Časopis Naučnog društva za pogonske mašine, traktore i održavanje
Journal of Scientific Society of Power Machines, Tractors and Maintenance

TRAKTORI I POGONSKE MAŠINE

1/2
UDK 631.372
ISSN 0354-9496
Godina 30
Nov. 2025.

TRACTORS AND POWER MACHINES

Novi Sad, Srbija

Trakt. i pog. maš., Trac. and pow. mach., Vol. 30, No. 1/2, p.1-61, Novi Sad, Nov. 2025.

Časopis Jurnal

TRAKTORI I POGONSKE MAŠINE TRACTORS AND POWER MACHINES

Izdavač – Publisher

Naučno društvo za pogonske mašine, traktore i održavanje
Scientific Society of Power Machines, Tractors and Maintenance

Suizdavač – Copublisher
Poljoprivredni fakultet, Departman za poljoprivrednu tehniku, Novi Sad
Faculty of Agriculture, Department of Agricultural Engineering, Novi Sad

Adresa – Adress
Poljoprivredni fakultet
Trg Dositeja Obradovića br. 8
Novi Sad, Srbija
Tel.: ++381(0)21 4853 391
Tel/Fax.: ++381(0)21 459 989
e-mail: milanto@polj.uns.ac.rs

Časopis izlazi svaka tri meseca
Godišnja pretplata za radne organizacije je 1500 din, za Inostranstvo 5000 din a za individualne predplatnike 1000 din
Žiro račun: 340-4148-96 kod Erste banke

Jurnal is published four times a year
Subscription price for organization is 40 EURO, for foreign organization 80 EURO and individual subscribes 15 EURO

Rešenjem Ministarstva za informacije Republike Srbije, Br.651-115/97-03 od 10.02.1997.god., časopis je upisan u registar pod brojem 2310
Prema Mišljenju Ministarstva za nauku, Republike Srbije ovaj časopis je "PUBLIKACIJA OD POSEBNOG INTERESA ZA NAUKU"

Štampa – Printed by
Štamparija "Birograf comp“ doo Zemun, Atanasija Pulje 22, 11080 Zemun
Tiraž 100 primeraka

