Journal of Manufacturing and Materials Processing

MDPI

Article

Prioritization of Process Improvement Measures in a Forging Process Using an IPF-AHP-Based SREM Framework

Nikola Kastratović [1], Dušan Arsić [1], Nikola Komatina [1], Marko Delić [1] and Dragan Marinković [2,3,4,*]

[1] Faculty of Engineering, University of Kragujevac, Sestre Janjić 6, 34000 Kragujevac, Serbia; nikolakastratovic93@gmail.com (N.K.); dusan.arsic@fink.rs (D.A.); nkomatina@kg.ac.rs (N.K.); marko.delic@kg.ac.rs (M.D.)
[2] Department of Structural Analysis, Technical University Berlin, 10623 Berlin, Germany
[3] Institute of Mechanical Science, Vilnius Gediminas Technical University, LT-10223 Vilnius, Lithuania
[4] University College, Korea University, 145 Anam-ro, Seoul 02841, Republic of Korea
* Correspondence: dragan.marinkovic@tu-berlin.de

Abstract

Forging represents a very important process in the metal processing industry, for which risk analysis and continuous improvement activities are required in order to satisfy customer requirements regarding product quality and mechanical properties. In practice, Process Failure Mode and Effects Analysis (PFMEA) is used for risk identification and assessment. However, since this analysis provides only recommended process improvement measures as an output, without prioritizing them according to technical, economic, and operational aspects, this study develops a Multi-Criteria Decision-Making (MCDM) approach based on the integration of the Analytic Hierarchy Process (AHP) and the Square-Root-Based Evaluation Method (SREM), extended through the application of Interval-Valued Pythagorean Fuzzy Numbers (IVPFNs) to model uncertainty in the assessments of the expert team. In this way, a decision-making framework was developed that enables the prioritization of process improvement measures identified through PFMEA in an exact and mathematically based manner. The proposed approach was tested through a case study conducted in a company primarily engaged in the production of forgings as a supplier to various industrial sectors. A total of six process improvement measures were considered and evaluated with respect to seven technical, economic, and operational criteria. The results of the study clearly demonstrated that the additional die-leading measure ranked first and represented the most stable solution, maintaining its leading position regardless of the changes introduced through the sensitivity analysis.

Keywords: forging; PFMEA; process improvement; MCDM; AHP; SREM; IVPFNs

Academic Editor: Andrea Ghiotti

Received: 20 June 2026
Revised: 16 July 2026
Accepted: 17 July 2026
Published: 19 July 2026

1. Introduction

Forging represents one of the oldest and most widely used metal processing technologies and remains one of the indispensable manufacturing processes in modern industry. The fundamental characteristic of forging is its ability to produce components with negligible material loss. Today, forging is widely applied in numerous high-precision industrial sectors, including the automotive, aerospace, and other manufacturing industries [1,2]. However, although modern forging processes rely on reliable and specially designed equipment, various process-related problems still occur. In fact, certain aspects of the forging process are particularly difficult to control due to the extreme operating conditions under which forging is performed. The most common problems include crack formation, die misalignment, incomplete die-cavity filling, tool wear, and similar defects [1,3–7].

Previous studies have shown that abrasive wear causes dimensional changes in up to 70% of forging dies. Moreover, tooling costs account for approximately 8% to 40% of total production costs in such manufacturing facilities [3]. Consequently, there is a need for continuous and systematic improvement of forging processes in order to reduce waiting times, production downtime, and the occurrence of defective products [1,3]. In fact, the optimization of complex manufacturing processes and the elimination of waste have been extensively addressed in the relevant literature [8]. Considering that no company, regardless of its financial stability, possesses unlimited resources, the prioritization of process improvement measures represents an important managerial task. Its objective is to align investments with those measures that provide the greatest overall benefit to the manufacturing process [9–11].

The most commonly used tool for identifying process improvement measures in practice is Process Failure Mode and Effects Analysis (PFMEA). It represents a standardized and widely accepted methodology in the automotive industry, where its application is prescribed by industry standard [12], as well as in other industries, particularly those requiring a high level of precision. PFMEA is based on the evaluation of three risk factors: the severity (S) of the consequence, the occurrence (O) of the failure, and the detection (D) capability [13–16]. In the traditional Failure Mode and Effects Analysis (FMEA) approach, as well as in its process-oriented variant (PFMEA), risk prioritization is performed using the Risk Priority Number (RPN), which is calculated as the product of these three risk factors [16,17].

Although PFMEA generates process improvement measures for the most critical risks, the conventional methodology has been subject to various criticisms [14,15]. The primary concern relates to the mechanism used to determine the risk level. Many authors argue that simply multiplying the values of the three risk factors cannot adequately represent the actual level of risk because the three risk factors rarely exhibit the same level of importance in real-world applications, despite this assumption being embedded in the traditional approach [14,15,17]. To address this limitation, the Action Priority (AP) approach was introduced [18]. However, even this improved framework does not provide a mechanism that would enable experts to evaluate the cost-effectiveness and overall efficiency of the proposed improvement measures [12].

Therefore, it can be concluded that PFMEA is a suitable tool for risk prioritization, on the basis of which process improvement measures are identified and proposed. However, in practice, decisions regarding their implementation depend on numerous factors. In a Multi-Criteria Decision-Making (MCDM) problem, such factors are referred to as criteria and may be either mutually dependent or conflicting in nature [19–22]. Consequently, MCDM approaches are particularly suitable for the systematic resolution of such engineering problems, where a balance must be achieved among technical, economic, and operational considerations [21,23,24], as demonstrated by previous studies that have confirmed the applicability of MCDM approaches to problems involving conflicting criteria [25,26].

