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# Booklet of Abstracts

## “2nd International Conference on Mathematical Modelling in Mechanics and Engineering”

**Mathematical Institute of the Serbian Academy of Sciences and Arts**
**Belgrade, 12.-14. September 2024.**

**Editor: Ivana Atanasovska**

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# PREFACE

It's my pleasure to introduce the "2nd International Conference on Mathematical Modelling in Mechanics and Engineering", organized by the Mathematical Institute of the Serbian Academy of Sciences and Arts and co-organized by the Faculty of Mechanical Engineering, University of Belgrade; the Faculty of Mechanical and Civil Engineering in Kraljevo, University of Kragujevac; and the Scientific Society for Engineering Design, Simulations and Innovations. The conference will be held in the hybrid form at the Mathematical Institute of the Serbian Academy of Sciences and Arts, Belgrade, Serbia, from the 12th to the 14th of September 2024.

This conference is planned as the second event in the series of conferences, which is planned to be held every two years and bring together leading academic scientists, researchers, and research scholars to exchange and share experience and research results on various aspects of mathematical modelling in mechanics and engineering. It will keep an interdisciplinary platform for researchers, practitioners and educators to present and discuss the most recent innovations, theories, and algorithms, as well as practical challenges encountered and solutions adopted in the fields of Classical Mechanics, Solid and Fluid Mechanics, Computational Mechanics, Biomechanics, Applied Mathematics and Physics, Structural Mechanics and Engineering. A considerable number of prominent scientists and professors submitted their abstracts and confirmed their attendance at the conference. The scientists and researchers from different countries in Europe and the world (Netherlands, Greece, Spain, Russia, USA, China, Kazakhstan, Italy, India, Malaysia, Slovenia, Bulgaria, Algeria etc.) also have confirmed participation at the conference. The conference presentations will cover development of analytical and numerical methods for effective simulations of different complex problems in mechanical engineering based on multiscale problems, from nano to macro-scales, analytical/numerical and data driven solutions to study complex media, composite aerospace and periodic structures and metamaterials, and capture essential features of linear and nonlinear dynamics that can lead to new designs of such systems. Some presentations will include new experimental setups to study engineering materials and novel control strategies based on classical or fractional derivative models used to control the dynamics of multibody, flexible and/or electromechanical systems. Finally, I believe that the sessions' discussions will have high potential to give significant contributions to the development of new and advanced mathematical models of real-world engineering mechanical systems.

On behalf of the Organizing Committee, I am very proud to announce that the number of accepted contributions to be presented at this Conference is 127, with 7 plenary and 7 invited lecture presentations. We would like to express our gratitude to the institutions that support the conference financially: The Ministry of Science, Technological Development and Innovation, Republic of Serbia; METALFER STEEL MILL, Serbia; SHIMADZU, Serbia; eCon Engineering Kft, Hungary; SVECOM, Beograd, Serbia; "PROJEKTINŽENJERING TIM", Niš, Serbia; AMING PROJEKT, Knjaževac, Serbia; BREGAVA, Beograd, Serbia. We are especially grateful to the members of the International Scientific Committee who contributed to this international scientific meeting with their advices and abstracts' reviews. We also thank the support of the co-organizers of this Conference.

I would also like to express special gratitude to the Department of Technical Sciences SANU, the Scientific Board of the Mathematical Institute of the Serbian Academy of Sciences and Arts, and the family of Academician Prof. Dr. Vladan Djordjevic for their support in organizing the Special session within this conference dedicated to the memory of Academician Prof. Dr. Vladan Djordjevic and establish the "Prof. Vladan Djordjevic" award for young scientists aged up to 35 contributing in the field of fluid mechanics.

I hope that this conference will be successful, at least as the first in this series of international conferences. I wish all participants a successful presentation of their scientific results.

Cordially,

Ivana Atanasovska, Conference Chair

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2nd International Conference on Mathematical
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Mathematical Institute SANU, 12-14. September, 2024.