Časopis Naučnog društva za pogonske mašine, traktore i održavanje

Journal of Scientific Society of Power Machines,Tractors and Maintenance

TRAKTORI I POGONSKE MAŠINE

1/2

UDK 631.372

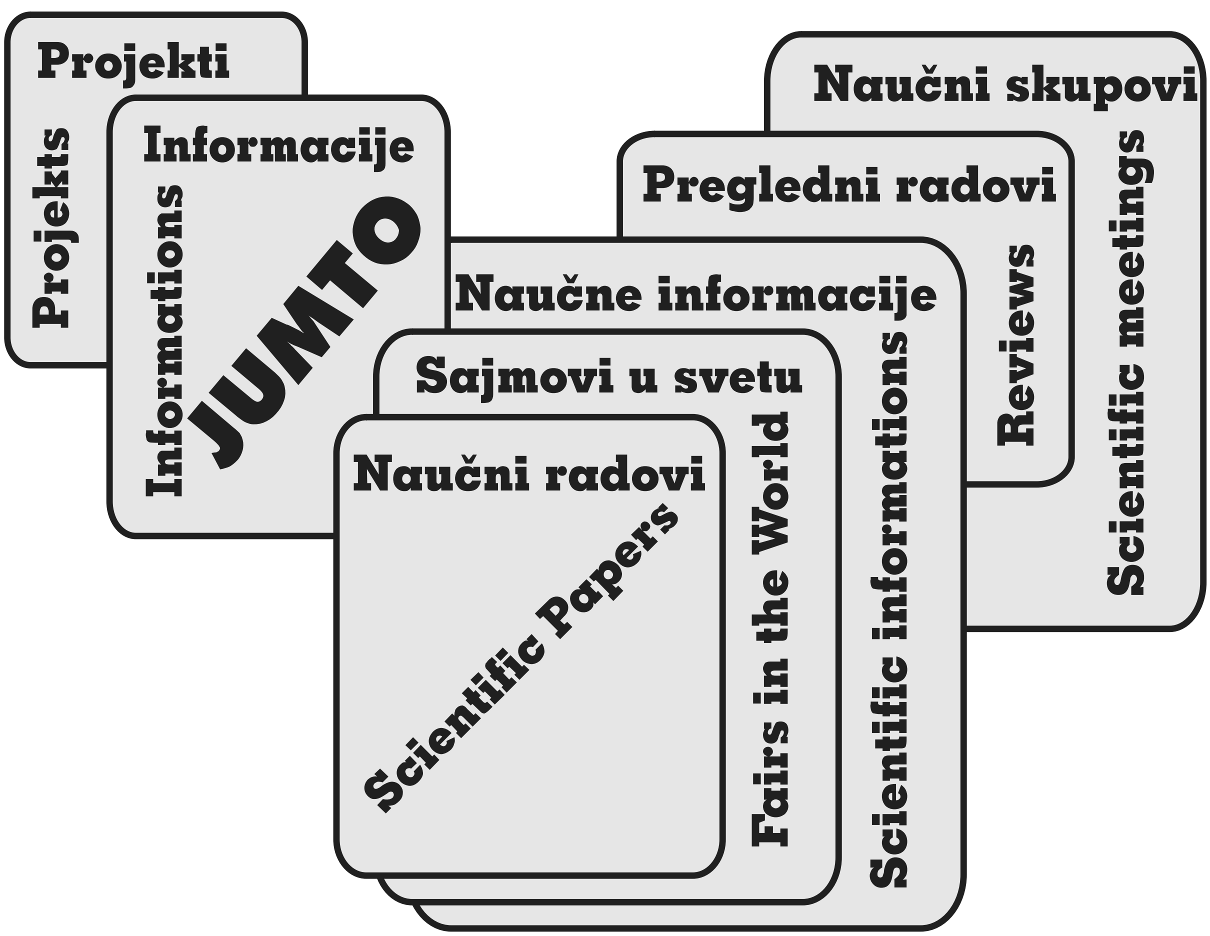
ISSN 0354-9496

Godina 30

Dec. 2025.

TRACTORS AND POWER MACHINES

SADRŽAJ - CONTENS



Novi Sad, Srbija

Trakt. i pog. maš., Trac. and pow. mach., Vol.30, No.1/2, p.1-61, Novi Sad, Nov. 2025.

Časopis Traktori i pogonske mašine broj 1/2 posvećen je XXXII-om naučnom skupu "Pravci razvoja traktora i obnovljivih izvora energije"

The journal Tractors and power machines number 1/2 is devoted to XXXII scientific meeting "Development of tractors and renewable energy resources"

JUMTO 2025

Programski odbor - Program board

- Prof. dr Lazar Savin, predsednik
- Prof. dr Ratko Nikolić
- Prof. dr Mirko Simikić
- Prof. dr Ivan Klinar
- Prof. dr Dragan Ružić
- Prof. dr Radojka Gligorić, sekretar
- Prof. dr Milan Tomić
- Dipl. inž. Milan Samardžija
- Prof. dr Zdenko Tkač

Pokrovitelji skupa - Godparent of meeting

- Ministarstvo prosvete, nauke i tehnološkog razvola Republike Srbije
- Pokrajinski sekretarijat za visoko obrazovanje i naučnoistraživačku delatnost
- Pokrajinski sekretarijat za poljoprivredu, vodoprivredu i šumarstvo AP Vojvodine

Organizatori skupa - Organizers of meeting

- Naučno društvo za pogonske mašine, traktore i održavanje

  JUMTO – Novi Sad
- Poljoprivredni fakultet, Departman za poljoprivrednu tehniku, Novi Sad
- Društvo za razvoj i korišćenje biogoriva – BIGO, Novi Sad
- Agencija za bezbednost saobraćaja, Beograd

Mesto održavanja - Place of meeting

Poljoprivredni fakultet, Novi Sad, 28.11.2025.

