ISSN 1330-3651 (Print), ISSN 1848-6339 (Online)
Received: 23 June 2025; Accepted: 9 September 2025

https://doi.org/10.17559/TV-20250623002770
Original scientific paper

Iterative Decoupler Design Method for TITO Processes

Saša PRODANOVIĆ*, Ljubiša DUBONJIĆ, Janani RAJARAMAN

Abstract: In this paper, possibilities for iterative tuning of the decoupler for TITO (two inputs and two outputs) process are investigated. The developed algorithm is based on getting decouplers terms iteratively, taking into account previously defined limits of response quality indicators. The effectiveness of the designed new method has been presented in four examples using simulations. It enables similar performances as computational methods for decoupler design from the literature, but without knowing of process mathematical model. So, that is a less complicated and less time-consuming method, which can be used for adding a decoupler to an already controlled process during its functioning and also for improving terms of an existing decoupler.

Keywords: decoupling; iterative algorithm; two inputs-two outputs process

1 INTRODUCTION

Interactions between system control loops cause significant difficulties in achieving the desired object performance, and even its stability in some cases. A more comprehensive approach to the process (object), based on taking into account its multivariability, has encouraged many researchers to deal with the development of control algorithms that should compensate for the aforementioned interaction. In pursuit of this, several types of decoupler have been designed in the past few decades. Many of them are designed for TITO (two inputs and two outputs) systems, because many multivariable processes in industry can be successfully presented and discussed as a TITO process or as a set of TITO subsystems. The basic postulates on multivariable systems are contained in [1].

The decoupler, as part of a centralized control system, has to provide that its product with a process transfer function matrix becomes a diagonal matrix, and thus simplify the design of a multivariable controller. So, for example, one of the decoupler types, which introduces as little dynamics as possible into the control loop, was designed after process description as the first order plus dead time model, with the fact that the largest common dead time and the common pole were eliminated from the decoupler columns $D = \mathrm{adj}(G)$ [2]. There are authors who perform static decoupling $D = G^{-1}(0)$. After defining interaction indices and maximum SISO (single input-single output) sensitivities for both control loops, they design decentralized controllers using integral gain maximization [3]. Decoupling can also be achieved by applying internal model control (IMC). In [4] IMC is applied to control MIMO (multi input-multi output) system using Butterworth filter. This filter was also used for setting the PI controller based on the characteristic equation of the Smith predictor structure [5]. It is possible to design the decoupler in the form of its minimal C+ version using only the matrix of transfer functions and at the same time ensure the stability of the system by means of a unity feedback loop at the output [6]. The usual approach is to form a multivariable controller composed of a decoupler, which makes the system dominantly diagonal, and a decentralized controller [7]. A specific approach is described in [8]. Here, an ideal decoupler with integral action is designed under the condition that the transfer function matrix of the open loop control system is diagonal. Centralized control was achieved by approximating the decoupler with four PID controllers, while the required system properties were obtained by correcting the proportional gain. Centralized control for the TITO process with inverted decoupling was developed in [9]. Here, the open-loop transfer matrix terms are determined based on a defined stability margin (gain margin and phase margin) or based on the natural frequency and damping factor. The design of PI or PID decoupling control for multivariable $n \times n$ processes has shown to be effective for cases where $n = 2$, 3, and 4 with multiple delay times [10]. As a useful set of achieved results in a specific field, it is important to highlight the extensive presentation and analysis of inverted decoupling as well as its comparison with conventional decoupling in [11]. Inverted decoupling has remained relevant in recent times, too. In this regard, a decoupling PID controller was designed to control process variables using an inverted decoupler [12]. The effectiveness of the inverted decoupler in overcoming the interaction between variables has been demonstrated in the control of MIMO systems in the process industry, also in cases where a multivariable PID controller is designed based on internal model control (IMC) [13]. Inverted decoupling, within the centralized control of the boiler as a TITO process, is also possible to achieve through four regulators, with the use of direct compensators to reject the effects of disturbances [14]. In addition to the above, it is important to note a simple method for designing an ideal diagonal decoupler that uses the equivalent transfer function matrix obtained via the normalized integrated error (NIE) [15]. Furthermore, when it comes to tuning PID controllers, it is important to say that methods based on desired time response are still used today. One of them [16] relates to tuning the PID controller for given pulse response characteristics. The relevance of this approach was also illustrated in [17], where the PID controller was tuned based on the required system performance, taking into account robustness constraints in the design. In addition, there is a significant contribution in terms of designing low-order controllers for SISO and MIMO systems with time delays [18]. State of the art analysis is greatly facilitated by a review of the history of PID controllers and the directions for the future [19], as well as a comprehensive review of decoupling control methods, including observations that are useful for researchers and engineers [20].

Regardless of whether the decoupling is achieved using multiple controllers or via a decoupler and a decentralized (diagonal) controller, a common feature of all above-mentioned design methods is that a mathematical model of the process (object) is necessary for their application. The mentioned methods of decoupler design with the appropriately tuned regulators give satisfactory and, from research to research, better control system characteristics. However, emphasis is also placed on the simplicity of designing decouplers and controllers. Therefore, many researchers focus on iterative methods in designing control systems because their usage does not require knowledge of the mathematical model of the process (object). In this regard, the following is a review of research that has dealt with this issue. An iterative approach was applied to design a decentralized PID controller, while the decoupler was formed from the conditions of the diagonality of the generalized process [21]. The possibilities of the iterative method are also investigated in [22]. Namely, in this study, three classical methods for tuning the PID controller were compared with iterative method for various processes control. It has been illustrated that the iterative approach provides the same or better results with a simpler procedure based on defined requirements regarding the overshoot and settling time of responses. There are also good results achieved in [23], but they added rise time as a criterion. Another significant method for designing PID controllers of this type involves frequency response as a quality indicator [24]. The characteristics of the system in the frequency domain as constraints when implementing the iterative method of tuning PID controllers for multivariable processes have also been used in other research [25]. The applicability of the iterative approach for a TITO system has also been presented using method called correlation-based tuning, where some controller elements serve to satisfy the set response performance, while others achieve decoupling [26]. In addition, a contribution to the control of MIMO systems was made by introducing a method for self-tuning a centralized multivariable PID controller [27]. The iterative approach was improved by introducing the Fruit Fly algorithm for optimizing PID controller parameters [28]. The field of application of this approach was expanded after research on the uncertainty and disturbance estimator setting using iterative feedback tuning [29]. The great possibilities of this approach have been demonstrated by the development of an algorithm for iterative tuning of the beam feedforward controller [30].

The common feature of the above iterative methods is that none of them is used for designing only the decoupler. In this regard, the goal of the research presented in this paper is to obtain a decoupler for a process without knowing the process transfer function matrix, that operates with a previously designed controller, i.e., to add a decoupler to the control system. For this purpose, an iterative method based on the desired (given) characteristics (quality indicators) of the process response was applied. Of course, the process responses should be of the same or similar quality as in the previously mentioned methods. The effectiveness and applicability of this method were tested on four different processes, i.e., their mathematical models, which served as a substitute for real processes during simulations. One model was formed in [31], then two models were formed by modifying the first one, and another model was taken from the literature [32]. The approach suggested in this paper is intended to tune decouplers for processes such as: distillation columns, flow tanks, boilers, chemical reactors, dispensers, etc.

