Energy of Interval-Valued Bipolar Neutrosophic Soft Sets: A New Tool for Medical Decision-Making

Nemanja Vučićević[1], Nenad Stojanović[1], Marina Svičević[1], Filip Andrić[1]

Faculty of Science, University of Kragujevac, Kragujevac, Serbia
nemanja.vucicevic@pmf.kg.ac.rs, nenad.stojanovic@pmf.kg.ac.rs,
marina.svicevic@pmf.kg.ac.rs, filip.andric@pmf.kg.ac.rs

Many applications of neutrosophic soft sets have been limited by insufficient parametric tools (Dalkılıç, 2021). However, the hybridization of interval-valued bipolar neutrosophic sets (Deli, 2017) with soft sets has overcome this limitation, resulting in interval-valued bipolar neutrosophic soft sets — a structure with enhanced parametric richness and wide-ranging applicability. These sets have attracted significant attention for both theoretical investigation and practical problem-solving, particularly in decision-making contexts. To formalize this notion, consider a universal set U, and denote by $IVBNS(U)$ the collection of all interval-valued bipolar neutrosophic sets over U. Let E be a set of parameters (attributes) used to describe the elements of U, and let $A \subseteq E$ be a chosen subset of parameters relevant to the problem at hand.

An **interval-valued bipolar neutrosophic soft set (IVBNSS)** over U is then an ordered pair $(\mathcal{B}, A)$, where $\mathcal{B}$ is a mapping from E to $IVBNS(U)$. For each parameter $e \in E$, the mapping $\mathcal{B}(e)$ associates a set of ordered tuples of the form:

$$\left(u, [T^{+}_{L(u)}, T^{+}_{R(u)}], [I^{+}_{L(u)}, I^{+}_{R(u)}], [F^{+}_{L(u)}, F^{+}_{R(u)}], [T^{-}_{L(u)}, T^{-}_{R(u)}], [I^{-}_{L(u)}, I^{-}_{R(u)}], [F^{-}_{L(u)}, F^{-}_{R(u)}]\right),$$

for all $u \in U$. Here, the functions $T^{+}_{L}, T^{+}_{R}, I^{+}_{L}, I^{+}_{R}, F^{+}_{L}, F^{+}_{R} : U \to [0,1]$, represent the lower and upper bounds of the degrees of truth, indeterminacy, and falsity in the positive domain, while $T^{-}_{L}, T^{-}_{R}, I^{-}_{L}, I^{-}_{R}, F^{-}_{L}, F^{-}_{R} : U \to [-1,0]$, capture the corresponding degrees in the negative domain.

In the work of Mudrić Staniškovski et al., the authors introduced an energy measure for fuzzy soft sets by constructing real-valued matrices based on membership degrees and computing their nuclear norm — the sum of singular values. This measure effectively captured the structural contribution of individual elements to the soft set system and was successfully applied in decision-making scenarios involving uncertainty. The method emphasized the spectral structure of the system rather than relying solely on aggregate membership scores, thereby providing more nuanced insights.

Building on this spectral perspective, we extend the energy concept to the interval-valued bipolar neutrosophic soft setting by incorporating both positive and negative information, represented through real-valued matrices constructed from the lower and upper bounds of the truth, indeterminacy, and falsity functions. The energy of an interval-valued bipolar neutrosophic soft set (IVBFNSS) is computed based on the singular values of matrices formed from the lower and upper bounds of the positive and negative truth, indeterminacy, and falsity functions. For each alternative, separate real-valued matrices are constructed for each component, including indeterminacy and falsity. Subsequently, we compute the optimistic and pessimistic energy values for each alternative (both based on combinations of the lower and upper bounds of all functions). The resulting energy is defined as the arithmetic mean of these two values. By performing these computations, we derive a ranking of the alternatives based on their energy values, allowing us to determine the relative ordering and identify the optimal solution.

The performance of the proposed algorithm is evaluated with a particular focus on medical diagnosis problems — a domain where precise and reliable decision-making is critically important. This is especially relevant in frequently studied scenarios, such as the assessment of individuals based on parameters like happiness, sadness, energy levels, and sleep rate, used to identify conditions such as bipolar disorder.

To briefly highlight the details of the proposed approach and the comparative analysis: the energy-based method within the interval-valued bipolar neutrosophic soft set framework demonstrates notable advantages, particularly in decision-making contexts. A comparative analysis with existing algorithms — especially the method by Abdel-Basset et al. (2019), a key contribution in the field — highlights the superior efficiency and robustness of our approach. The algorithm was applied to several medical diagnosis examples to determine the best alternative, where it performed successfully. Unlike the Abdel-Basset et al. algorithm, which failed to identify the best alternative in a specific case, our method consistently selected the optimal alternative and successfully determined the correct solution.

Moreover, the proposed algorithm shows strong potential for handling large-scale, high-dimensional medical data.

Keywords: Interval-valued bipolar neutrosophic soft set, Decision making, Singular value, Nuclear norm

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