Štampanje ove publikacije pomoglo je:

Ministarstvo prosvete, nauke i tehnološkog razvoja Republike Srbije

SADRŽAJ – CONTENS

Biblid: 0354-9496(2025) 30: 1/2, p. 5-12
UDK: 631.3

Naučni rad
Scientific paper

RELIABILITY MODELING OF THE BRAKE DRUM IN AUTOMOTIVE BRAKING SYSTEMS

Ćatić D.[1], *Ćatić V.*[2]

SUMMARY

This paper presents research findings related to the reliability analysis of the brake drum in the braking system of light commercial vehicles. The introductory section highlights the importance of motor vehicle braking systems for the safety of both people and vehicles in traffic. In heavy motor vehicles, drum brakes are used as the actuators of both service and parking brakes. In order to assess the reliability of the brake drum, it is essential to understand the operating principles of drum brakes, their operating parameters, working conditions, failure causes and mechanisms, as well as other relevant information regarding the operation of the considered component. Reliability modeling of the brake drum was conducted based on failure data collected during the operational phase. For the purpose of reliability modeling, the three-parameter Weibull distribution was used due to its generality and flexibility in fitting empirical data. The theoretical distribution model was tested using graphical methods and non-parametric statistical hypothesis tests. The conclusion emphasizes the importance of reliability assessment, the potential applications of the obtained results, and directions for future research in this field.

Key words: reliability, motor vehicles, brake drum, Weibull distribution

1. INTRODUCTION

Braking systems in motor vehicles represent a typical example of complex vehicle systems whose structure is determined by the complex target function defined by the applicable international and national regulations on vehicle traffic safety [1, 2]. The main subsystems of the braking system are [3, 4]: the service brake, the auxiliary brake, the parking brake, and, in heavier motor vehicles, the supplementary brake or retarder. Structurally, the subsystems of the braking system are generally designed in the same way. They consist of the control unit, the transmission mechanism, and the brake itself. Brakes, as the actuators of individual subsystems, have a particularly important role in considerations of the reliability of motor vehicle braking systems due to their operating conditions and their influence on vehicle safety. Despite sharing the same fundamental physical operating principle, the design solutions for brakes can differ. In motor vehicles, two basic types are used: drum (radial) brakes and disc (axial) brakes. As actuators for the brakes on the rear wheels of commercial vehicles, drum brakes are most commonly used. Due to their favorable characteristics, these brakes are also

1 Dobrivoje Ćatić, Full Profesor, Faculty of engineering, University of Kragujevac, 34000 Kragujevac, 6 Sestre Janjić Street, Serbia, caticd@kg.ac.rs

2 Vladimir Ćatić, Software engineer, HTEC Group, Bulevar Milutina Milankovica 7DJ, Belgrade 11070, Serbia, vladimir.catic@htecgroup.com

used on the front wheels of heavy motor vehicles. Since the rear-wheel brakes also serve as the actuators of the parking (auxiliary) brake, the importance of drum brakes for the reliable and safe operation of motor vehicle braking systems is evident.

For modeling the reliability of the brake drum in the braking system of light commercial vehicles, the Weibull distribution was applied. The mathematical form of this three-parameter distribution enables its use for modeling reliability in all three periods of the service life of machine elements [5]. Analytical determination of the parameters of the three-parameter Weibull distribution was carried out using the least squares method and appropriate computer software [6]. In addition to graphical tests, the Kolmogorov and Pearson non-parametric statistical hypothesis tests were used to evaluate the approximate model.

2. STRUCTURE AND FAILURE ANALYSIS OF DRUM BRAKES

Figure 1 [7] shows the parts of the drum brakes that are installations on the front wheels of light commercial vehicle manufactured by "Zastava Iveco trucks" in Kragujevac. Similar solutions of drum brakes are applied also on the rear wheels. The difference between these constructions is in existence of a lever mechanism on the rear brakes, which allows the use of these brakes as the parking (auxiliary) brakes.

