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Suppression of Torsional Oscillations in Systems with Flexible Couplings and Repeated Iterative Operations

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***Abstract*— This paper addresses the problem of mechanical resonance in high-performance servo drives operating under repeated iterative tasks. The elastic coupling between the motor and the load can cause mechanical resonance, leading to forced torsional oscillations in the system. While various solutions have been proposed to suppress such oscillations, the effectiveness of Iterative Learning Control (ILC)-based structures for this issue has not yet been thoroughly investigated. This paper provides a brief overview of anti-resonant servo system architectures and evaluates the potential of ILC-based serial compensation as a viable anti-resonant control strategy.**

Keywords — Torsional resonance, Oscillation suppression, Iterative Learning Control (ILC).

I. Introduction

In most machine centers, industrial robots, servomechanisms, and other rotating machinery, geared reduction mechanisms are used between motor output shafts and driven components. However, the limited torsional stiffness of these mechanisms often leads to transient vibrations, particularly at lower frequencies associated with the mechanical system's eigenvalues, during motor start-up or shutdown. Elastic couplings and joints within the system pose a significant challenge to performance enhancement, as high loop gains can destabilize torsional resonance modes caused by transmission flexibility. Suppressing vibrations in rotating machinery remains a critical engineering challenge [1-7].

Namely, in a typical industrial environment, most drive subsystems are connected to the load through transmission mechanisms with finite stiffness. This factor is often overlooked in the design of control algorithms for various types of servomechanisms. Consequently, torsional resonance may arise in the low-frequency range (typically between 200 Hz and 400 Hz) or in the high-frequency range (between 500 Hz and 1200 Hz), potentially leading to control system instability [3]. Specifically, elastically coupled masses—motor and load—introduce a finite number of zeros and a pair of conjugate complex poles into the system's transfer function, which can result in mechanical resonance. This issue is more pronounced in servo systems where the position sensor is located on the load shaft [3, 5]. In such cases, the closed-loop system can excite unmodeled torsional resonance modes [3].

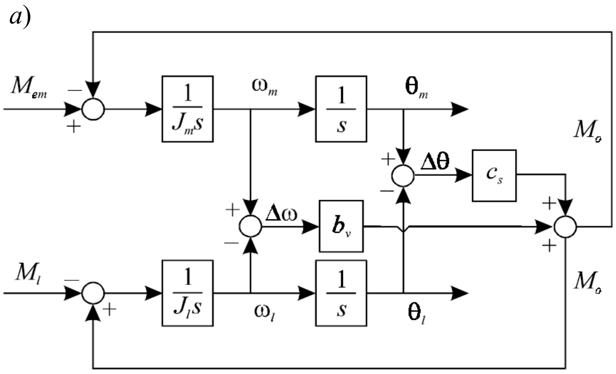
This phenomenon limits system performance and leads to insufficiently damped oscillations in the system's response due to excitation from the reference input or load [3-8]. Often overlooked in the design of conventional servo systems, resonant modes can overlap with the system's bandwidth, inducing forced vibrations in the machine and its interconnected components [3,6,7]. This phenomenon is often accompanied by noise and may result in joint damage, machine component failures (due to metal fatigue, etc. [8]), potential tool wear and breakage, as well as possible consequences for the quality of the workpiece. A comprehensive review of anti-resonant control strategies is provided in [9]. So far, the anti-resonant potential of Iterative Learning Control (ILC) algorithms has not been studied, and this will be the focus of this paper.

Iterative Learning Control (ILC) is a data-driven control method that has become increasingly important with technological advancements in repetitive manufacturing processes, such as car-body painting, where robots perform repeated tasks [10, 11]. A comprehensive overview of synthesis methods for Iterative Learning Control (ILC) algorithms and their applications can be found in several research articles, review papers, and PhD theses [10–18].