The relevant literature has already demonstrated that FMEA can be effectively integrated with MCDM approaches [9,12,20,27,28], thereby significantly increasing the objectivity of the decision-making process in industrial environments. However, the application of MCDM methods is often based on expert assessments, which are difficult to express using precise numerical values [10,29,30]. To address this issue, fuzzy set theory was developed [31], enabling the modeling of uncertain and imprecise information. Over time, this concept evolved into several distinct theoretical directions [30,32–36]. One of the most widely recognized approaches for modeling uncertainty is represented by intuitionistic fuzzy sets (IFSs) [37,38]. Unlike classical fuzzy sets, IFSs are characterized by both a degree of membership and a degree of non-membership, whose sum cannot exceed the value of one [32,39,40].

Further development of this concept led to the introduction of Pythagorean fuzzy sets (PFSs) [41,42], in which this restriction was slightly relaxed by allowing the sum of the squares of the membership and non-membership degrees to be less than or equal to one. In this way, a broader space for modeling uncertainty was created.

In this study, Interval-Valued Pythagorean Fuzzy Numbers (IVPFNs) were employed, as they represent one of the most advanced approaches for modeling uncertainty, allowing membership and non-membership degrees to be defined as intervals rather than fixed values [29,43,44]. In other words, IVPFNs provide greater flexibility in representing uncertainty compared to IFSs and PFSs, which can be particularly important in problems related to risk assessment and reliability in industrial environments [15,45].

The objective of this paper is to determine the priority of process improvement measures, previously identified through PFMEA, in a mathematically based and exact manner. The aim is to consider, in addition to the qualitative aspect that is an integral part of PFMEA, several other aspects, such as technical, economic, and operational ones. Therefore, the evaluation criteria were defined in such a way as to satisfy the aforementioned aspects.

The Analytic Hierarchy Process (AHP) method was used to determine the criteria weights, as it has been very frequently applied in the literature [19,43]. As previously stated, IVPFNs were employed to model uncertainty in the considered problem, and their application represents a well-known and reliable extension of the AHP method (IPF-AHP) [10,33,43]. Unlike conventional AHP and fuzzy AHP, which are based on classical (triangular) fuzzy numbers, IPF-AHP enables experts to express their judgments through membership and non-membership degrees represented as intervals [13,43]. In this way, greater flexibility is achieved in modeling uncertainty and imprecision [46].

For ranking the identified process improvement measures, which in this case represent the alternatives, the Square-Root-Based Evaluation Method (SREM) [9] was employed. The method is relatively new and has so far been applied only to the problem of evaluating and ranking industrial presses, taking into account their operational performance. In the original study, through comparison with other MCDM approaches, it proved to be a very stable and reliable method.

In addition, its advantage lies in its mechanism that enables decision-makers to change the decision-making perspective through the application of square and root transformations. Such an approach can give preference to alternatives that have extremely high values with respect to certain (particularly important) criteria, while on the other hand it can favor those alternatives that are stable with respect to a larger number of criteria. The method even allows fine adjustment through the transformation parameter α and the examination of the obtained solution from multiple perspectives.

In this study, the original SREM was employed, with the exception that linguistic expressions modeled by IVPFNs were used to evaluate the performance of the alternatives with respect to qualitative criteria. Subsequently, the experts' assessments were aggregated according to the fuzzy algebra rules and defuzzified into a single crisp value.

An analysis of the available relevant literature revealed several research gaps that are addressed by this study: (1) the integration of AHP and SREM within an IVPFN environment is performed for the first time; (2) no studies were identified in which the authors considered the problem of prioritizing process improvement measures identified through PFMEA in a forging process; and (3) the SREM is applied for the first time to solve this type of problem.

https://doi.org/10.3390/jmmp10070250

The main scientific contributions of the study can be summarized as follows: (1) the development of an IPF-AHP-based SREM decision-making framework; (2) the integration of IVPFNs with the aim of reducing subjectivity in industrial decision-making; and (3) the implementation of a sensitivity analysis through testing different values of the α transformation parameter and examining the influence of criteria weights defined by different experts. The main practical contributions of the study are: (1) the enhancement of PFMEA through the prioritization of process improvement measures generated as a result of its implementation; and (2) the execution of a real-world case study in an industrial environment.

The remainder of the paper is organized as follows. Section 2 presents the literature review. The methodology, including the problem description and the proposed mathematical model, is presented in Section 3. Section 4 presents the case study conducted in a manufacturing facility located in Central Serbia. The analysis and discussion of the obtained results are provided in Section 5. Finally, the conclusions of the study are presented in Section 6.

2. Literature Review

FMEA has been widely used as a tool for process and system optimization and improvement. In study [27], the authors modified the method and applied it for risk assessment in the operation of a turbine and alternator unit. In the automotive industry domain, PFMEA was used as a basis for identifying critical phases in automotive component manufacturing [12]. Furthermore, in the domain of forging processes in the automotive industry, the authors in [47] extended FMEA through the use of Pareto analysis and the 8D methodology.

In the relevant literature, various extensions of the conventional AHP approach have been proposed through the application of different methods for modeling uncertainty in pair-wise comparisons. The AHP method has been extended by applying fuzzy, hesitant fuzzy, and intuitionistic fuzzy approaches [48–50]. In this study, the IPF-AHP approach was employed, which is almost standardized in the relevant literature [10,33,43,44,51], since the algebra of IVPFNs itself does not provide much room for modifications and methodological variations. Such an approach was employed in this study as well, with the only modifications being related to the number of criteria considered and the number of experts participating in the research.

FMEA is frequently integrated with various MCDM approaches in order to overcome the limitations of the conventional methodology (see [21,52–55]). In this study, the objective is to enhance PFMEA through the application of an MCDM approach in order to determine the priority of process improvement measures generated as a result of the PFMEA conducted for the forging process in the considered manufacturing facility.