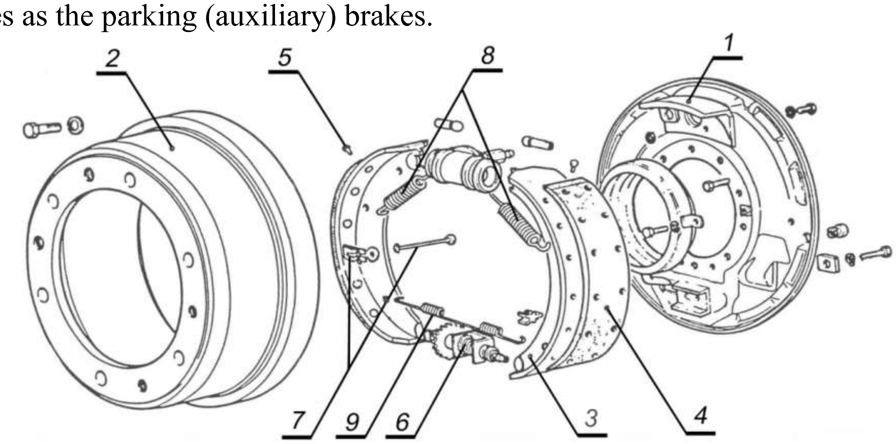


Fig. 1. *Front wheel drum brake parts: 1 - Baking plate, 2 - Drum, 3 - Carrier part shoe, 4 - Friction lining, 5 - Rivet, 6 - Brake adjuster, 7 - Elements for holding the shoes, 8 - Shorter return springs, 9 - Longer return spring*

The drum brake consists of fixed and moving components [2]. The fixed components are attached to the vehicle's supporting structure via the backing plate (1), while the movable drum (2) is mounted on the wheel hub and secured with bolts. Friction elements of drum brakes are two symmetrically placed brake shoes and drum. The shoes of drum brakes consist of a metal carrier (3) and a lining made of friction material (4). The connection between the metal part and friction lining can be achieved by riveting (rivets (5)) or bonding. Brake shoes of simplex brakes are supported at lower ends, and in particular, at drum brakes shown in Figure 1, on clamping of the brake adjuster (6). Activation forces act at the upper ends of shoes and break them apart, which results in friction force between the drum and brake linings. Main task of elements for holding shoes (7) is to ensure the specific position of shoes, so during brakes activating, the brake is as quickly and better brought into contact with the drum. Return springs (8) and (9) hold the shoes attached to the supports and during releasing they return shoes to the starting position. Drum brakes can be activated either hydraulically or mechanically. When a hydraulic transmission mechanism is used for the service brake on motor vehicles, the shoes are actuated by a hydraulic cylinder, which is screwed to the backing plate.

In drum brake systems for motor vehicles, as with disc brakes, either complete or partial failures may occur [8, 9]. Complete failures, which are rare, arise when the brake is unable to generate any braking torque. Partial failures, on the other hand, lead to a significant deterioration in the drum brake's operating characteristics, including braking torque, operating temperature, braking uniformity, and the intensity of noise generated during braking.

Failures that lead to a reduction in braking torque are commonly referred to as friction failures. These can be either permanent or transient. Permanent friction failures of a drum brake may result, among other causes, from damage to the drum and the brake shoe linings. The most common types of damage to the brake drum of motor vehicle braking systems occurring during operation are [10]: cracked drum, cracks caused by overheating, contamination of the friction surface with grease, occurrence of martensitic spots, deep grooves on the sliding surface, blue discoloration of the sliding surface, polished surface, drum deformation, excessive wear, thinning of the drum, circumferential cracking of the drum's mounting ring and radial cracking of the mounting ring at the bolt holes.

3. ASSESSMENT OF RELIABILITY INDICATORS AND DETERMINATION OF WEIBULL DISTRIBUTION PARAMETERS

As part of research into the reliability of the braking system of light commercial vehicles, operational testing of the brake drum was carried out. By testing machine elements and systems for reliability assessment under actual operating conditions, a realistic picture of their behavior with respect to the loss of operational capability is obtained. Based on the results of a detailed analysis of brake drum damage and its effects on braking system performance, it can be concluded that, according to the possibility of restoring operational capability, damaged brake drums can be classified as: irreparable, conditionally repairable depending on the extent of damage, and repairable. In cases where repair is possible, appropriate maintenance measures are used to restore the drum's operational capability. It should be emphasized that the collected data refer to the operating time corresponding to the total service life of the brake drum, i.e., the time until its decommissioning. Since the degree of damage to motor vehicle components is, in most cases, a function of the total distance traveled, the "operating time" until brake drum failure is expressed in kilometers traveled. During the testing, a total of n = 55 data points were collected on the operating time until failure of the brake drum in the braking system. Based on the expression $z = 1 + 3.3 \log n$ for calculating the number of intervals for grouping the values of a random variable in the case of a large sample, the rounded result is 7 intervals. The calculated interval width is Δt = 80,000 km, determined from the maximum and minimum values of the random variable. The obtained operating times until brake drum failure are grouped into time intervals and presented in Table 1.

