4 \tag{22}$$

$$t_5(q) = 2{,}15563914488 \cdot q^{0{,}0331059257} \tag{23}$$

$$t_6(q) = 2{,}75278178767 \cdot 0{,}9998309645^q \tag{24}$$

Data for production trends and corresponding regression curves are presented in Figs. 1-4.

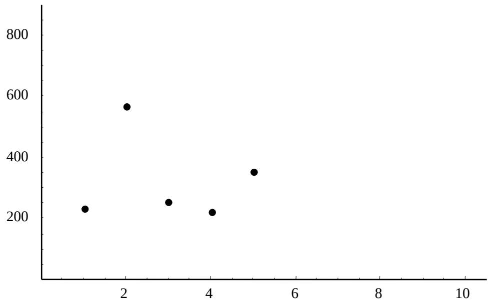

Fig. 1 Production trends in experimental area, in accordance with relation (12)

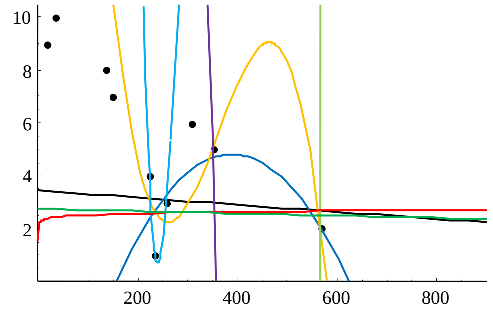

Fig. 3 Regression functions $t_i(q)$ presented in experimental and prediction areas

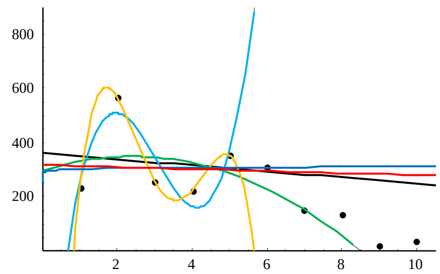

Fig. 2 Regression functions $q_i(t)$ presented in experimental and prediction areas

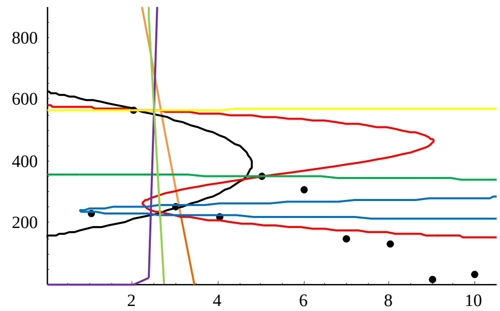

Fig. 4 Multi-functions $p_i(t)$ inverse to regress. functions $t_i(q)$ presented in both areas

Six adopted types of approximation curves define a set of alternatives $q_i = f_i(t)$ and $t_i = f_i(q)$ in a game matrix and they can be used to determine the states of the systems S_1-S_5 in the experimental area (5 years), while by trend extrapolation the state of the system S_6 in the prediction area (1 year) can be determined. Fig. 1 shows the achieved dynamics of the representative product production analyzed for 5 years. Fig. 2 displays production trends and regression curves $q_i = f_i(t)$ described by relations (13)-(18). Fig. 3 gives production dynamics and regression curves $t_i = f_i(q)$ described by relations (19)-(24) and Fig. 4 multi-functions $p_i = f_i(t)$ that are inverse to functions $t_i = f_i(q)$. Situations where little is known about the future states of the system and where the probability distribution of the occurrence that decision is made about is unknown are referred to as uncertain. Solving a prediction model implies the choice of one of the alternatives offered (regression curve). The choice of optimal alternatives (alternative) will be performed using the Savage criterion (K_1). Integrated mathematical model for predicting and decision-making is shown in Tab. 3.

Linear (t_i – relation 19), geometric (t_5 – relation 23) and exponential (t_6 –relation 24) regressions define the states of the system by the functions p_1, p_9 and p_{10}. Regression by the second-degree polynomial (t_2 –relation 20) is not involved by the model because it does not define the likely states (S_6) in the prediction area. Regression by the third-degree polynomial (t_3 – relation 21) defines three values of the state (S_6) by the functions p_2, p_3 and p_4. Regression by the fourth-degree polynomial (t_4 – relation 22) defines four values of the state (S_6) by the multi-functions p_5, p_6, p_7 and p_8. Given the fact that real values of the volume of production (Q) range from zero to ideal exploitation capacity (C_e), the states of the system (S_6) defined by the functions q_3, q_4, p_1, p_9 and p_{10} should be omitted in decision-making. This means that the prediction model consists of two game matrices of the formats (4x6) and (7x6). The Savage criterion will be applied to the alternatives defined by relation (25). A set of optimal alternatives (q^*, p^*) is defined, based on data from Tab.3, using the software package [4], relation (26).