Section 2 defines the structure of the control system that includes the controller and provides guidelines for the iterative design method. Section 3 presents the procedure according to which the proposed method is applied. Testing of the method on representative process models and its comparison with other control systems is carried out in Section 4. Conclusions are given in Section 5.

2 PROBLEM DEFINITION

Problems in the decoupler design are mainly related to the shortcomings of the mathematical model of the process. Researchers are continuously investing energy in improving identification methods with the aim of taking into account the dominant process variables during mathematical modeling. However, due to unavoidable approximations and simplifications, unmodeled process dynamics are almost always present and cause errors in the decoupler's parameters. Furthermore, it is evident that the formation of the process transfer function matrix also prolongs the procedure for determining the decoupler's parameters. Different types of decouplers, such as ideal and simplified, direct and inverted, have been introduced to improve response characteristics and simplify the overall design of the control system. In the aforementioned methods, when it comes to the TITO process, the following process model is used, Eq. (1):

$$G(s)=\begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \tag{1}$$

where g_{ij} - are terms of the process transfer matrix $G(s)$.

A decoupler is formed for it and given by Eq. (2):

$$D(s)=\begin{bmatrix} d_{11}(s) & d_{12}(s) \\ d_{21}(s) & d_{22}(s) \end{bmatrix} \tag{2}$$

where d_{ij} - are terms of the decoupler $D(s)$.

The decoupling is achieved when the product $G(s)D(s)$ is a diagonal matrix. Therefore, Eq. (3) holds:

$$\begin{gathered} Q=\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}\times\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}= \\ =\begin{bmatrix} g_{11}d_{11}+g_{12}d_{21} & g_{11}d_{12}+g_{12}d_{22} \\ g_{21}d_{11}+g_{22}d_{21} & g_{21}d_{12}+g_{22}d_{22} \end{bmatrix}=\begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix} \end{gathered} \tag{3}$$

where q_1 and q_2 - are terms of the diagonal matrix Q.

In Eq. (3) and Figs. 1 and 2, the complex variable s is intentionally omitted for the sake of brevity. Fig. 1 shows the structural diagram of the multivariable automatic control system considered in this paper.

In Figs. 1 and 2, the meaning of the labels is as follows: r_1 and r_2 - reference values, e_1 and e_2 - error signals, v_1 and

v_2 - controller outputs, u_1 and u_2 - manipulative values, y_1 and y_2 - process responses.

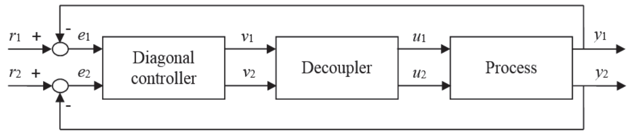

Figure 1 Control strategy: decoupler and decentralized (diagonal) controller [33]

In this approach, centralized control is achieved using a decoupler and a decentralized (diagonal) controller. A more detailed presentation of this control strategy is given in Fig. 2.

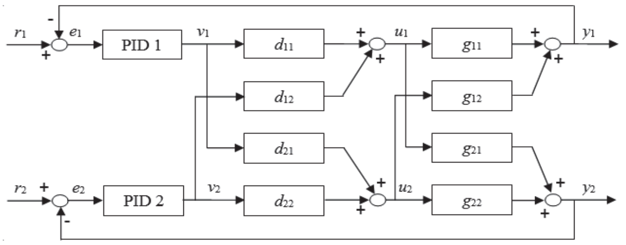

Figure 2 Centralized control using decoupler and decentralized PID controller [8]

Since in this paper the decoupler is tuned using an iterative method, it is important to observe the following guidelines:

- The decoupler terms will in most cases contain only gain. In other words, they will be of zero order and proportional type of action. This leads to their simpler design and the intention of introducing as little dynamics as possible into the control system.
- For pre-tuned PID controllers, the parameters of the decoupler finaly must provide better process responses from iteration to iteration, i.e. they must converge to the final value. This approach is important because it prevents the process from possibly becoming unstable. This allows the proposed decoupler tuning method to be applicable during the operation of the process (plant), i.e. not to be limited only to its mathematical model.
- From the aspect of response quality, this principle of decoupler design is based on fulfilling the set condition. In other words, for a given quantitative limit of one of the response quality indicators, the other indicators should be brought to the best possible level.
- If it happens that any indicator stops tending towards the defined (set) value during more iterations and in unacceptable amount, the operator must interrupt the tuning and correct the set conditions (limit values) of the corresponding response indicators.

3 DECOUPLER DESIGN

According to previous experience, it is evident that in the multivariable processes control, the use of a decoupler enables significantly better system behavior than a control strategy without it. To ensure comparability in both cases, the PID controller parameters should be the same, i.e., the process must be controlled by an identical controller. This design method is planned to be implemented on a real process in real time with both control loops closed, but its testing and validation in this research were carried out using simulations. As a precondition, it is necessary to achieve a computer-controlled process, which is increasingly the case today. Furthermore, when carrying out iterative decoupler design, it is necessary to simulate the functioning of the entire control system. Therefore, in this research, in fact, all components except the control system (computer) were replaced by their mathematical models so that the entire system could be represented in the computer. The reliability and accuracy of the used mathematical models of the process have been illustrated several times in earlier research. Of the four variants of the decoupler presented in [33], the second one, Eq. (4) was chosen, in which it is necessary to tune the two off-diagonal terms $d_{12}(s)$ and $d_{21}(s)$, while the diagonal terms are ones. Based on this, we have:

$$D(s)=\begin{bmatrix} 1 & d_{12}(s) \\ d_{21}(s) & 1 \end{bmatrix}=\begin{bmatrix} 1 & \dfrac{-g_{12}(s)}{g_{11}(s)} \\ \dfrac{-g_{21}(s)}{g_{22}(s)} & 1 \end{bmatrix} \tag{4}$$

When the process control system is designed without a decoupler, following the configuration as in Fig. 2, at the beginning of the tuning, the values of the decoupler terms should be $d_{12}(s) = 0$ and $d_{21}(s) = 0$, and the terms $d_{11}(s) = 1$ and $d_{22}(s) = 1$. The incremental change in the gain of the decoupler terms $d_{12}(s) = 0$ and $d_{21}(s) = 0$ can be in a positive or negative direction. That depends on the gain value for which we obtain better response quality indicators, which are collected in each iteration and compared with the desired (defined) one. Therefore, the random number method is not applicable for real-time tuning, i.e., during system operation, while it can be applied for decoupler design using simulations on a known process model. This also applies to the well-known bisection method. In general, no method can be applied that, in the first iteration, uses values of the decoupler terms (gains) significantly different from those found in the control unit. More precisely, values significantly different from zero cannot be taken in the case of the initial introduction of the decoupler or from some other current values that need to be corrected in the already present decoupler. This ensures that at the beginning of the tuning, the current response does not change significantly, because that change in the first iteration could be to the side that makes the response significantly worse. Since the goal is to tune the decoupler as precisely as possible, a larger increment is used in the first part of the adjustment (to save time), and a smaller one in the second part to make a finer tuning. Therefore, the tuning can be made on a real system (in exploitation or laboratory) or on its mathematical model. The iterative method proposed in this paper, as mentioned above, can also be applied for additional tuning of an existing decoupler if it does not provide satisfactory decoupling. In that case, it is necessary to start with the current values of the terms $d_{12}(s)$ and $d_{21}(s)$.