Tab. 1. Number failure of brake drum of braking system in time intervals

Distance travelled [10^3 km]	40÷120	120÷200	200÷280	280÷360	360÷440	440÷520	520÷600
Number of failure	7	16	18	8	3	1	2

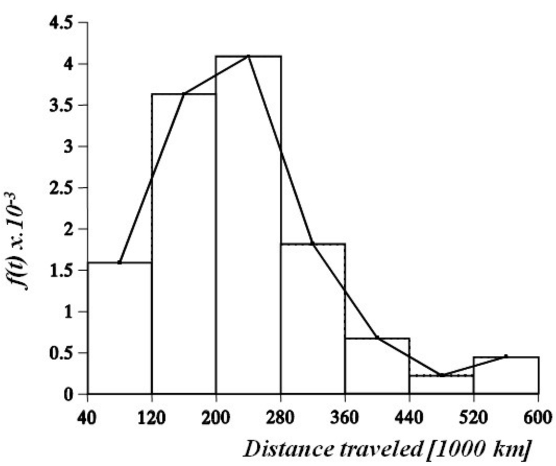
Using the software for determining the theoretical model of an empirical distribution, described in detail in [5], the following numerical characteristics of the statistical series were obtained: mean value t_{MTTF} = 232,727 km, standard deviation σ = 110,296 km, median t_{50} = 220,000 km, mode t_m = 213,333 km, coefficient of skewness C_s = 0.9959, and coefficient of kurtosis C_k = 4.1465. The relationship between the mean value, median, and mode of the random variable operating time until brake drum failure, indicates a positively skewed empirical distribution. This is confirmed by the calculated value of the skewness coefficient. The calculated value of the standard deviation of the random variable indicates a large dispersion in the operating time until brake drum failure.

In continuation of the program, based on the procedures for assessment of functional indicators of the distribution of the random variable for a big sample (n > 30), for number of failures $m(t_i)$, estimated values of the number of correct objects $n(t_i)$, the reliability $R(t_i)$, the unreliability $F(t_i)$, density of operation time until failure $f(t_i)$ and failure intensity $h(t_i)$ of brake drum of braking system are gained for middles of time intervals t_i and given in Table 2.

Tab. 2. Estimated values of functional indicators of the distribution of the random variable

i	m_i	t_i	$n(t_i)$	$R(t_i)$	$F(t_i)$	$f(t_i)$	$h(t_i)$
1	7	80,000	51.5	0.93636	0.06364	0.15909E-02	0.16990E-02
2	16	160,000	40.0	0.72727	0.27273	0.36364E-02	0.50000E-02
3	18	240,000	23.0	0.41818	0.58180	0.40909E-02	0.97826E-02
4	8	320,000	10.0	0.18182	0.81818	0.18182E-02	0.10000E-01
5	3	400,000	4.5	0.08182	0.91818	0.68182E-03	0.83333E-02
6	1	480,000	2.5	0.04545	0.95455	0.22727E-03	0.50000E-02
7	2	560,000	1.0	0.01818	0.98182	0.45455E-03	0.25000E-01

Illustrations of graph charts of estimated values of density of operation time until failure $f(t)$, and failure intensity $h(t)$, of brake drum, in the form of histograms and polygons, are given in Figure 2. In rough assessments, theses graph charts may serve for determination of hypothetical distribution models. The positively skewed shape of the probability density function (Figure 2a)) is characteristic of the Weibull, Rayleigh, and lognormal distributions. Based on the graph of the estimated failure rate (Figure 2b)), it can be concluded that the failure rate is increasing, which is characteristic of the aging period of components.