$$Q \in [0, Ce) \Leftrightarrow \left\{ q \in (q_1, q_2, q_5, q_6) \wedge p \in \left(p_k \middle| k = \overline{2,8} \right) \right\} \tag{25}$$

$$K_1 \Rightarrow \left(q^* = q_5 \wedge p^* = p_8 \right) \tag{26}$$

Table 3. Integrated mathematical model (alternatives-states of systems-criteria for decision-making)

q_i, p_k \ S_j, K_s		SYSTEMS STATES (S_j , j = 1,2,...,6) – CRITERIA (K)						
		S_1	S_2	S_3	S_4	S_5	**S_6**	K_1
ALTERNATIVES $q_i(t)$	**q_1**	348	337	326	315	304	**293**	1059
	q_2	330	347	345	324	285	**227**	1125
	q_3	249	507	345	164	365	**1352**	160
	q_4	234,3	566,4	256,3	222,5	350,6	**-507**	1859
	q_5	301,7	304,3	305,8	306,9	307,8	**308,5**	1043
	q_6	313	309	305	301	298	**294**	1058
	PRINCIPLE "**MIN-MAX**":							**MIN**
	VALUE OF CRITERIA **K_s**:							**1043**
	OPTIMAL ALTERNATIVE **q_i (t)**:							**q_5**
ALTERNATIVES $p_k(t)$	**p_1** (t_1)	1806	1066	326	-414	-1154	**-1894**	92893
	p_2 (t_3)	573	566,39	224	206	349,5	**184**	90815
	p_3 (t_3)	573	566,39	224	206	349,5	**369**	90630
	p_4 (t_3)	573	566,39	224	206	349,5	**534**	90465
	p_5 (t_4)	234,3	566,4	256,3	222,5	350,6	**218**	90781
	p_6 (t_4)	234,3	566,4	256,3	222,5	350,6	**267**	90732
	p_7 (t_4)	234,3	566,4	256,3	222,5	350,6	**349**	90650
	p_8 (t_4)	234,3	566,4	256,3	222,5	350,6	**567**	90432
	p_9 (t_5)	-0,008	0,104	21674	33333	66666	**90999**	5990
	p_{10} (t_6)	5990	1890	-509	-2210	-3530	**-4609**	95608
	PRINCIPLE "**MIN-MAX**":							**MIN**
	VALUE OF CRITERIA **K_s**:							**90432**
	OPTIMAL ALTERNATIVE **p_k (t)** :							**p_8**
IDEAL EXPLOITATION CAPACITY:				***Ce* = 600 000 kom/god**				

5. CONCLUSION

On the grounds of data from Tab. 3, graph in Fig. 5 and relation (26), the limits within which the volume of future production trends for a representative product (Q) relation (27) can be predicted.

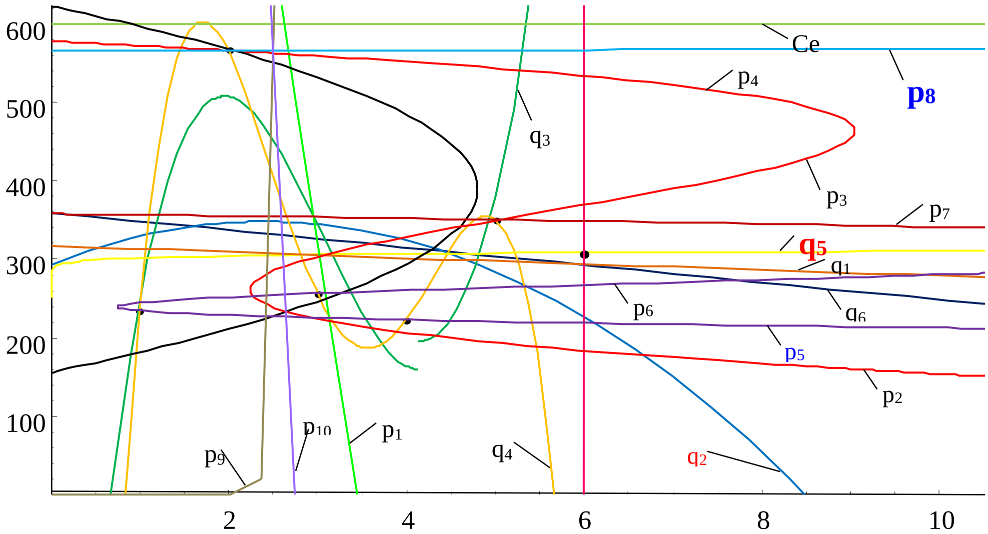

Fig. 5 Exploitation capacities (C_e), achieved production level S_1-S_5, , regression curves $q_i(t)$, multi-functions $p_k(t)$ and achieved volume of production in the prediction area (S_6)

$$\{(q^*, p^*) \Leftrightarrow (q_5, p_8)\} \Rightarrow 308500 \leq Q \leq 567000 \qquad (27)$$

Using a 5-year analysis of business operations, the production program was designed for the next (6th) year that ranges, based on adopted criteria, within the limits defined by relation (27). With time flow, it is found that the volume of production of Q = 306 700 pieces/year (designated on a straight line t=6 in Fig. 5) has been achieved.

REFERENCES

/1/ Jovanović R. J., Milanović D. D., Đukić D. R., „*Applying the criteria of decision making trends under uncertainty situations*“, XIII International symposium, SymOrg 2012, Facultiy of Organizational Sciences, Zlatibor, 2012.,

/2/ Đukić R., Žižović M., Jovanović R. J., „*The impact of the time horizon on the research of the production trends*“, Journal - Tehnika i praksa, No 1, VŠTSS Čačak, Čačak, 2010.,

/3/ Đukić R., Fetahović A., Trivković S., Jevremović B., „*Production menagement based on predicting future situation and trends*“, Journal - Tehnika i praksa, No 2, VŠTSS Čačak, Čačak, 2010.,

/4/ Đukić D. R., Jovanović R. J., „*Program for rating alternatives according to selected criteria of decision - making*“, YU INFO 09 Conference, Information Society of Serbia, Kopaonik, 2009.