In this paper, the proposed method, i.e., the procedure of tuning the decoupler, is explained for the case when the control system does not contain a decoupler and it needs to be added. The procedure consists of the next steps:

1. Creating a configuration for centralized control in the computer (control system) as in Fig. 2.
2. Defining and entering the appropriate data into the program:
- Setting the decoupler terms to zero value, $d_{12}(s) = 0$ and $d_{21}(s) = 0$.
- Determining the response quality indicator whose value is the least acceptable and defining its limit value.
- Determining the increment size in the first part for coarse and the second part for fine tuning the decoupler terms.
- Defining other response quality indicators, which also serve to tune decoupler terms and determine their limit values.
3. If the set combination of conditions (limit values of response quality indicators) is not achievable (i.e., if any of the response parameters starts to increase indefinitely for more iterations), it is necessary to return to step 2 and correct them.

An example of a flowchart based on which iterative tuning of the decoupler for the TITO process was programmed is given in the Fig. 3.

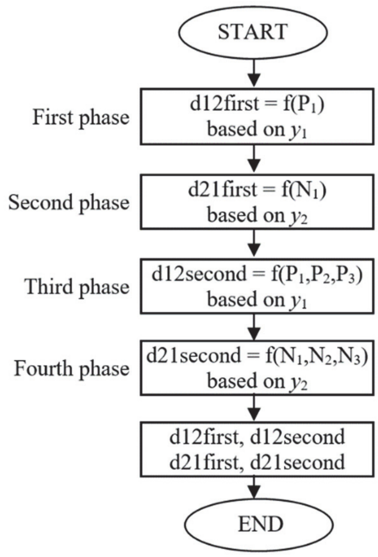

Figure 3 Flowchart for decoupler design

It is important to note that the output variables are tuned alternately in the program. Namely, the first response quality indicator P_1 is set as a dominant condition (constraint) in the first phase of the program for the first output variable (response), and then the same type of quality indicator N_1 is set for the second output variable in the second phase. In the third and fourth phases of the program, other quality indicators of interest (P_2, P_3 and N_2, N_3) are also set as conditions, and they serve for additional (fine) tuning of the decoupler terms. As a response quality indicators, the following can be considered: overshoot, rise time, settling time, static error or some others depending on operators' requests. Their limits are determined based on heuristic, i.e. in accordance with the system dynamic behavior that is desired to be achieved.

In order to more clearly illustrate the previously presented iterative method, Fig. 3 shows its flowchart, while Fig. 4 shows its more precise representation for the first phase of tuning. The meaning of the labels in the aforementioned flowcharts is as follows: P_1 - parameter of response y_1, which is set as a dominant condition; P_2 and P_3 - parameters of response y_1, which are set as additional conditions; N_1 - parameter of response y_2, which is set as a dominant condition; N_2 and N_3 - parameters of response y_2, which are set as additional conditions; *inkr* - value of the increment of the decoupler terms (in the considered examples in the first and second phase, it was taken that *inkr* = 0,1 , and in the third and fourth phases it was taken that *inkr* = 0,01); $P_{1\ 0}$ - response parameter value at the beginning of the tuning (without decoupler); $P_{1\ \mathrm{inkr}}$ - response parameter value at certain increment values; $P_{1\ \max}$ - limit value of the response parameter; d12first - value of the decoupler term d_{12} after the first tuning phase; d21first - value of the decoupler term d_{21} after the second tuning phase; d12second - value of the decoupler term d_{12} after the third tuning phase; d21second - value of the decoupler term d_{21} after the fourth tuning phase.

It is important to emphasize that, according to the block diagram in Fig. 2, the decoupler term d_{12} is tuned based on the parameters (P_1, P_2 and P_3) of the response y_1, and the decoupler term d_{21} is tuned based on the parameters (N_1, N_2 and N_3) of the response y_2.

Fig. 3 shows that the method is carried out in four phases. The first of them is shown in detail in Fig. 4.

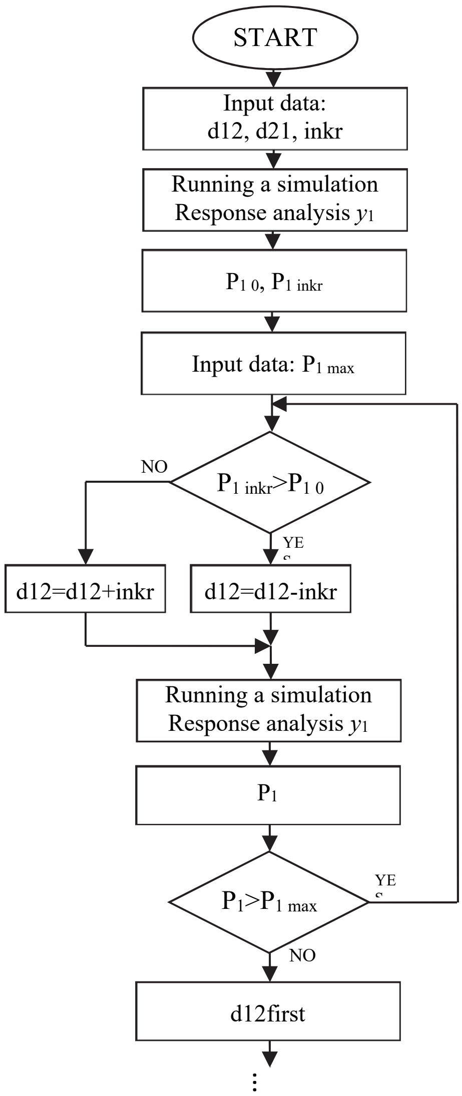

Figure 4 First phase of the flowchart for decoupler design

It is understood that the presented tuning algorithm must be adapted to the software in the control unit (computer) that controls the process in order to perform step 1, i.e., it is necessary to form the configuration as in Fig. 2.

The advantages of the proposed iterative method for decoupler tuning (design), whose effectiveness has been presented in this research, are:

- A mathematical model of the process is not needed. Therefore, there are no obstacles due to model errors.
- The possibility of setting a limit for the values of the response quality indicator so that a compromise can be purposefully made between them. This extends its applicability in addition to real systems (online) and to designing exclusively using simulations on the mathematical model of the process (offline), because as previously explained, it gives the possibility to predetermine the mentioned indicators that are of interest in a specific case.
- Time reduction and automation of the tuning (design) process.

The limitation of this method is that for its online applicability it is necessary that the controlled process has several consecutive transient states and steady states in its functioning. Therefore, in each iteration, an analysis of the response is required, i.e. its quality indicators in which the user is interested. This restriction does not apply to offline setup.