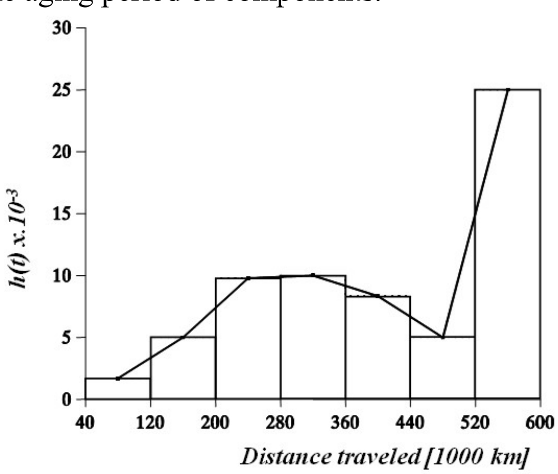


Fig. 2. Diagrams of estimated values for distribution of: a) density and b) intensity of brake drum failure

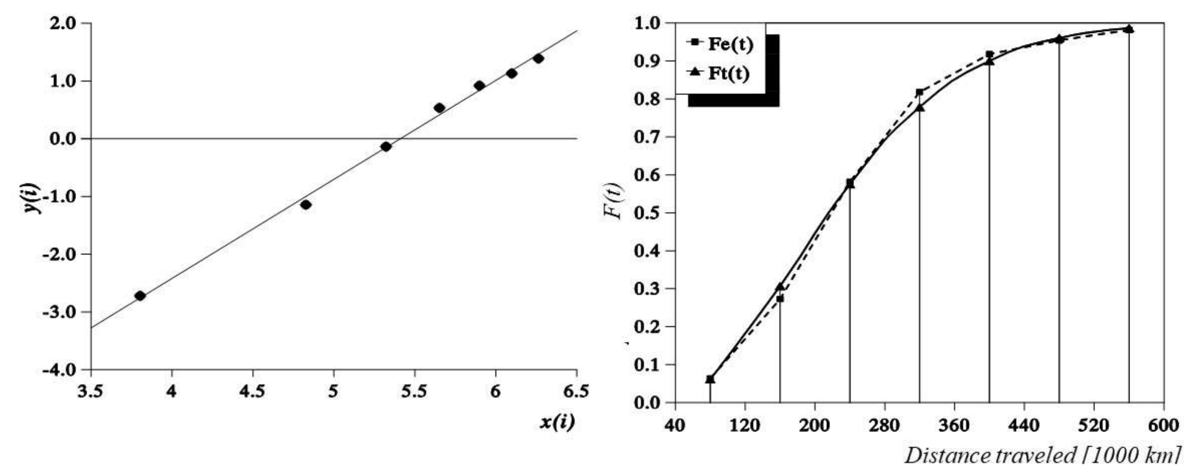
Analytical determination of the parameters of the Weibull distribution was performed using the least squares method with the aid of computer software. By approximation of empirical distribution of operation time until failure of brake drum of braking system by the three-parameter Weibull distribution and by using a computer program whose algorithm is presented at work [6], after 17 iterations by halving the intervals and determination of a sign of second derivative a_2 equations of the approximate parabola $y = a_0 + a_1 \cdot x + a_2 \cdot x^2$, the parameters of the distribution are obtained: location parameter γ = 35,186 km, scale parameter η = 224,085 km and shape parameter β = 1.714. Based on this, the expression for probability of faultless operation of brake drum is:

$$R(t) = e^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta}} = e^{-\left(\frac{t-35{,}186}{224{,}085}\right)^{1.714}} \qquad (1)$$

During determination of proper function probability of brake drum and all other functional indicators of reliability derived from expression (1), time t, is expressed in kilometres of the distance passed.