To the best of the authors' knowledge, such a problem has not been addressed in the literature so far. In addition, there are very few approaches that combine IVPFNs, AHP, and FMEA in general. In study [51], the authors employed IPF-AHP to determine the weights of the risk factors S, O, and D. Furthermore, when determining the composition of a PFMEA team, the authors in [56] used IVPFNs to model the uncertainty and imprecision existing in the considered problem.

Based on the review of the existing approaches, it can be clearly concluded that no methodology for prioritizing process improvement measures based on the proposed framework currently exists. In addition, this study employs the SREM, which represents a relatively new approach and has not yet been widely utilized. Collectively, these aspects constitute both the motivation for conducting this research and the scientific contribution arising from it.

https://doi.org/10.3390/jmmp10070250

3. Methodology

This section presents a description of the considered MCDM problem, including the sets of criteria used and the alternatives evaluated based on them. The basic characteristics and mathematical operations of IVPFNs are provided, as well as the approach used to model the linguistic expressions employed by the experts in this study to express their assessments. Figure 1 presents the research framework developed for the purposes of this study.

Figure 1. Research framework.

In the remainder of this section, the presented research framework is explained in detail. In addition to the description, the mathematical formulation of the decision-making problem is also provided.

3.1. Problem Formulation

This section presents the fundamentals of the considered decision-making problem. The following subsections provide a description of the considered alternatives and criteria, introduce the panel of experts involved in the study, and present the basic mathematical operations and formulation of the employed IVPFNs.

3.1.1. Definition of the Decision Alternatives

In this study, the alternatives represent the measures most frequently employed for process improvement in the considered forging production facility. The most significant measures were identified based on the company's PFMEA report, which cannot be presented in this paper due to data confidentiality constraints.

As previously stated, the objective of the study is to determine the implementation priority of process improvement measures identified through the PFMEA. In this context, these measures can be formally regarded as alternatives and represented by the index set, i, where $i = 1, \dots, I$.

The alternatives considered in this study are defined as follows:

- Additional die leading ($i = 1$) refers to additional die-leading elements that facilitate more accurate alignment of the upper and lower die halves during forging operations. This measure is implemented in the tool shop together with a solution designed and specified by the process engineering department.
- Computer Numerical Control (CNC) of the cavity and gutter ($i = 2$) is the machining of die cavities and flash gutters by means of CNC processing and it provides higher precision compared to conventional manufacturing methods. This procedure is carried out by the tool shop using CNC machines, resulting in improved cavity filling and a reduced risk of defects in the final product. The CNC machining program is provided by the process engineering department.
- Special construction of the first operation ($i = 3$) is a special design intended for the first forging operation and is used to improve material flow in the subsequent stages of production. This measure is implemented by the tool shop using a solution developed and specified by the process engineering department. The effects achieved are a more uniform material distribution as well as a reduced likelihood of defect occurrence.
- Double die leading ($i = 4$) in a double-die-leading system is an approach that provides greater stability during press operation. This system is particularly used for complex forgings, enabling higher process accuracy and a longer die service life.
- Integral die leading ($i = 5$) is a solution in which the guiding elements are permanently integrated into the die design. The effect of this solution is a reduction in misalignment between the die halves, which significantly contributes to improving the reliability of the forging process.
- Parts pressing in the trimming operation ($i = 6$) is carried out in production using appropriate tools designed for flash trimming, resulting in reduced deformations and improved removal of flash remaining after forging, thereby improving the quality of the final product.

3.1.2. Definition of the Evaluation Criteria

The defined process improvement measures are evaluated and ranked based on a total of seven criteria. The criteria were defined in cooperation with the company's expert team. The expert team involved in this study consisted of a design engineer ($e = 1$), a quality engineer ($e = 2$), and a production engineer ($e = 3$). Formally, the set of experts can be represented as $e, e = 1, \ldots, E$, where e denotes the expert index, and E denotes the total number of experts.

The criteria used to evaluate the process improvement measures are as follows:

- Tool design ($j = 1$) represents the time required by design engineers to develop the tool design and prepare the corresponding technical documentation. A shorter design time enables faster implementation of changes in production and reduces the overall project lead time.
- Technology development ($j = 2$) refers to the time required by process engineers to define and verify the manufacturing process for forging-tool production. This stage includes the selection of machines and tooling, as well as the generation of CNC programs for tool manufacturing.
- Tool manufacturing ($j = 3$) represents the time required for the production or modification of the tool. A long manufacturing time may lead to production delays and increased indirect costs.

https://doi.org/10.3390/jmmp10070250

- Cost ($j = 4$) represents the direct financial cost associated with the implementation of the measure. Due to its significant impact on production economics, this criterion is assigned the highest importance.
- Risk reduction potential ($j = 5$) represents the expected effect of the measure on process risks and the likelihood of their occurrence.
- Sustainability of the effect ($j = 6$) denotes the potential of the measure to maintain its effectiveness even after changes in process conditions, such as changes in technology, operators, and other relevant factors.
- Potential to address multiple failure modes ($j = 7$) represents the estimated scope of the measure's impact on the identified critical failure modes. In other words, it reflects the ability of the measure to simultaneously influence multiple failure modes or their underlying causes and consequences.

The first four criteria are quantitative in nature and belong to the cost-type category, meaning that lower values are preferred. The remaining three criteria are qualitative in nature and belong to the benefit-type category, meaning that higher values are preferred.