4 EXAMPLES

Testing of the proposed decoupler design method was carried out numerically, i.e., by simulating the functioning of the entire control system. Therefore, the difference between this approach and the real conditions is that the considered processes are represented by their mathematical model (transfer function matrices). Four examples are tested in setpoint tracking and analysed below. For this purpose, verified mathematical models of the process were taken in order to satisfy the reliability of the research.

4.1 Example 1

A flow tank that has been modelled in [31] was taken and is given by Eq. (5). Water is supplied to this tank via two valves at different temperatures of 15 °C and 70 °C, and is drained periodically via an on/off valve. Such flow tanks in which two liquids are mixed are very often used in industry. The level with a setpoint of 1 m and temperature with a setpoint of 30 °C are the outputs of the control system, or more precisely, outputs of the process. In order to simplify the model, both components were assumed to be water and the mathematical model was formed based on generally known physical laws.

$$G(s)=\begin{bmatrix}\dfrac{0,01}{63s+1} & \dfrac{0,01}{63s+1}\\ \dfrac{-0,15}{10s+1}\mathrm{e}^{-3s} & \dfrac{0,4}{10s+1}\mathrm{e}^{-2s}\end{bmatrix} \tag{5}$$

For the considered process, PI controllers in the following form $G(s) = K_P+K_I/s$ were designed using the method in [34]. The first regulator is $K_P = 2{,}875$ and $K_I = 0{,}05096$, and the second one is $K_P = 0{,}125$ and $K_I = 0{,}01096$. For the process presented by Eq. (5), a decoupler Eq. (4) was designed based on the computational method [33], with parameters $d_{12}(s) = -1$ and $d_{21}(s) = 0{,}375\cdot\mathrm{e}^{-s}$. The need to introduce a decoupler into the control system of process Eq. (5) was investigated in [35], where a significant level of coupling was shown. It is important to note that in this research, in all variants of the decoupler design, for comparability, the same regulator was applied.

There are no restrictions on the process nature if the tuning is done exclusively using simulations (offline). However, for the feasibility of this decoupler designing method, for the case of a flow tank during exploitation (online), it is necessary that it has more frequent filling and emptying in its operation (e.g. at certain time intervals). One example of such a process could be a dispenser.

For the following set conditions:

- for level response $Oh < 0{,}23$ %, $Tuh < 104$ s, $Tsh < 171$ s,
- for temperature response $Ot < 0{,}002$ %, $Tut < 5{,}6$ s, $Tst < 23$ s.

By applying the considered iterative method, the following decoupler parameters were obtained $d_{12} = -1{,}07$ $d_{21} = 0{,}42$. Where are: O - overshoot, Tu - rise time, Ts - settling time, while the labels h and t refer to level and temperature, respectively.

The limits of the response quality indicators are determined by taking into account the existing process responses in order to have some guidance to obtain better responses by introducing a decoupler. In the considered example, the goal was to minimize overshoot, while at the same time, the rise time and the settling time have the best (lowest) achievable value. For this occasion, process response analysis with a computationally derived decoupler was used to show that a similar response quality can be obtained using an iterative method. It is also important to say that, first, individual settings of the decoupler parameters are made that dominantly affect the corresponding outputs, and then the parameter values are obtained $d_{12}(s)$ and $d_{21}(s)$. The process responses (level and temperature) obtained in each phase (step) of decoupler tuning are shown in Fig. 5. In order to more precisely observe the difference between the lines, which represent the responses, an enlarged part of the diagram from Fig. 5 is shown in Fig. 6.

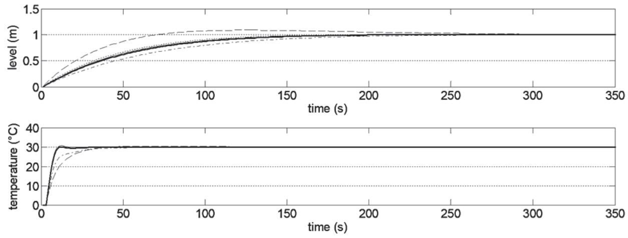

Figure 5 Process responses (level and temperature) in each phase of decoupler tuning (example 1)

Here, the corresponding decoupler terms in those phases are also given. Of course, the legend of Fig. 6 applies to Fig. 5 as well, with the reference (desired) value being marked with R. The responses in Figs. 5 and 6 from phase to phase of the tuning have better characteristics. More precisely, the basic requirement that the overshot at both outputs be insignificant is met. The efficiency of the decoupler tuned using a suggested iterative method is demonstrated by a comparative display of the process responses: without a decoupler, with a computationally designed decoupler, and with this iteratively designed (tuned) decoupler in Fig. 7.

Fig. 7 shows that the responses obtained by the iterative method are very close to the responses obtained by the computational method according to [33]. This, as already mentioned, is one of the main goals of the investigated and proposed method. More precisely, the difference in level is imperceptible, and in temperature it is still slightly noticeable. Those minor differences are illustrated by the response quality indicators and are given in Tab. 1.

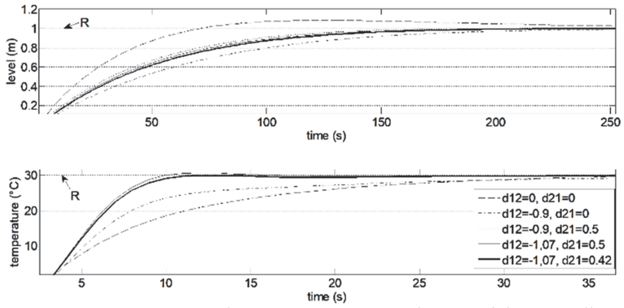

Figure 6 Enlarged view of the process responses from Fig. 3 (example 1)

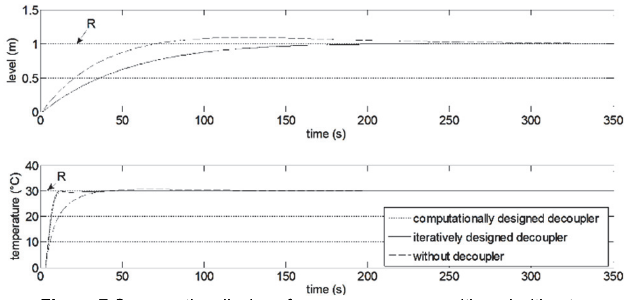

Figure 7 Comparative display of process responses with and without a decoupler (example 1)

In Tab. 1, all times are expressed in seconds, and overshoots are expressed in percentages. Considering the values in this table, it is evident that the decoupler designed using the presented iterative method provides satisfactory process responses, and moreover, some parameters, such as the rise time and settling time for temperature, are significantly better compared to the control system containing a computationally designed decoupler.

In order to enhance clarification, primary time response parameters (dominant conditions) P_1 (overshoot in level *Oh*) and N_1 (overshoot in temperature *Ot*), during the iterations are given in Fig. 8 and 9, respectively.