II. Servo System with Flexible Coupling

The model of a typical mechatronic subsystem, serving as the controlled plant, is shown in Fig. 1.

a)



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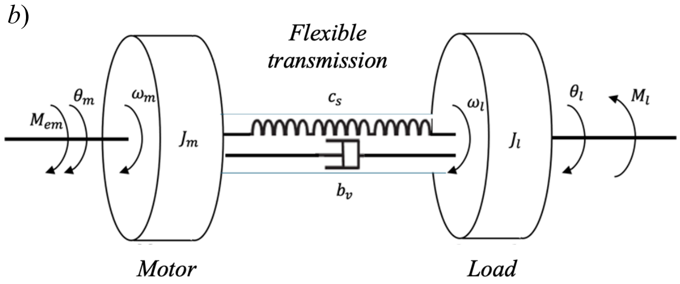

Fig. 1. AC motor with a load flexibly coupled via an elastic shaft [3]

The driving torque M_{em} is the control variable, while the load shaft torque M_l represents a disturbance. The motor inertia J_m and the load inertia J_l are coupled through a transmission mechanism (such as a shaft, gearbox, toothed belt, etc.) with a stiffness coefficient c_s. The viscous friction coefficient b_v typically has very low values, which facilitates the occurrence of weakly damped torsional oscillations [3]. The torsional torque M_o is equal to the load torque M_l only in steady-state conditions, while during transient processes — when the motor speed ω_m and the load speed ω_l differ—it is described by the relation [3, 8, 9]:

$$M_o = c_s \Delta\theta + b_v \Delta\omega. \qquad (1)$$

Conventional approaches to the synthesis of speed or position servo systems are based on a simple plant model.

$$G^0(s) = \frac{\theta_l(s)}{M_{em}(s)} = \frac{1}{(J_m+J_l)s^2}, \ \omega_l(s) = s\theta_l(s). \qquad (2)$$

However, by considering the elastic coupling between the motor and the load, a more accurate model of the plant is given by the transfer function.

$$G_m(s) = \frac{\theta_m(s)}{M_{em}(s)} = \frac{1}{(J_m+J_l)s^2} \frac{1+\frac{2\zeta_z}{\omega_z}s+\frac{1}{\omega_z^2}s^2}{1+\frac{2\zeta_p}{\omega_p}s+\frac{1}{\omega_p^2}s^2},$$
$$G_l(s) = \frac{\theta_l(s)}{M_{em}(s)} = \frac{1}{(J_m+J_l)s^2} \frac{1+\frac{2\zeta_z}{\omega_z}s}{1+\frac{2\zeta_p}{\omega_p}s+\frac{1}{\omega_p^2}s^2}, \qquad (3)$$

where resonance and antiresonance undamped natural frequencies ω_p and ω_z, and relative damping coefficients ζ_p and ζ_z are given by

$$\omega_p = \sqrt{\frac{c_s(J_m+J_l)}{J_m J_l}}, \ \omega_z = \sqrt{\frac{c_s}{J_l}},$$
$$\varsigma_p = \sqrt{\frac{b_v^2(J_m+J_l)}{4c_s J_m J_l}}, \ \varsigma_z = \sqrt{\frac{b_v^2}{4c_s J_l}} \qquad (4)$$

The undamped natural frequencies ω_p and ω_z are referred to as the resonant and antiresonant frequencies, respectively [2,5], and their ratio is known as the resonant ratio.

$$R_r = \frac{\omega_p}{\omega_z} = \sqrt{1+\frac{J_l}{J_m}}. \qquad (5)$$

When the sensor is placed on the motor shaft, low values of the resonant ratio reduce the impact of the torsional load on the speed servo system dynamics. The high motor inertia prevents the propagation of torsional oscillations from the load to the motor. However, in this configuration ($J_m \gg J_l$), the control performance is biased toward the motor-side variables — angular velocity ω_m and position θ_m — although most applications require accurate control of the load-side variables (ω_l and θ_l).

It is often not straightforward to identify the parameters of the flexible coupling (ω_p, ω_z, ζ_p, and ζ_z). Moreover, the effects of flexible coupling are frequently overlooked or neglected, and model (2) is typically used as the nominal model of the controlled plant. Such an approach may lead to torsional oscillations in the system.