3.1.3. Mathematical Representation of IVPFNs

The foundation of IVPFNs originates from PFSs, which were introduced in [41], as an extension of IFSs [37,38]. The main characteristic of PFSs is their dual nature, represented by the membership degree $\mu_{\tilde{A}}(x)$ and the non-membership degree $\nu_{\tilde{A}}(x)$ of the function $\tilde{A}(x)$. While a PFS can be regarded as a general concept, an individual element of this set can be defined as a PFN, which is formally represented as follows [42,57]:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x); x \in X)\} \tag{1}$$

A characteristic feature of PFNs is that the sum of the squares of $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ must be less than or equal to one. This distinguishes them from intuitionistic fuzzy numbers (IFNs), which also possess $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$, but for which the sum of these values must be less than or equal to one, whereas this is not necessarily the case for PFNs. Therefore, the hesitation degree of $\tilde{A}$ in the set X can be represented for a PFN as follows [39]:

$$\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x) \tag{2}$$

For which the following holds:

$$0 \leq \pi_{\tilde{A}}(x) \leq 1 \tag{3}$$

An extension of PFNs in which $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ are defined within specific intervals, i.e., possess lower and upper bounds, is referred to as an IVPFN. It can be formally represented as follows [29]:

$$\tilde{A} = \left\{x, \left[\, \mu_{\tilde{A}_L}(x), \mu_{\tilde{A}_U}(x)\right], \left[\nu_{\tilde{A}_L}(x), \nu_{\tilde{A}_U}(x)\right]; x \in X\right\} \tag{4}$$

For which the following holds:

$$0 \leq \mu_{\tilde{A}_L}(x), \mu_{\tilde{A}_U}(x), \nu_{\tilde{A}_L}(x), \nu_{\tilde{A}_U}(x) \leq 1 \tag{5}$$

where

$$\mu_{\tilde{A}_L}(x)^2 + \nu_{\tilde{A}_L}(x)^2 \leq 1; x \in X \tag{6}$$

The algebra of IVPFNs was also developed on the foundations of the algebra of PFNs. Let two PFNs be formally represented as $\tilde{A} = (\mu_1, \nu_1)$ and $\tilde{B} = (\mu_2, \nu_2)$ and two IVPFNs as $\tilde{A} = \{[\mu_1^L, \mu_1^U], [\nu_1^L, \nu_1^U]\}$ and $\tilde{B} = \{[\mu_2^L, \mu_2^U], [\nu_2^L, \nu_2^U]\}$, with c denoting a non-negative crisp number. The following mathematical operations are defined:

- Addition of two PFNs [57,58]:

$$\tilde{A} \oplus \tilde{B} = \left(\sqrt{\mu_1^2 + \mu_2^2 - \mu_1^2\mu_2^2}, \nu_1, \nu_2\right) \tag{7}$$

- Addition of two IVPFNs [29,46]:

$$\tilde{A} \oplus \tilde{B} = \left(\left[\sqrt{(\mu_1^L)^2 + (\mu_2^L)^2 - (\mu_1^L)^2(\mu_2^L)^2}, \sqrt{(\mu_1^U)^2 + (\mu_2^U)^2 - (\mu_1^U)^2(\mu_2^U)^2}\right], [\nu_1^L\nu_2^L, \nu_1^U\nu_2^U]\right) \tag{8}$$

- Multiplication of two PFNs [57,58]:

$$\tilde{A} \otimes \tilde{B} = \left(\mu_1\mu_2, \sqrt{\nu_1^2 + \nu_2^2 - \nu_1^2\nu_2^2}\right) \tag{9}$$

- Multiplication of two IVPFNs [29,46]:

$$\tilde{A} \otimes \tilde{B} = \left([\mu_1^L\mu_2^L, \mu_1^U\mu_2^U], \left[\sqrt{(\nu_1^L)^2 + (\nu_2^L)^2 - (\nu_1^L)^2(\nu_2^L)^2}, \sqrt{(\nu_1^U)^2 + (\nu_2^U)^2 - (\nu_1^U)^2(\nu_2^U)^2}\right]\right) \tag{10}$$

- Multiplication of a PFN by a crisp number [57,58]:

$$c\tilde{A} = \left(\sqrt{1 - (1 - \mu^2)^c}, \nu^c\right) \tag{11}$$

- Multiplication of an IVPFN by a crisp number [29,46]:

$$c\tilde{A} = \left(\left[\sqrt{1 - (1 - (\mu_1^L)^2)^c}, \sqrt{1 - (1 - (\mu_1^U)^2)^c}\right], [(\nu_1^L)^c, (\nu_1^U)^c]\right) \tag{12}$$

- Exponentiation of a PFN by a crisp number [57,58]:

$$\tilde{A}^c = \left(\mu^c, \sqrt{1 - (1 - \nu^2)^c}\right) \tag{13}$$

- Exponentiation of an IVPFN by a crisp number [29,46]:

$$\tilde{A}^c = \left([(\mu_1^L)^c, (\mu_1^U)^c], \left[\sqrt{1 - (1 - (\nu_1^L)^2)^c}, \sqrt{1 - (1 - (\nu_1^U)^2)^c}\right]\right) \tag{14}$$

In addition to the aforementioned operations, other important mathematical operations for IVPFNs include:

- Distance between these two IVPFNs [46]:

$$d(\tilde{A}, \tilde{B}) = \frac{1}{2}\sqrt{\left(((\mu_1^L)^2 - (\mu_2^L)^2)\left(1 - \frac{\pi_1^L - \pi_2^L}{2}\right) + \sqrt{((\mu_1^U)^2 - (\mu_2^U)^2)\left(1 - \frac{\pi_1^U - \pi_2^U}{2}\right)}\right)} \tag{15}$$

where the hesitation degrees corresponding to the lower and upper bounds are defined as follows [46]:

$$\pi^L = \sqrt{1 - (\mu_U^2 + \nu_U^2)} \tag{16}$$

$$\pi^U = \sqrt{1 - (\mu_L^2 + \nu_L^2)} \tag{17}$$

- Transformation of an IVPFN into a crisp value (defuzzification) [39]:

$$defuzz(\tilde{A}) = \tag{18}$$

$$= \frac{1}{6}\left(\mu_L^2 + \mu_U^2 + (1 - \pi_L^4 - \nu_L^2) + (1 - \pi_U^4 - \nu_U^2) + \mu_L\mu_U + \sqrt[4]{(1 - \pi_L^4 - \nu_L^2)(1 - \pi_U^4 - \nu_U^2)}\right)$$

In this section, the basic characteristics and mathematical operations of IVPFNs are presented, namely those that are useful for the purposes of this study. More detailed explanations of the concept and mathematical proofs can be found in the relevant literature [29,39,42,46,57,58].