Table 1 Comparative display of response quality indicators (level and temperature), for example 1

Controller parameters		LEVEL	TEMPERATURE
	Defined conditions	*Oh* < 0,23, *Tuh* < 104, *Tsh* < 171	*Ot* < 0,002, *Tut* < 5,6, *Tst* < 23
PI controller 1 $K_P = 2,875$ $K_I = 0,05096$ PI controller 2 $K_P = 0,125$ $K_I = 0,01096$	Without decoupler $d_{12} = 0$; $d_{21}=0$	Rise Time *Tuh*: 49,39 Settling Time *Tsh*: 266,94 Overshoot *Oh*: 8,72	Rise Time *Tut*: 18,77 Settling Time *Tst*: 33,39 Overshoot *Ot*: 1,71
	Computationally designed decoupler $d_{12} = -1$; $d_{21} = 0,375 \cdot e^{-s}$	Rise Time *Tuh*: 102,66 Settling Time *Tsh*: 166,43 Overshoot *Oh*: 0,23	Rise Time *Tut*: 5,43 Settling Time *Tst*: 18,31 Overshoot *Ot*: $8,42 \cdot 10^{-11}$
	Iteratively designed decoupler $d_{12} = -1,07$; $d_{21} = 0,42$	Rise Time *Tuh*: 103,13 Settling Time *Tsh*: 168,12 Overshoot *Oh*: 0,19	Rise Time *Tut*: 5,08 Settling Time *Tst*: 9,44 Overshoot *Ot*: $2,25 \cdot 10^{-06}$

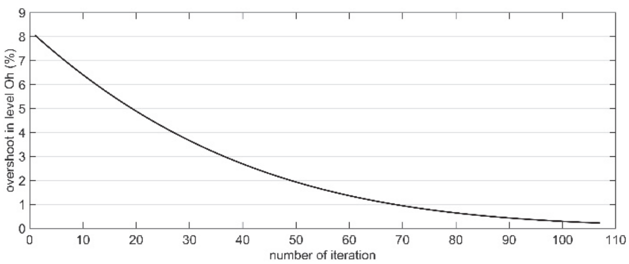

Figure 8 Dominant condition P_1 (*Oh*) during the iterations

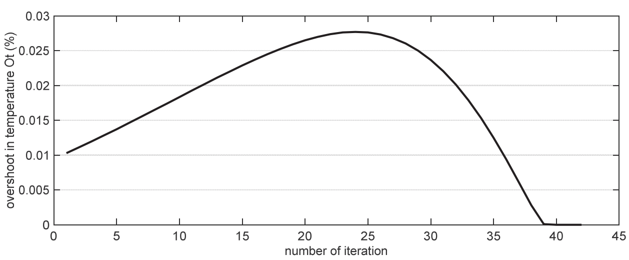

Figure 9 Dominant condition N_1 (*Ot*) during the iterations

Here it is noticeable that the convergation of both dominant conditions is good, with one significance being that the overshoot in *Ot* is a little bit (negligibly) rising up to the 24th iteration, and after that rapidly decreases to a set limit.

4.2 Example 2

This example, as well as the next one, was taken into consideration to display the effectiveness of the proposed iterative method in managing different objects. The aim is to introduce generality and show the applicability of the suggested method to various processes whose inputs and outputs are not flows, levels or temperatures, as in example 1. Setpoint is 1 for output y_1 and 30 for output y_2. The process model is given by Eq. (6).

PI regulators are taken the same as in example 1. So, according to [34], parameters of the first controller are K_P = 2,875 and K_I = 0,05096, and the second controller K_P = 0,125 and K_I = 0,01096. This aims to examine the effectiveness of the proposed method in the presence of model uncertainty or errors because, as can be seen in the transfer matrix Eq. (6), the off-diagonal elements have been changed compared to the transfer matrix Eq. (5). Respecting the order of investigation, the decoupler was designed using the calculation method according to Eq. (4), as in the previous example. The following values of its terms were obtained: $d_{12}(s) = -(63s + 1)/(30s + 1)$; $d_{21}(s) = 0{,}625 \cdot e^{-4s}$. After the simulations were performed, the terms of the decoupler were obtained using the iterative method, and their values are: $d_{12}(s) = -0{,}16$ and $d_{21}(s) = 0{,}62$. Both process responses y_1 and y_2 as well as the corresponding decoupler terms are shown in Fig. 10 for decoupler tuning phases during the application of the iterative method.

$$G(s) = \begin{bmatrix} \dfrac{0{,}01}{63s+1} & \dfrac{0{,}01}{30s+1} \\ \dfrac{-0{,}25}{10s+1}e^{-6s} & \dfrac{0{,}4}{10s+1}e^{-2s} \end{bmatrix} \qquad (6)$$

Based on the responses in Fig. 10, it is evident that there are no obstacles to online decoupler design, because from phase to phase the system works better and better.

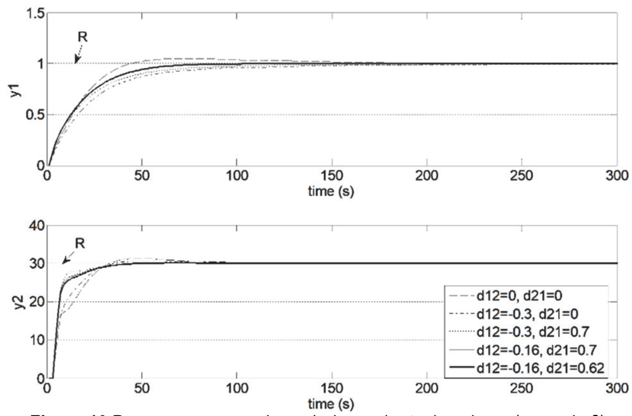

Figure 10 Process responses in each decoupler tuning phase (example 2)

After that, it is very important to compare the responses of three control system cases: without a decoupler, with a computationally designed decoupler, and with an iteratively designed decoupler, which are shown in Fig. 11. This figure shows that for the process described by model Eq. (6), the application of the decoupler obtained by the computational method according to [33] based on Eq. (4) does not provide a good response y_2. Of course, this does not mean that there are no other computational methods that successfully overcome this problem. Moreover, many papers have displayed that if the mathematical model of the process is known and the decoupler is designed computationally, no method is universal and that the most appropriate method should be chosen for each specific process. It is important to note again that in this study, the calculation of the decoupler according to Eq. (4) was taken for all examples. The shortcomings of computationally designed decouplers were not addressed in this research because, as previously stated, efforts were focused on achieving good response characteristics using an iteratively designed decoupler. Having that in mind, it is obvious from Fig. 11 that, for the defined conditions (limit values) for overshoot, rise time and settling time, the iteratively designed decoupler fulfills its goal, that is, in other words, provides good processes responses y_1 and y_2.