III. Anti-resonant systems and strategies

The most commonly used structure for compensating torsional oscillations due to mechanical resonance is shown in Fig. 2, while a notch filter:

$$W_{notch}(s) = \frac{s^2+2\varsigma_p\omega_p s+\omega_p^2}{s^2+2\varsigma\omega_p s+\omega_p^2}, \ \varsigma \gg \varsigma_p. \qquad (6)$$

is most frequently used in practice as an anti-resonant compensator, with the aim of precisely canceling the resonant poles of the plant. Successful synthesis of such a filter requires accurate knowledge of the values of ω_p and ζ_p. This assumption poses a significant challenge in the implementation of practical applications.

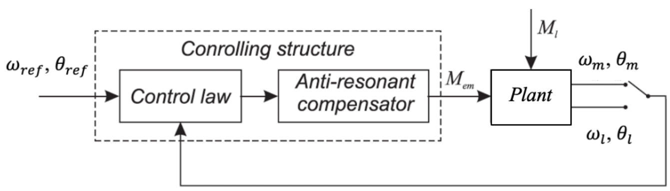

Fig. 2. System with an anti-resonant compensator

In [5, 8, 9], robust control methods based on the RST and IMPACT (Internal Model Principle and Control Together) structures were proposed to address the anti-resonance issue in high-performance servo systems. These methods consider potential modeling uncertainties arising from the use of a nominal model (2) in controller synthesis, as well as from torsional resonance within the servo system's operating range. In [3], an anti-resonant filter was introduced to achieve good performance in the presence of weakly damped torsional oscillations. However, this filter is sensitive to modeling uncertainties, while the robust methods offer improved resilience at the cost of certain performance limitations.

IV. Iterative Learning Control – ILC

Iterative Learning Control (ILC) is a control strategy designed to enhance the performance of systems that execute repetitive tasks by leveraging information from previous iterations [13, 11]. Unlike conventional controllers, which do not utilize data from past executions, ILC integrates previous tracking errors into its control law. This allows for accurate reference tracking even in the presence of model uncertainties or unknown but repetitive disturbances. The ILC controller seeks to compute a compensatory control input, based on stored data from prior iterations, in order to mitigate the impact of such disturbances and reduce tracking error in subsequent runs [10, 18]. In contrast to feedback controllers that react only after disturbances occur, ILC acts predictively by exploiting experience from earlier iterations [10].

Fig. 3. illustrates the integration of the ILC signal into a feedback control system, where the index j represents the j-th repetition or iteration of system operation. After each iteration j, the ILC signal f_{j+1} is updated for the subsequent iteration $j+1$, using data from the previous cycle. Through this iterative process, the ILC signal is progressively refined, leading to improvements in the control input and overall system performance.

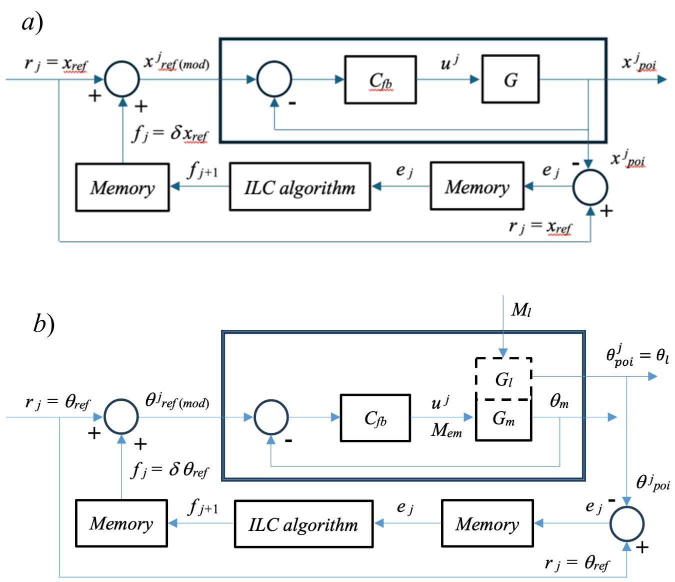

Fig. 3. System with serial ILC compensator: a) general case, b) positional servo system case