4. TESTING OF THE WEIBUL APPROXIMATE DISTRIBUTION

In order to determine validity of approximation, graphical testing over probability paper for Weibull distribution and nonparametric testing were conducted using tests of Kolmogorov and Pearson. Figure 3 a) presents arrangement of points with transformed x and y coordinated for particular value γ = 35,186 km on probability paper for Weibull distribution. Linear arrangement of the points in Figure 3 a) suggests that the approximate model satisfies conditions of graphical testing.



a) b)

Fig. 3. a) Arrangement of points on probability paper for Weibull distribution, b) Graphical representation of deviations of Weibull approximate distribution from empirical distribution

For testing of hypothetical distribution model, according to Kolmogorov test, it is necessary to determine the greatest absolute value of difference between theoretical model and estimated values of distribution functions of operation time until failure. Table 3 contains a segment of output list of a program that relates to this part. Figure 3 b) presents graphical representation of deviations of theoretical approximate model $F_t(t)$, from empirical distribution $F_e(t)$.

Tab. 3. Deviations of Weibull approximate curve $F_t(t)$ from estimated values of distribution function $F_e(t)$ of operation time until failure

i	t_i	$F_e(t_i)$	$F_t(t_i)$	delta
1	80,000	0.0636	0.0614	0.0022
2	160,000	0.2727	0.3070	0.0343
3	240,000	0.5818	0.5756	0.0062
4	320,000	0.8182	0.7787	0.0395
5	400,000	0.9182	0.9003	0.0179
6	480,000	0.9545	0.9608	0.0062
7	560,000	0.9818	0.9864	0.0046

As it may be seen from Table 3, the largest deviation of theoretical model from empirical distribution is for the result No. 4 and amounts to 0.0395. For number of samples, n = 55 and given level of significance for Kolmogorov's test, α = 0.20, λ_α = 1.07 permitted value of difference is [11]:

$$D_n = \frac{\lambda_\alpha}{\sqrt{n}} = \frac{1.07}{\sqrt{55}} = 0.1443. \tag{2}$$

Since the maximal deviation is less than the permitted value of the difference, the Weibull approximate distribution satisfies the Kolmogorov test for the adopted level of significance. Based on the Kolmogorov test table, it can be concluded that the considered theoretical model could also be accepted for a much higher significance level α.

For non-parametric testing of the Weibull approximate distribution, the Pearson test (χ^2 test) was also applied [11]. In the case of a large sample, the data in Table 1 are already grouped into intervals of equal length. The condition for applying the Pearson test is that each interval (class) contains at least five values of the random variable. Therefore, intervals 5, 6, and 7 were combined. The cumulative frequency for these three intervals is 6. Table 4 presents the empirical and theoretical frequencies of the random variable by class. The theoretical frequencies for each class were calculated based on the sample size n and the theoretical probabilities of the random variable within the intervals (p_i). The calculated comparative measure of deviation between the theoretical and empirical models for this test is χ^2 = 2.88.

Tab. 4. Empirical frequencies f_i, theoretical frequencies f_{ti}, and the calculated χ^2 test value

Class number	Value intervals x 10^3	f_i	p_i	$f_{ti} = n \cdot p_i$	$\frac{(f_i - f_{ti})^2}{f_{ti}}$
1	40÷120	7	0.170947	9.4021	0.6137
2	120÷200	16	0.273695	15.0532	0.0595
3	200÷280	18	0.241661	13.2910	1.6684
4	280÷360	8	0.161158	8.8637	0.0842
5	360÷600	6	0.143500	7.8945	0.4546
		$n = \sum_{i=1}^{r} f_i = 55$			$\chi^2 = 2.88$

In non-parametric testing of approximate theoretical models using the χ^2 test, the first step is to establish the hypotheses. The null hypothesis is usually taken to be that the approximate distribution satisfies the test, while the alternative hypothesis is that it does not. The rejection criterion for the null hypothesis is as follows: if the calculated value of the statistic test χ^2 is greater than the tabulated value $\chi^2_{\alpha,\nu}$ for the adopted risk level α and the calculated number of degrees of freedom ν, the theoretical model is rejected as inadequate. The number of degrees of freedom for the Pearson test is calculated using the formula:

$$\nu = r - l - 1, \tag{3}$$

where r is the number of classes into which the values are grouped, and l is the number of parameters of the tested theoretical model.