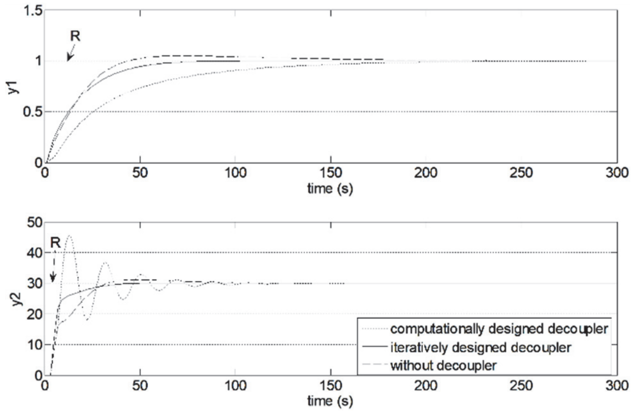

Figure 11 Comparative display of process responses with and without a decoupler (example 2)

4.3 Example 3

Mathematical model of the process is given by Eq. (7). This time the changes were introduced in the diagonal members of the process model in relation to the process Eq. (5).

$$G(s) = \begin{bmatrix} \dfrac{0{,}02}{63s+1} & \dfrac{0{,}01}{63s+1} \\ \dfrac{-0{,}15}{10s+1}e^{-3s} & \dfrac{0{,}4}{10s+1}e^{-7s} \end{bmatrix} \qquad (7)$$

PI controllers are designed by Dalin's λ method in the following form $G(s) = K_P + K_I/s$. Their parameters are: first controller $K_P = 1{,}4375$ and $K_I = 0{,}02548$, and second controller $K_P = 0{,}049$ and $K_I = 0{,}0049$. The decoupler is also designed using computational method according to the Eq. (4), as well as in previous examples. Its terms are: $d_{12}(s) = -0{,}5$ and $d_{21}(s) = 0{,}375 \cdot e^{4s}$. The proposed iterative method for decoupler design gives the following parameters: $d_{12}(s) = -0{,}44$ and $d_{21}(s) = 0{,}11$. After the simulations of the iterative procedure, Fig. 12 shows the responses and corresponding decoupler terms in all tuning phases.

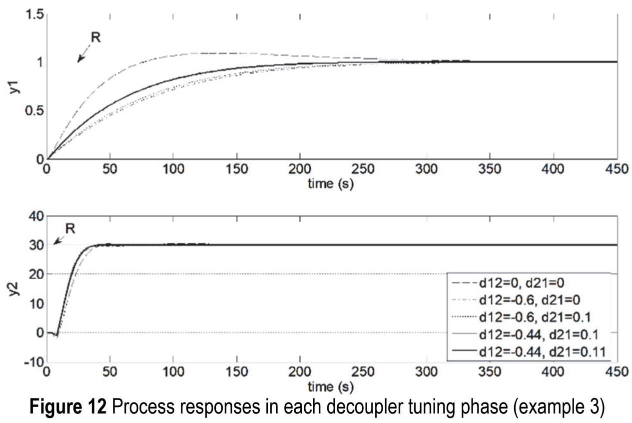

Figure 12 Process responses in each decoupler tuning phase (example 3)

A comparative display of both responses y1 and y2 for three cases: without a decoupler, with a computationally designed decoupler, and with an iteratively designed decoupler is given in Fig. 13. It is noted that the computationally designed decoupler does not give good results for the output y_2. The reason is explained in the previous example.

The iteratively designed decoupler satisfies the set conditions of reducing (minimizing) the overshoot, while respecting limitations in rise time and settling time. That is also obvious in Fig. 13.

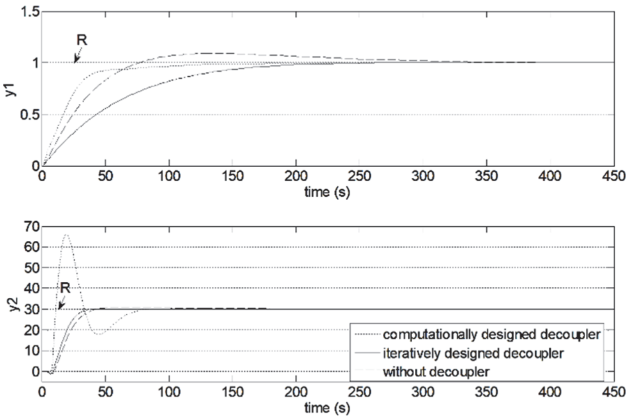

Figure 13 Comparative display of process responses with and without a decoupler (example 3)

4.4 Example 4

A binary distillation column (water-methanol) [32] is described by Eq. (8).

$$G(s)=\begin{bmatrix} \dfrac{12,8}{16.7s+1}\mathrm{e}^{-s} & \dfrac{-18,9}{21s+1}\mathrm{e}^{-3s} \\ \dfrac{6,6}{10.9s+1}\mathrm{e}^{-7s} & \dfrac{-19,4}{14.4s+1}\mathrm{e}^{-3s} \end{bmatrix} \tag{8}$$

For this process, PID controllers in the following form $G(s) = K_P + K_I/s + K_D{\cdot}s$ were designed according to [21]. And that is the first controller $K_P = 0{,}216$, $K_I = 0{,}0757$ and $K_D = 0{,}0174$, and the second one $K_P = -0{,}0675$, $K_I = -0{,}0192$ and $K_D = -0{,}0634$. The decoupler was first designed using the computational method, according to the Eq. (4). Its terms are given in Eq. (9).

$$\begin{aligned} d_{12}(s) &= 1{,}47\frac{16{,}7s+1}{21s+1}\mathrm{e}^{-2s} \\ d_{21}(s) &= 0{,}34\frac{14{,}4s+1}{10{,}9s+1}\mathrm{e}^{-4s} \end{aligned} \tag{9}$$

Then, the decoupler was tuned using an iterative method and the following parameters were obtained: $d_{12}(s) = 1{,}74$ and $d_{21}(s) = 0{,}32$. By analogy with the previous examples, Fig. 14 shows the process responses and corresponding terms of the decoupler according to the phases of its tuning during the implementation of the iterative method. The responses obtained in the last phase of the iterative design method are good. Responses obtained in some previous phases of tuning show that the system is not stable, which is unacceptable in online design. This means that for this process it is possible to apply only an offline procedure using simulations on a mathematical model of the process. In such cases, the possibility remains to determine in advance the aforementioned response indicators that are of interest for specific process control. Furthermore, in this example, the tuning process needs to be repeated several times until the appropriate delay times in the decoupler members are obtained. So in the decoupler, which is used for process Eq. (8), the delay times are: 2 seconds for d_{12}, and 4 seconds for d_{21}. This does not complicate the situation in this case, because the proposed iterative method for process Eq. (8) can anyhow only be carried out offline.