The ILC system operates in two domains: the time (or frequency) domain and the iteration domain. The time or frequency domain captures the dynamics of the plant (G) and the behavior of the feedback control system, as illustrated by the boxed section in Fig 3. The iteration domain describes the convergence of the ILC signal f_{j+1} and the evolution of system behavior across iterations, by monitoring the complete set of measured signal sequences during the j-th iteration such as the tracking error signal e_j. This domain operates in an offline manner with respect to the system's dynamic execution, meaning that the ILC signal f_{j+1} is computed between iterations j and $j+1$.

The fundamental problem in ILC is to design an update algorithm (for calculation of the ILC signal f_{j+1} in iteration $(j+1)$) such that the tracking error signal (e_{j+1}) of the closed-loop system is minimised in some appropriate sense. A general updating equation is given by

$$f_{j+1}(z) = Q(z)\left(f_j(z) + L(z)E_j(z)\right), \tag{7}$$

where $L(z)$ is the learning filter, and $Q(z)$ is the robustness filter. Filtering using $L(z)$ and $Q(z)$ is performed off-line using non-causal filtering methods; time-shifting is used after filtering with $L(z)$; filtering with $Q(z)$ is based on magnitude characteristics of $|Q(z)|$ only (zero-phase). Typically, $Q(z)$ is a low-pass filter.

In Fig.3, C_{fb} and G denote the feedback controller and the plant, respectively. If

$$T_{cs}(z) = \left(1 + C_{fb}(z)G(z)\right)^{-1} C_{fb}(z)G(z) \tag{8}$$

is the complementary sensitivity function, the output signals of the systems from Fig. 3(a) are

$$x_{poi}^j(z) = T_{cs}(z)f_j(z) + T_{cs}(z)r(z). \tag{9}$$

Based on Fig. 3 and relations (7) and (9), we obtain

$$E_{j+1}(z) = Q(z)[1 - T_{cs}(z)L(z)]\,E_j(z) + [1 - Q(z)]r(z). \tag{10}$$

The tracking error converges monotonically if and only if

$$|Q(z)[1 - T_{cs}(z)L(z)]| < 1 \tag{11}$$

at all frequencies f_h with $z = e^{i2\pi f_h T}$ $(\omega T = 2\pi f_h T,\ \ \omega T \in [-\pi,\ \pi])$. Namely, the robustness filter $Q(z)$ should ensure the convergence condition (11). Based on the tracking error propagation equation (10), design the learning filter $L(z)$ should be

$$L(z) \approx \left(T_{cs}(z)\right)^{-1}. \tag{12}$$

The design of the learning filter $L(z)$ is based on an adequate model inversion. For calculation of learning filter $L(z)$ ZPETC methodology can be used [19]. The design of the robustness filter $Q(z)$ is based on convergence condition (11) to ensure the convergence of the ILC algorithm (7).

If exact computation of the learning filter (12) is not feasible, whether it is causal or not, it is necessary to verify the convergence condition (11) for $Q(z) \equiv 1$. If this condition is not satisfied, a robustness filter ($Q(z) \not\equiv 1$) should be designed to ensure compliance with the convergence criterion (11). Through graphical-analytical analysis of condition (11) with $Q(z) \equiv 1$, one can determine the maximum frequency bandwidth and minimum selectivity required for the robustness filter $Q(z)$ [10].

V. Ilustrative Example

In this paper, we will test the effectiveness of applying ILC algorithms on the subsystem shown in Fig. 3(b). We will assume that during the controller design phase, it is possible to mount angular velocity and position sensors at the point of interest on the loaded shaft. During the operational phase, the sensor on the loaded shaft will not be used. The system will use the ILC feedforward signal $f_{j\,(=p)}$ (in Fig. 3, p is the number of iterations for which the signal f_j converges) and feedback controller based on the motor shaft signals (θ_m, ω_m). The design of the ILC feedforward signal is based on the load shaft signals (θ_l, ω_l) and the defined reference trajectory $\theta_{ref}(t)$.