In the specific case of the three-parameter Weibull distribution, the result is: ν=5−3−1=1. The relatively small number of classes into which the values of the random variable are grouped, combined with the three distribution parameters, has led to a significant tightening of the test criterion. Primarily for this reason, the maximum risk level at which the rejection criterion for the null hypothesis is not met is only α = 0.05. The corresponding tabulated test value is $\chi^2_{0.05,\ 1} = 3.841$. This means that if values of α greater than 0.05 are adopted, the Weibull approximate distribution is rejected as inadequate. For values of α less than or equal to 0.05, the rejection criterion for the null hypothesis is not satisfied.

5. CONCLUSIONS

Research on the reliability of drum brake components is of great importance for improving vehicle safety in traffic. Determining the theoretical model of the reliability distribution law for the brake drum enables the planning of maintenance measures, the scheduling of spare parts production, the determination of the optimal warranty period for the component in question, and more.

Determination of the parameters of the Weibull approximate distribution using a computer and suitable software, unlike graphical or graph-analytical methods, allows for achieving satisfactory accuracy of results at high speed. Due to this, and owing to the well-known properties of this distribution in interpreting various laws of random variables, the Weibull approximate distribution often proves to be the optimal solution compared to other theoretical models.

Based on the values of the random variable “operating time until brake drum failure,” it can be concluded that the failures occur during both normal operation and aging periods. This means that, based on the causes of brake drum failures, the data should ideally be separated according to the respective period and subjected to separate statistical analyses. This would require determining the direct cause of each failure, which is practically difficult to achieve.

In future work, it is possible to perform a program-based approximation of the empirical distribution of brake drum operating times until failure using a larger number of hypothetical theoretical models. By comparing the results of graphical and analytical non-parametric hypothesis testing, it is possible to select the theoretical distribution model that shows the smallest deviation from the empirical distribution according to all or most criteria. Additionally, systematic collection of data on brake drum failures during operation would enable the planning and execution of accelerated tests for the reliability assessment of the considered component. Furthermore, future research could include an analysis of the impact of human factors, such as operating regimes and maintenance practices, on the reliability and service life of the drum brake.

ACKNOWLEDGEMENT

This paper is the result of research conducted under the author's contract No. 451-03-137/2025-03/200107, dated February 4, 2025.

REFERENCES

[1.] Edited by Crolla, A. D.: Automotive Engineering Powertrain, Chassis System and Vehicle Body, Published by Elsevier Inc., 2009.
[2.] Day, A.: Braking of Road Vehicles, Published by Elsevier Inc., 2014.
[3.] Mikulić, D.: Motorna vozila, Teorija kretanja i konstrukcija, Veleučilište Velika Gorica, 2020.
[4.] Todorović, J.: Braking of motor vehicle, (in Serbian), Institute for textbooks and teaching aids, Beograd, 1988.
[5.] Ćatić, D.: Development and application of reliability theory methods, (in Serbian), Monograph, The Faculty of Mechanical Engineering from Kragujevac, Kragujevac, 2005.
[6.] Ćatić, D., Glišović, J., Veličković, S., Blagojević, J., Delić, M.: Determination of parameters of the Weibull distribution by applying the method of least squares, Mobility and Vehicle Mechanics, Vol. 41, No. 1, pp. 63-77, 2015
[7.] Service manuals, "Zastava Iveco trucks", Kragujevac, 1991.
[8.] Ćatić, D., Glišović, J., Miković, J., Veličković, S.: Analysis of failure causes and the criticality degree of elements of motor vehicle's drum brakes, Tribology in Industry, 2014, Vol. 36, No. 3, pp. 316-325, ISSN 0354-8996
[9.] Vasiljević, S., Ćatić, D., Glišović, J., Fault tree analysis of motor vehicle's disc brake, Tractors and power machines, Vol. 24, No. 1/2 pp. 72-81, Novi Sad, Dec. 2019.
[10.] Web page: https://www.accuridecorp.com/sites/default/files/resources/2021-6/WE3.000%20 Rev.%202%20Gunite%20Brake%20Drum%20Service%20Manual.pdf, Maintenance and Insta-llation Manual, Accessed on: 25. 04. 2025.
[11.] Vukadinović, S.: Elements of probability theory and mathematical statistics, (in Serbian), Economic review, Beograd. 1990.

Rad primljen: 18.08.2025. Rad prihvaćen: 17.09.2025.