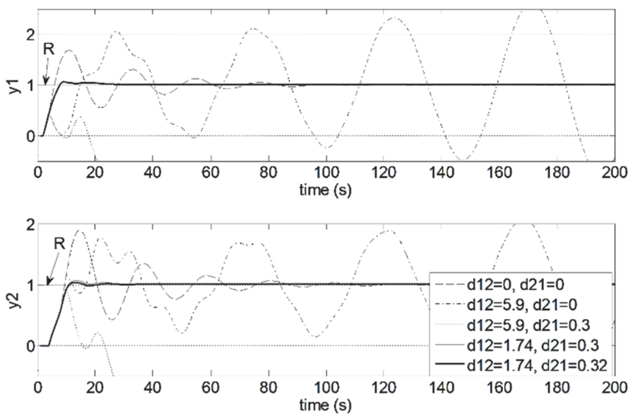

Figure 14 Process responses in each decoupler tuning phase (example 4)

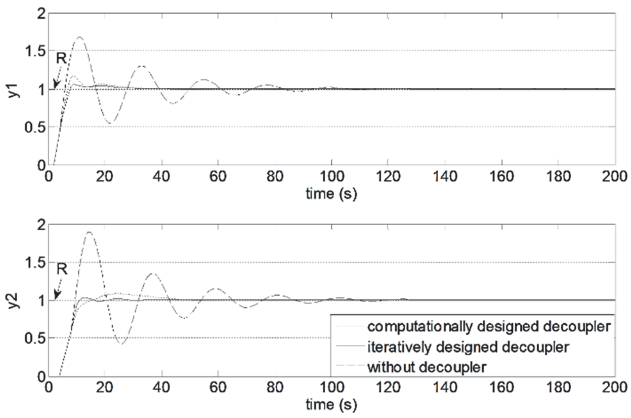

Figure 15 Comparative display of process responses with and without a decoupler (example 4)

The responses for three control cases: without a decoupler, with a computationally designed decoupler, and with an iteratively designed decoupler are given in Fig. 15. In this example, the iteratively tuned decoupler provides better system performance than the computationally designed decoupler. This is illustrated by adequate fulfillment of the defined limit values of the overshoot with optimal rise time and settling time.

5 CONCLUSION

The ever-present problem of TITO processes decoupling was solved in this research using an iterative approach. Its application eliminates the need to know a mathematical model for a large group of linear TITO processes with and without time delay, which is necessary for many other methods. This method is suitable for making a compromise between individual response quality indicators, i.e., setting and fulfilling limit values for those indicators that the user defines as the most important, while not allowing other parameters to go beyond acceptable and also defined limits. The effectiveness of two ways of applying the proposed method has been illustrated. The first way, in the case of systems that have several consecutive identical periods in their functioning and in the case of systems where the functioning is not disturbed by step changes in the reference (desired) value. For them, the decouplers can be designed during their functioning (online). This is shown in the first three examples. Another way, for systems that are not of the above-mentioned nature and that can enter the area of instability in the intermediate phases of decoupler design. For this kind of systems, decouplers can only be designed (offline). Since the offline tuning method requires knowledge of the mathematical model, it is important to note that the advantages related to tuning speed are lost, but the possibility of targeted achievement of response quality indicators is retained. This method of application is shown in the fourth example. As a measure of the quality of the obtained responses, the responses of the process obtained by the application of control with the computationally obtained decoupler were used, because their efficiency was researched in earlier research. The focus is on iterative tuning of the decoupler, which will give the same or similar good process responses as computational methods. Then the main advantages come into play, which are reflected in the tuning without knowledge of the mathematical model of the process, thereby simplifying the design procedure and reducing time consumption, and this has been presented in this paper. The specificity and advantage of the proposed method are also reflected in the possibility of special tuning of the decoupler for processes that are already controlled by PID controllers or their shorter versions, i.e. subsequent decoupling of the process, as well as for improving the current values of existing decouplers, all with the aim of improving the responses characteristics. When examining both of the aforementioned ways of applying this method, the functioning of the process was simulated. In the first case (online), the transfer functions play the role of the real process to which this method is planned to be applied in practice, while in the second case (offline), tuning is carried out on a mathematical model of process. That is why the choice of mathematical models in the four considered examples was of great importance, so the first three were formed using generally known physical laws, while the fourth one was taken from the literature, having in mind that it has been used very often in research so far. In accordance with the above, it is understood that the proposed iterative approach to decoupler tuning can also be applied in the case when a mathematical model of the process is obtained using one of the identification methods. Future research may focus on iterative tuning of the decoupler, whose terms do not contain only gains, and on investigation of the robustness of the proposed method to disturbance (perform sensitivity analyses), taking into account actuator limits and stability margins. It would also be useful to check the effectiveness of fractional order PID controllers in combination with iteratively designed decoupler. Moreover, a natural extension of the presented research should be the consideration and overcoming of theoretical and computational barriers for the application of the suggested iterative method to the MIMO systems. That would certainly lead to a more complex algorithm for iterative tuning of the decoupler.

6 REFERENCES

[1] Skogestad, S. & Postlethwaite, I. (2005). *Multivariable Feedback Control: Analysis and Design, 2nd ed.* John Wiley & Sons, Chichester.

[2] Nordfeldt, P. & Hägglund, T. (2006). Decoupler and PID controller design of TITO systems. *Journal of Process Control*, *16*, 923-936. https://doi.org/10.1016/j.jprocont.2006.06.002

[3] Åström, K. J., Johansson, K. H., & Wang, Q. G. (2002) Design of decoupled PI controller for two-by-two systems. *IEE Proceedings on Control Theory and Applications*, *149*, 74-81. https://doi.org/10.1016/S0009-2509(01)00365-7

[4] Wang, H., Zhu, Y-q., & Chen, J. (2017). A design method of decoupling IMC controller for multi-variable system based on Butterworth filter. *American Control Conference*, 5714-5719. https://doi.org/10.23919/ACC.2017.7963845

[5] Huang, C., Gui, W-h., Yang, C-h., & Xie, Y-f. (2011). Design of decoupling Smith control for multivariable system with time delays. *Journal of Central South University of Technology*, *18*, 473-478. https://doi.org/10.1007/s11771-011-0720-8

[6] Wang, Q. G. & Yang, Y. (2002). Transfer function matrix approach to decoupling problem with stability. *Systems & Control Letters*, *47*, 103-110. https://doi.org/10.1016/S0167-6911(02)00179-2

[7] Jevtović, B. T. & Mataušek, M. R. (2010) PID controller design of TITO system based on ideal decoupler. *Journal of Process Control*, *20*, 869-876. https://doi.org/10.1016/j.jprocont.2010.05.006

[8] Morilla, F., Vázquez, F., & Garrido, J. (2008). Centralized PID control by decoupling for TITO processes. *Proceedings of the 13th IEEE International Conference on Emerging Technologies and Factory Automation*, 1318-1325. https://doi.org/10.1109/ETFA.2008.4638570

[9] Garrido, J., Vázquez, F., & Morilla, F. (2010). Centralized inverted decoupling for TITO processes, *Proceedings of the 15th IEEE International Conference on Emerging Technologies and Factory Automation*, 1-8. https://doi.org/10.1109/ETFA.2010.5641287

[10] Vu, T. N. L. & Lee, M. (2013). An extended method of simplified decoupling for multivariable processes with multiple time delays. *Journal of Chemical Engineering of Japan*, *46*(4), 279-293. https://doi.org/10.1252/jcej.12we213

[11] Garrido, J., Vázquez, F., Morilla, F., & Hägglund, T. (2010). Practical advantages of inverted decoupling. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, *225*, 977-992. https://doi.org/10.1177/2041304110394556

[12] Üstüner, M. A. & Taşkin, S. (2019). Inverted decoupling PID controller design for a MIMO system. *Sigma Journal of Engineering and Natural Sciences*, *37*(4), 1139-1151.