3.1.4. Linguistic Assessment Framework

For the purposes of this study, two linguistic scales were employed. The first scale was used to assess the relative importance of the criteria, while the second scale was used to evaluate the performance of the alternatives with respect to criteria $j = 5$, $j = 6$, and $j = 7$. The linguistic terms were modeled using IVPFNs.

The following linguistic terms were used to assess the relative importance of the criteria, serving as input to the IPF-AHP method (defined in accordance with recommendations from the relevant literature [10,16,19,33,39,51]):

- Equally important (E): $\{[0.50,0.50],[0.50,0.50]\}$;
- Absolutely less important (AL): $\{[0.00,0.00],[0.90,1.00]\}$;
- Significantly less important (SL): $\{[0.10,0.20],[0.80,0.90]\}$;
- Less important (L): $\{[0.20,0.35],[0.65,0.80]\}$;
- Moderately less important (ML): $\{[0.35,0.45],[0.55,0.65]\}$;
- Nearly equally important (NE): $\{[0.45,0.55],[0.45,0.55]\}$;
- Moderately more important (MM): $\{[0.55,0.65],[0.35,0.45]\}$;
- More important (M): $\{[0.65,0.80],[0.20,0.35]\}$;
- Significantly more important (SM): $\{[0.80,0.90],[0.10,0.20]\}$;
- Absolutely more important (AM): $\{[0.90,1.00],[0.00,0.00]\}$.

The following linguistic terms were used to evaluate the alternatives with respect to the aforementioned three criteria:

- Very low (V1): $\{[0.0, 0.2],[0.8, 1.0]\}$;
- Low (V2): $\{[0.2, 0.4],[0.6, 0.8]\}$;
- Moderate (V3): $\{[0.4, 0.6],[0.4, 0.6]\}$;
- High (V4): $\{[0.6, 0.8],[0.2, 0.4]\}$;
- Very high (V5): $\{[0.8, 1.0],[0.0, 0.2]\}$.

In this case, the domain of IVPFNs is defined within the interval ([0,1]). Ten linguistic expressions were used to assess the relative importance of the criteria in order to obtain a more precise determination of the criteria weights. Among them, one term denotes that the criteria are of equal importance (E), one denotes that the criteria are of nearly equal importance (NE), four expressions indicate that the considered criterion is less important than the compared criterion (AL, SL, L, and ML), and four expressions indicate that the considered criterion is more important than the compared criterion (MM, M, SM, and AM). The evaluation of alternatives was performed using five linguistic expressions, which was considered sufficient to provide an appropriate level of scaling for the considered decision-making problem.

3.2. The Proposed IPF-AHP-Based SREM Framework

In this study, the IPF-AHP method was applied to determine the criteria weights. It represents a standard variant of this approach and has been defined and utilized in numerous studies [10,16,19,33,39,44,45,51,59]. Therefore, the IPF-AHP approach is implemented through the following steps:

Step 1. Formation of the fuzzy pair-wise comparison matrix for evaluating the relative importance of criteria at the level of each expert separately:

$$\left[W_{jj'}^{e}\right]_{J\times J'} \tag{19}$$

In this case, the criteria can be formally represented as follows: $j, j' = 1, \ldots, J$; $j \neq j'$. The experts can be formally represented as follows: $e = 1, \ldots, E$.

Step 2. The consistency of the assessments is checked at the level of each expert. For this purpose, the eigenvector method [60] is employed. To evaluate consistency, the elements of the fuzzy pair-wise comparison matrix are mapped onto a crisp scale ranging from 1 to 5. The reciprocal values are determined in accordance with the reciprocal property of the conventional AHP method.

As in the conventional AHP method, the experts' assessments are considered consistent if the consistency ratio (CR) is less than or equal to 0.10 (10%).

Step 3. The differences matrix, D, is formed:

$$D = \left[d_{jj'}\right]_{J\times J'} \tag{20}$$

For which the following holds:

$$\begin{gathered} d_{jj'L} = \mu_{jj'L}^{2} - \nu_{jj'U}^{2} \\ d_{jj'U} = \mu_{jj'U}^{2} - \nu_{jj'L}^{2} \end{gathered} \tag{21}$$

Step 4. The interval multiplicative matrix, G, is formed:

$$G = \left[g_{jj'}\right]_{J\times J'} \tag{22}$$

For which the following holds:

$$\begin{gathered} g_{jj'L} = \sqrt{1000^{d_{jj'L}}} \\ g_{jj'U} = \sqrt{1000^{d_{jj'U}}} \end{gathered} \tag{23}$$

Step 5. The indeterminacy value matrix, H, is formed:

$$H = \left[h_{jj'}\right]_{J\times J'} \tag{24}$$

For which the following holds:

$$h_{jj'} = 1 - \left(\mu_{jj'U}^{2} - \mu_{jj'L}^{2}\right) - (\nu_{jj'U}^{2} - \nu_{jj'L}^{2}) \tag{25}$$

Step 6. The matrix of unnormalized weights, T, is formed:

$$t_{jj'} = \left(\frac{g_{jj'L} + g_{jj'U}}{2}\right) \cdot h_{jj'} \tag{26}$$

Step 7. The criteria weights for each expert are determined as follows:

$$\omega_{j}^{e} = \frac{\sum_{j=1}^{J} t_{jj'}}{\sum_{j=1}^{J} t_{jj'} \sum_{j=1}^{J} t_{jj'}} \tag{27}$$

Step 8. The aggregated criteria weights are determined as follows:

$$\omega_{j} = \frac{1}{E} \cdot \sum_{e=1}^{E} \omega_{j}^{e} \tag{28}$$

To preserve the structure and consistency of the criteria weights, the arithmetic mean operator was employed.