Our aim is to control the load shaft position in the absence of a dedicated load side position sensor for motion control systems with flexible coupling. In our simulation analysis, we use a model of flexible coupling between the motor axle and the load as the actual plant model [3]. The important data are as follows (see Fig. 1): J_m=0.000620 kgm^2, J_l=0.000220 kgm^2, c_s=350 Nm/rad, b_v=0.004 Nms/rad. The sample rate is T=0.5 ms. The desired close-loop system transfer function is specified by undamped natural frequency ω_n=400 rad/s and relative damping coefficient ζ=1.

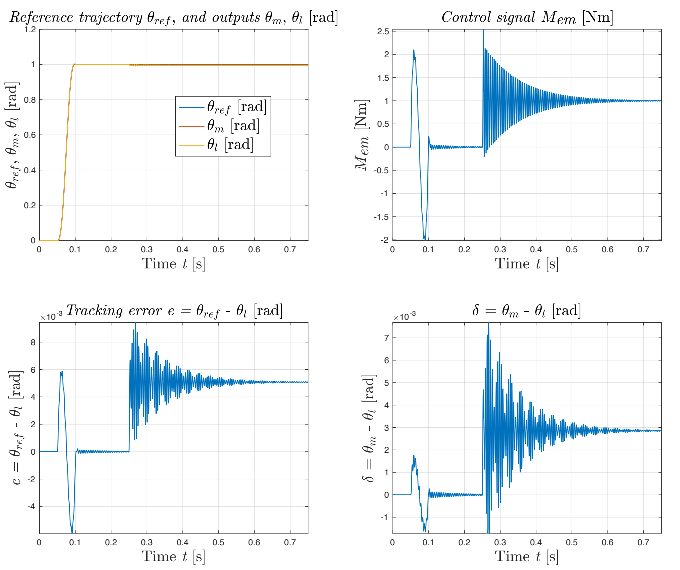

Fig. 4. Simulation-based validation of the feedback controller prior to ILC implementation

The digital feedback controller $C_{fb}(z)$ is designed in the frequency domain by shaping the sensitivity functions, with the goal of achieving a closed-loop bandwidth of 300 Hz. The plant has a bandwidth of approximately 6 Hz and exhibits resonant modes at 233 Hz. The controller must enable the system to reach the desired 300 Hz bandwidth and respond accurately to step disturbances and reference inputs, without steady-state error. Fig. 4 shows the closed-loop system response without ILC implementation. The effect of a load torque disturbance $M_l(t) = h(t - 0.25)$ was also simulated. The reference trajectory is an S-curve with a rise time of 0.05 s.

In the illustrative example (Fig. 3(b)), the complementary sensitivity function

$$T_{cs}(z) = \frac{C_{fb}(z)G_l(z)}{1 + C_{fb}(z)G_m(z)} \tag{12}$$

contains a non-minimum phase zero, which implies that the computation of the learning filter L can only be approximate, since the exact inversion of (12) is not feasible.

The ZPETC (Zero Phase Error Tracking Control) methodology for inverting a model represented by the discrete-time transfer function

$$\frac{z^{-d}b(z^{-1})}{a(z^{-1})} = \frac{z^{-d}b_s(z^{-1})b_u(z^{-1})}{a(z^{-1})} \tag{13}$$

leads to the learning filter

$$L(z, z^{-1}) = \frac{z^d a(z^{-1})b_u(z)}{\beta b_s(z^{-1})}, \tag{14}$$

where $b_s(z^{-1})$ and $b_u(z^{-1})$ are polynomials with stable and unstable zeros, respectively, and $\beta = (b_u(1))^2$.

The convergence of the ILC algorithm (7) and the effects of applying the ILC signal on reference trajectory tracking accuracy are illustrated in Fig. 6. It is important to note that the use of the ILC control algorithm also contributes to the suppression of oscillations in systems with flexible coupling (see Fig. 5). Suppressing torsional oscillations in such systems is of great practical significance.