[13] Azizah, Z., Sutikno, J.P., Handogo, R., & Hikmadiyar, R. A. (2020). Inverted decoupling MIMO internal model control using Mp tuning. *IOP Conference Series: Materials Science and Engineering*, *736*(022101), 1-14. https://doi.org/10.1088/1757-899X/736/2/022101

[14] Garrido, J., Vázquez, F., & Morilla, F. (2012). Multivariable PID control by inverted decoupling: Application to the benchmark PID 2012. *IFAC Conference on Advances in PID Control PID'12.* https://doi.org/10.3182/20120328-3-IT-3014.00060

[15] Cai, W. J., Ni, W., He, M. J., & Ni, C. Y. (2008). Normalized decoupling - a new approach for MIMO process control system design. *Industrial and Engineering Chemistry Research*, *47*, 7347-7356. https://doi.org/10.1021/ie8006165

[16] Lee, J. G., Jeong, J., & Kim, Y. C. (2024). A novel PID controller tuning for desired time response using a pulse response. *Journal of Electrical Engineering and Technology*, *19*, 1861-1879. https://doi.org/10.1007/s42835-023-01673-y

[17] Cammann, L. & Jäschke, J. (2024). Robust PID controller tuning using bilevel optimization. *4th IFAC Conference on Advances in Proportional-Integral-Derivative Control*, 382-387. https://doi.org/10.1016/j.ifacol.2024.08.081

[18] Yu, X., Ding, P., Ou, L., & Zhang, W. (2024). General stabilization approach of the low-order controller for both SISO systems and MIMO systems with multiple time delays. *Automatica*, *166*(111639), 1-9. https://doi.org/10.1016/j.automatica.2024.111639

[19] Hägglund, T., & Guzmán, J. L. (2024). Give us PID controllers and we can control the world. *IFAC Proceedings*, *2024*, 99-104. https://doi.org/10.1016/j.ifacol.2024.08.018

[20] Liu, L., Tian, S., Xue, D., Zhang, T., Chen, Y. Q., & Zhang, S. (2019). A review of industrial MIMO decoupling control. *International Journal of Control, Automation and Systems*, *17*, 1246-1254. https://doi.org/10.1007/s12555-018-0367-4

[21] Wang, Q. G., Huang, B., & Guo, X. (2000). Auto-tuning of TITO decoupling controllers from step tests. *ISA Transactions*, *39*, 407-418. https://doi.org/10.1016/S0019-0578(00)00028-8

[22] Lequin, O., Gevers, M., Mossberg, M., Bosmans, E., & Triest, L. (2003). Iterative feedback tuning of PID parameters: comparison with classical tuning rules. *Control Engineering Practice*, *11*, 1023-1033. https://doi.org/10.1016/S0967-0661(02)00303-9

[23] Xu, J-X., Huang, D., & Pindi, S. (2008). Optimal tuning of PID parameters using iterative learning approach. *SICE Journal of Control, Measurement, and System Integration*, *1*(2), 143-154. https://doi.org/10.9746/jcmsi.1.143

[24] Vázquez, F., Morilla, F., & Dormido, S. (1999). An iterative method for tuning decentralized PID controllers. *Proceedings of the 14th IFAC World Congress*, 1501-1506. https://doi.org/10.1016/S1474-6670(17)56254-0

[25] Garrido, J., Ruz, M. L., Morilla, F. & Vázquez, F. (2021). Iterative method for tuning multiloop PID controllers based on single loop robustness specifications in the frequency domain. *Processes 2021*, *9*(140), 1-25. https://doi.org/10.3390/pr9010140

[26] Mišković, L., Karimia, A., Bonvina, D., & Gevers, M. (2007). Correlation-based tuning of decoupling multivariable controllers. *Automatica*, *43*, 1481-1494. https://doi.org/10.1016/j.automatica.2007.02.006

[27] Sommer, S. & Kienle, A. (2012). Auto-tuning of multivariable PID controllers using iterative feedback tuning. *Automatisierungstechnik*, *60*(1), 20-27. https://doi.org/10.1524/auto.2012.0969

[28] Han, J., Wang, P., & Yang, X. (2012). Tuning of PID controller based on fruit fly optimization algorithm. *Proceedings of 2012 IEEE International Conference on Mechatronics and Automation*, 409-413. https://doi.org/10.1109/ICMA.2012.6282878

[29] Zhang, Y., Sun, L., Shen, J., Lee, K. Y., & Zhong, Q-C. (2019). Iterative tuning of modified uncertainty and disturbance estimator for time-delay processes: A data-driven approach. *ISA Transactions*, *84*, 164-177. https://doi.org/10.1016/j.isatra.2018.08.028

[30] Kwon, S., Archuleta, A., Castellano, L., Prokop, M., Van Rooy, P., Rose, C. R., & Torrez, P. (2022). Iterative tuning of the beam feedforward controller for LANSCE LINAC digital low level RF control system. *Proceedings, 5th North American Particle Accelerator Conference, NAPAC2022*, 475-477.

[31] Prodanović, S. Lj., Nedić, N. N., & Brašić, V. S. (2014). Some considerations of mutual coupling in multivariable processes. *XII International SAUM Conference on Systems, Automatic Control and Measurements*, 257-260.

[32] Wood, R. K. & Berry, M. W. (1973). Terminal composition control of binary distillation column. *Chemical Engineering Science*, *28*, 1707-1717. https://doi.org/10.1016/0009-2509(73)80025-9

[33] Morilla, F., Garrido, J., & Vázquez, F. (2013). Control multivariable por desacoplo. *Revista Iberoamericana de Automática e Informática industrial*, *10*, 3-17. https://doi.org/10.1016/j.riai.2012.11.001

[34] Dubonjić, LJ., Nedić, N., Filipović, V., & Pršić, N. (2013) Design of PI controllers for hydraulic control systems. *Mathematical Problems in Engineering*, *2013*, 1-10. https://doi.org/10.1155/2013/451312

[35] Prodanović, S. & Nedić, N. (2014). Non-conventional control of level and temperature in the flow tank. *Proceedings of 2. International Scientific Conference on Mechanical Engineering Technologies and Applications - COMETa2014*, 421-428.

Contact information:

Saša PRODANOVIĆ, PhD, Associate Professor
(Corresponding author)
University of East Sarajevo, Faculty of Mechanical Engineering,
Vuka Karadžića 30, 71123 East Sarajevo, Bosnia & Herzegovina
E-mail: sasa.prodanovic@ues.rs.ba

Ljubiša DUBONJIĆ, PhD, Associate Professor
University of Kragujevac,
Faculty of Mechanical and Civil Engineering in Kraljevo,
Dositejeva 19, 36000 Kraljevo, Serbia
E-mail: dubonjic.lj@mfkv.kg.ac.rs

Janani RAJARAMAN, PhD, Assistant Professor
Department of Mechanical Engineering,
Sri Chandrasekharendra Saraswathi Viswa Mahavidyalaya University,
Sri Jayendra Saraswathi Street, Enathur, Kanchipuram 631561, India
E-mail: janani.rajaraman@kanchiuniv.ac.in