The SREM supported by IVPFN-based assessments was employed for ranking the alternatives through the following steps:

Step 1. The decision matrix is formed:

$$[X_{ij}]_{I\times J} \tag{29}$$

where x_{ij} denotes the element of the decision matrix representing the value of alternative i with respect to criterion j.

The first four criteria are defined based on the prior experience and knowledge of the company experts. The remaining three criteria are qualitative in nature, and their values are assessed by the experts using predefined linguistic terms modeled by IVPFNs.

Due to the nature of the original SREM, as well as the insufficiently developed algebra of IVPFNs, the linguistic assessments provided by the experts are used solely for the construction of the decision matrix.

The experts' assessments are aggregated using the addition operation for IVPFNs, after which the values are defuzzified according to the procedure presented in the previous section.

Step 2. The normalized decision matrix is formed according to the procedure defined in the original SREM:

$$[M_{ij}]_{I\times J} \tag{30}$$

Normalization for benefit-type criteria is performed as follows:

$$m_{ij} = \frac{x_{ij}}{\sum_{i=1}^{I} x_{ij}} \cdot 100 \tag{31}$$

Normalization for cost-type criteria is performed as follows:

$$m_{ij} = \frac{\frac{1}{x_{ij}}}{\sum_{i=1}^{I} \frac{1}{x_{ij}}} \cdot 100 \tag{32}$$

Step 3. The square transformation of the normalized values is performed, and the squared-transformed normalized matrix is formed:

$$[S_{ij}]_{I\times J} \tag{33}$$

For which the following holds:

$$S_{ij} = \frac{m_{ij}^2}{\sum_{i=1}^{I} m_{ij}^2} \tag{34}$$

Step 4. The root transformation of the normalized values is performed, and the root-transformed normalized matrix is formed:

$$[R_{ij}]_{I\times J} \tag{35}$$

For which the following holds:

$$R_{ij} = \frac{\sqrt{m_{ij}}}{\sum_{i=1}^{I} \sqrt{m_{ij}}} \tag{36}$$

Step 5. The transformation matrix, or the α-fused matrix, is formed:

$$[A_{ij}]_{I\times J} \tag{37}$$

For which the following holds:

$$a_{ij} = \alpha \cdot S_{ij} + (1-\alpha) \cdot R_{ij} \tag{38}$$

Step 6. The values are weighted using the criteria weights obtained through the IPF-AHP approach, and the weighted α-fused matrix is formed:

https://doi.org/10.3390/jmmp10070250

$$[N_{ij}]_{I\times J} \tag{39}$$

For which the following holds:

$$n_{ij} = a_{ij} \cdot \omega_j \tag{40}$$

Step 7. The values of the weighted α-fused matrix are summed for each alternative separately in order to determine the final ranking index:

$$F_i = \sum_{j=1}^{J} n_{ij} \tag{41}$$

The alternative with the lowest value of the final ranking index is ranked last, whereas the alternative with the highest value of this parameter is ranked first.

4. Case Study

The proposed IPF-AHP-based SREM framework was applied to a case study conducted in a metal forming company engaged in the production of forged components. The company's headquarters and production facility are located in Central Serbia. This section is divided into three parts. The first part provides a description of the processes carried out in the company. The second part presents the application of the IPF-AHP method for determining the criteria weights, while the third part ranks the process improvement measures using the SREM.

4.1. Description of the Forging Production Process

In the considered production facility, forgings are manufactured using several hydraulic and mechanical presses/hammers of different capacities, together with supporting equipment for material cutting, heating, heat treatment, and finishing operations. The production process itself consists of a total of six phases, which are presented in Figure 2.

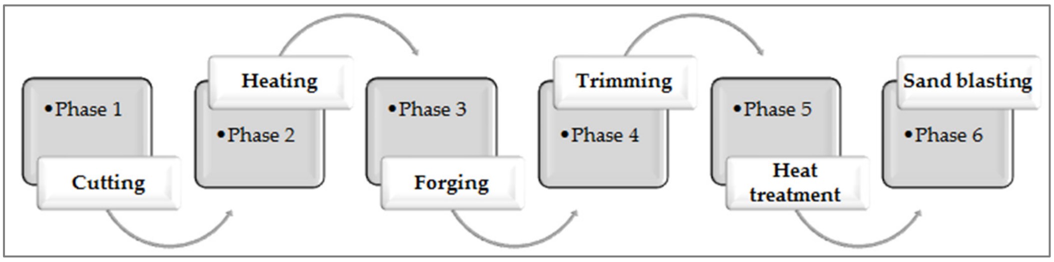

Figure 2. The production process phases.

As can be seen in Figure 2, the production of forgings begins with the cutting phase. This operation is performed using saws, shearing machines, or CNC machines. The objective is to cut the material in order to obtain a sufficient amount for further processing. After that, in the second phase, the workpieces are transported to a furnace, where they are heated to a temperature suitable for plastic deformation.