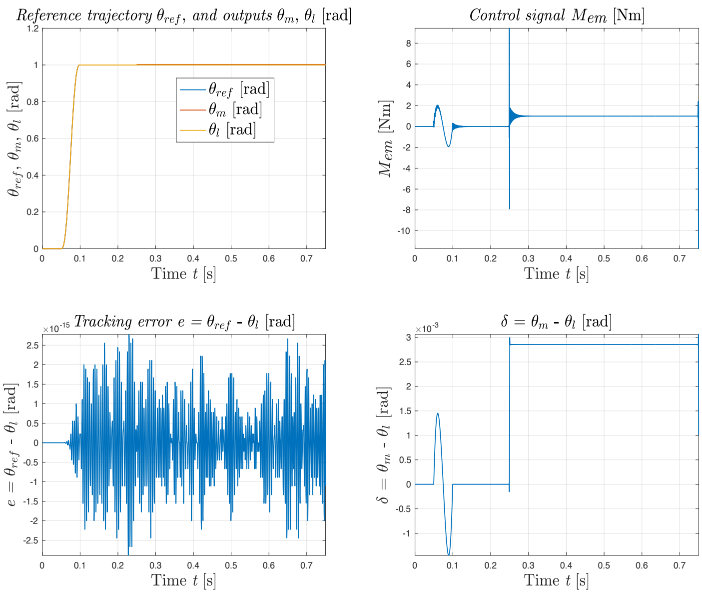

Fig. 5. Simulation results validating the effects of serial ILC implementation

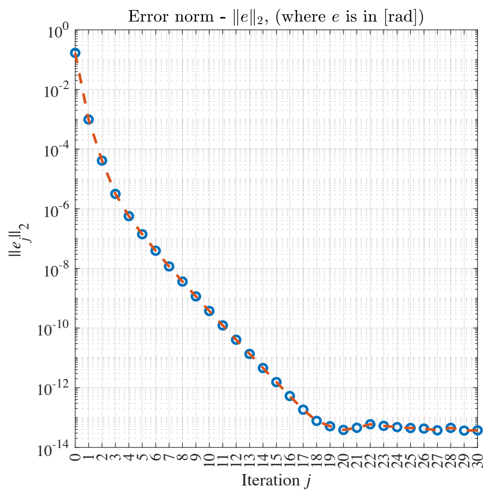

Fig. 6. Illustration of iterative learning of the ILC signal

It should be noted that the control signals presented in Fig. 4 and Fig. 5 are shown with different *y*-axis scales. This is due to the increased control effort (i.e., higher peak control values) in the ILC-enhanced system, which responds more strongly to the load torque disturbance.

Furthermore, the introduction of ILC significantly reduces the tracking error between the reference and load shaft angles. This improvement is reflected in a difference of approximately 10^{-7} in the *y*-axis scale of the tracking error plots, indicating enhanced precision, even though the curves appear visually overlapped. In fact, the achieved high-precision tracking performance directly reflects the effectiveness of the proposed method in suppressing internal oscillations within the system.

VI. Conclusion

This paper presents a concept for applying Iterative Learning Control (ILC) in the synthesis of high performance servo systems. The proposed approach has proven highly effective in suppressing torsional oscillations that may arise due to mechanical resonance. A limiting factor of this methodology lies in the fundamental assumptions underlying the use of ILC - namely, that the reference trajectory, disturbances, and initial conditions remain unchanged across successive iterations of system operation.

Unlike other anti-resonant control approaches, ILC is a data-driven method capable of compensating for limited *a priori* knowledge of the controlled system. For example, unlike implementations based on anti-resonant filters such as notch filters, the success of the method does not depend on accurately identifying the filter parameters, i.e., the precise value of the torsional resonance frequency.

It has also been shown that the learning phase of ILC requires sensors at all relevant points, whereas during the exploitation phase, only the precomputed ILC signal is needed. The results presented in this paper confirm the effectiveness of ILC in mitigating the effects of torsional mechanical resonance, provided that the necessary conditions for its synthesis and implementation are satisfied.

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