# IMPROVEMENT OF A PHYSICAL FIELD IN FEA BY APPLYING A SMOOTHING METHOD

**Marija M. Rafailović[1], Miroslav M. Živković[2], Vladimir P. Milovanović[3], Jelena M. Živković[4] and Gordana R. Jovičić[5]**

*Faculty of Engineering, University of Kragujevac, 34000 Kragujevac, Serbia*

*[1]marija.rafailovic@kg.ac.rs; [2]zile@kg.ac.rs; [3]vladicka@kg.ac.rs; [4]jelena.zivkovic@kg.ac.rs; [5]gordana.jovicic@kg.ac.rs*

ORCID iD: [1]0009-0001-2132-9249; [2]0000-0002-0752-6289; [3]0000-0003-3071-4728; [4]0000-0002-4398-7365; [5]0000-0002-9799-5555

**Keywords:** Finite element method, Extended finite element method, Strain smoothing, Smoothed finite element method (S-FEM), Strain-smoothed element (SSE) method, Low-order elements.

## ABSTRACT

Due to its reliability and efficacy, the finite element method (FEM) is considered to be one of the most successful numerical methods, which is widely applied in science and engineering for structural analysis. Besides its wide application in engineering practice, the standard FEM has certain limitations. Linear triangular and tetrahedral finite elements result in very stiff behavior and a very low rate of convergence which stems from the constant stress field. In addition, the FEM model exhibits sensitivity to distorted isoparametric elements, during the use of which the Jacobian matrix is badly conditioned, leading to poor solutions and even the breakdown of the computation process. Moreover, considering the $C^0$ continuity of shape functions, the stress field across the element boundaries is discontinuous.

Great efforts have been invested to develop numerical strategies that may be used to overcome the abovementioned problems. In this manner, the smoothed finite element method (S-FEM) [1] is created, and it is used to modify the compatible strain field, by applying the gradient smoothing approach in the FEM, originally employed to stabilize the nodal integrated Galerkin meshfree methods [2]. Depending on the applied scheme for creating smoothing domains on top of the element mesh, a series of S-FEM methods has been developed [1]. Integration in the S-FEM methods has significantly been simplified by applying Gauss's divergence theorem, by means of which the domain integration is transformed into the boundary integration. The formulation of S-FEM methods requires the shape function values only, not their derivatives. Therefore, the Jacobian matrix is not needed, due to which isoparametric mapping is avoided. Significant properties of the S-FEM methods complete with their diverse application were discussed thoroughly in [1].

Bearing in mind all the advantages of the S-FEM methods, including the ones related to enhancing the convergence behaviour of 3-node triangular and 4-node tetrahedral finite elements [1], the only elements the mesh of which can be automatically generated even when it comes to extremely complex geometry, the S-FEM methods are expanded to solve problems involving discontinuous and singular physical fields. Coupling the strain smoothing with the partition of unity enrichment led to the formation of a series of new methods [3,4], which exploit the advantages of not only the S-FEM method, but the extended finite element method

(X-FEM) as well. By transforming the internal integration into the boundary integration, the subdivision of elements intersected by discontinuities was not found to be necessary [3]. In addition, since the derivatives of shape functions in the S-FEM method are replaced by the shape functions multiplied by the component of the outward unit vector on the boundary, the singular term integration $1/\sqrt{r}$, which occurs in the derivatives of near-tip enrichment functions (branch functions), is eliminated during the computation of the stiffness matrix in the fracture mechanics.

To improve the accuracy of 3-node triangular and 4-node tetrahedral finite elements, a new strain smoothing method, coined the strain-smoothed element (SSE) method [5], has recently been proposed. The strain field of such elements is constant, whereas by applying the S-FEM methods the piecewise constant strain fields are constructed through smoothing domains. In the context of the SSE method, a linear strain field is constructed in the elements themselves, by utilizing the constant strains of the adjacent elements. A significant characteristic of the SSE method is the lack of creation of special smoothing domains. Therefore, the standard FEM framework is maintained. To the best of the authors' knowledge, the SSE method has been successfully developed for the polygonal finite elements at last [6].

A numerical analysis was conducted within this particular research in order to provide a comparative presentation of the accuracy of linear and quadratic tetrahedral finite elements, implemented within the PAK and Nastran program packages, compared to the 4-node tetrahedral finite element of the corrected strain field [5]. The 3D problem, known as the Lame problem in the literature [5], was analyzed. The obtained results demonstrated that the corrected elements, even in case of extremely coarse mesh, were achieving a considerably higher level of accuracy compared to the linear elements. When comparing the quadratic and corrected tetrahedral finite elements, a slightly enhanced convergency of the quadratic element was observed. Since the shape functions of these two elements were not in the same range, the elements were described as the ones with various degrees of freedom, meaning that it was completely expected to see the quadratic element converging in a slightly faster manner towards the reference solution. The results of the program packages were well-aligned.

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