The third phase involves the forging operation (see Figure 3), which represents the most important part of the process. Depending on the complexity of the part, i.e., the forging, the number of operations, or deformation stages, may vary. The forging operation itself is performed on presses or hammers, during which the desired shape of the part is formed. It is precisely in this phase that various problems may occur, such as forging cracks, tool wear, machine downtime, incomplete die-cavity alignment, and similar defects.

Figure 3. Forging of parts in the considered production facility.

In the fourth phase, the trimming operation is performed. During this phase, excess material generated during the forging process is removed in order to ensure the specified dimensions of the part.

In the fifth phase, the parts undergo heat treatment in order to improve their mechanical properties. This process is carried out in vacuum and chamber furnaces.

In the sixth phase, sand blasting is performed to remove oxides and impurities from the parts. This phase has a significant influence on the final appearance of the product. The specialized forging dies, as well as the finished forgings, are shown in Figure 4.

(**a**) (**b**)

Figure 4. Specialized forging die (**a**) and finished forging (**b**).

Only a brief description of the production process is provided in this section. More detailed information cannot be disclosed due to the company's data protection and confidentiality policy. Nevertheless, the presented material is sufficient to explain the considered problem.

4.2. Determination of Criteria Weights Using the IPF-AHP Method

In this section, the weights of the considered criteria were determined using the proposed IPF-AHP method. As a starting point for the application of this method, assessments of the relative importance of the criteria were obtained from three experts from the company:

- A design engineer ($e = 1$):

$$\begin{bmatrix} \boldsymbol{E} & NE & ML & L & E & M & M \\ NE & \boldsymbol{E} & ML & L & NE & M & M \\ MM & MM & \boldsymbol{E} & ML & M & M & M \\ M & M & MM & \boldsymbol{E} & M & MM & MM \\ E & NE & L & L & \boldsymbol{E} & M & M \\ L & L & L & ML & L & \boldsymbol{E} & NE \\ L & L & L & ML & L & NE & \boldsymbol{E} \end{bmatrix}$$

- A quality engineer (e = 2):

$$\begin{bmatrix} \boldsymbol{E} & NE & L & L & E & ML & ML \\ NE & \boldsymbol{E} & L & L & NE & L & L \\ M & M & \boldsymbol{E} & M & M & NE & NE \\ M & M & L & \boldsymbol{E} & M & ML & ML \\ E & NE & L & L & \boldsymbol{E} & ML & ML \\ MM & M & NE & MM & M & \boldsymbol{E} & NE \\ MM & M & NE & MM & M & NE & \boldsymbol{E} \end{bmatrix}$$

- A production engineer (e = 3):

$$\begin{bmatrix} \boldsymbol{E} & NE & L & SL & E & M & M \\ NE & \boldsymbol{E} & L & SL & NE & M & M \\ M & M & \boldsymbol{E} & L & M & SM & SM \\ SM & SM & M & \boldsymbol{E} & SM & AM & AM \\ E & NE & L & SL & \boldsymbol{E} & M & M \\ L & L & SL & AL & L & \boldsymbol{E} & E \\ L & L & SL & AL & L & E & \boldsymbol{E} \end{bmatrix}$$

To verify the consistency of the assessments, the matrix was transformed so that each linguistic term corresponded to a crisp value, as explained in the proposed model. By applying the eigenvector method [61], it was found that all three matrices were consistent, with consistency ratio (CR) values of 0.05, 0.08, and 0.07, respectively. Therefore, the experts' assessments can be accepted as final.

Since the IPF-AHP methodology is well established in the relevant literature and was applied in its conventional form in this study, a detailed computational procedure is not presented. The criteria weights obtained for each expert are as follows:

- A design engineer (e = 1):

$\omega_1^1 = 0.15$ $\omega_2^1 = 0.15$ $\omega_3^1 = 0.22$ $\omega_4^1 = 0.24$ $\omega_5^1 = 0.15$ $\omega_6^1 = 0.04$ $\omega_7^1 = 0.04$

- A quality engineer (e = 2):

$\omega_1^2 = 0.06$ $\omega_2^2 = 0.05$ $\omega_3^2 = 0.25$ $\omega_4^2 = 0.19$ $\omega_5^2 = 0.06$ $\omega_6^2 = 0.20$ $\omega_7^2 = 0.20$

- A production engineer ($e = 3$):

$\omega_1^3 = 0.08$ $\omega_2^3 = 0.07$ $\omega_3^3 = 0.22$ $\omega_4^3 = 0.51$ $\omega_5^3 = 0.08$ $\omega_6^3 = 0.02$ $\omega_7^3 = 0.02$

By applying the arithmetic mean operator, the following aggregated criteria weights were obtained:

$\omega_1 = 0.10$ $\omega_2 = 0.09$ $\omega_3 = 0.23$ $\omega_4 = 0.32$ $\omega_5 = 0.09$ $\omega_6 = 0.08$ $\omega_7 = 0.08$

Based on the aggregated criteria weights, it was determined that Cost ($j = 4$) is the most important criterion for the expert team of the considered company. Tool manufacturing ($j = 3$) ranks second in importance, while the remaining criteria have very similar levels of importance.

4.3. Prioritization of Process Improvement Measures Using the SREM

This section presents the application of the proposed SREM approach for ranking and prioritizing process improvement measures. Table 1 presents the decision matrix obtained from the data and assessments provided by the company experts. As already stated in this study, the first four criteria are crisp and of the cost type, while the remaining three are of the benefit type and were evaluated by the expert team using linguistic expressions modeled by IVPFNs.

Table 1. The decision matrix with individual expert assessments.

Process Improvement Measure, i	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$	$j = 7$
$i = 1$	6	3	5	1000	V3, V2, V2	V4, V4, V5	V3, V2, V4
$i = 2$	20	15	30	6000	V5, V4, V5	V5, V5, V5	V4, V5, V4
$i = 3$	4	2	8	1600	V3, V4, V3	V4, V4, V5	V4, V4, V5
$i = 4$	5	3	7	1400	V5, V4, V4	V4, V5, V5	V4, V4, V4
$i = 5$	4	2	9	1800	V5, V5, V5	V3, V3, V3	V5, V5, V5
$i = 6$	8	5	6	1200	V1, V2, V1	V5, V3, V4	V2, V2, V1

After that, the experts' assessments were aggregated into a single evaluation (see Table 2). For this purpose, the IVPFN addition operation was employed, while the approach proposed in [46], defined within the methodology of this study, was used for the defuzzification of values.

Table 2. Aggregated and defuzzified decision matrix.

i	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$	$j = 7$
$i = 1$	6	3	5	1000	0.483	0.717	0.663
$i = 2$	20	15	30	6000	0.728	0.795	0.717
$i = 3$	4	2	8	1600	0.678	0.717	0.717
$i = 4$	5	3	7	1400	0.717	0.728	0.697
$i = 5$	4	2	9	1800	0.795	0.616	0.795
$i = 6$	8	5	6	1200	0.254	0.702	0.322

After the aggregation and defuzzification of the experts' assessments modeled using IVPFNs, the original SREM was applied to determine the priorities of the process improvement measures. The ranking of the alternatives is presented in Table 3.

Table 3. Ranking of process improvement measures.

i	F_i ($\alpha = 0$)	**Rank**	F_i ($\alpha = 0.25$)	**Rank**	F_i ($\alpha = 0.5$)	**Rank**	F_i ($\alpha = 0.75$)	**Rank**	F_i ($\alpha = 1$)	**Rank**
$i = 1$	0.177	1	0.191	1	0.206	1	0.221	1	0.236	1

$i = 2$	0.095	6	0.082	6	0.069	6	0.056	6	0.043	6
$i = 3$	0.162	3	0.162	2–3	0.162	2–3	0.162	2	0.163	2
$i = 4$	0.163	2	0.162	2–3	0.162	2–3	0.161	3	0.161	3
$i = 5$	0.157	4–5	0.155	5	0.154	5	0.152	5	0.150	5
$i = 6$	0.157	4–5	0.157	4	0.157	4	0.157	4	0.158	4

It can be consistently shown that the process improvement measure of additional die-leading ($i = 1$) ranks first regardless of whether and to what extent the root or quadratic transformation is favored, i.e., regardless of whether the objective is to identify an alternative with overall stability or an alternative with specific advantages with respect to the most important criteria.

The second and third positions in the compromise ranking are occupied by the alternatives of double die leading ($i = 4$) and the special construction of the first operation ($i = 3$). The values of the final ranking index for these two alternatives are very similar and exhibit only minor variations as the α parameter changes. In fact, alternative $i = 4$ performs better with respect to the two most important criteria, which is why the quadratic transformation places this alternative in second position.

The fourth position in the ranking is predominantly occupied by the measure of parts pressing in the trimming operation ($i = 6$). Only for $\alpha = 0$ does integral die leading ($i = 5$) appear in fourth position, while in all other cases this measure is ranked fifth. The alternative CNC cavity and gutter ($i = 2$) consistently remains in the last position.

Although the SREM contains a mechanism for conducting a form of sensitivity analysis, the obtained results can be further examined by considering the criteria weights derived from the individual expert assessments. The ranking results obtained in this manner are presented in Tables 4–6.

Table 4. Ranking of process improvement measures (expert 1 criteria weights).

i	F_i $(\alpha = 0)$	Rank	F_i $(\alpha = 0.25)$	Rank	F_i $(\alpha = 0.5)$	Rank	F_i $(\alpha = 0.75)$	Rank	F_i $(\alpha = 1)$	Rank
$i = 1$	0.179	1	0.189	1	0.199	1	0.209	1	0.219	1
$i = 2$	0.100	6	0.087	6	0.074	6	0.061	6	0.048	6
$i = 3$	0.174	2	0.178	2	0.182	2	0.186	2	0.190	2
$i = 4$	0.171	3–4	0.170	4	0.170	4	0.169	4	0.169	4
$i = 5$	0.171	3–4	0.175	3	0.179	3	0.182	3	0.186	3
$i = 6$	0.155	5	0.151	5	0.146	5	0.142	5	0.138	5

Table 5. Ranking of process improvement measures (expert 2 criteria weights).

i	F_i $(\alpha = 0)$	Rank	F_i $(\alpha = 0.25)$	Rank	F_i $(\alpha = 0.5)$	Rank	F_i $(\alpha = 0.75)$	Rank	F_i $(\alpha = 1)$	Rank
$i = 1$	0.155	1	0.167	1	0.180	1	0.192	1	0.205	1
$i = 2$	0.094	6	0.085	6	0.077	6	0.068	6	0.060	6
$i = 3$	0.141	3–4	0.140	4	0.139	4	0.139	4	0.138	4
$i = 4$	0.144	2	0.143	2	0.143	2–3	0.143	3	0.142	3
$i = 5$	0.135	5	0.132	5	0.128	5	0.124	5	0.121	5
$i = 6$	0.141	3–4	0.142	3	0.143	2–3	0.144	2	0.145	2

Table 6. Ranking of process improvement measures (expert 3 criteria weights).

i	F_i $(\alpha = 0)$	Rank	F_i $(\alpha = 0.25)$	Rank	F_i $(\alpha = 0.5)$	Rank	F_i $(\alpha = 0.75)$	Rank	F_i $(\alpha = 1)$	Rank
$i = 1$	0.196	1	0.217	1	0.239	1	0.260	1	0.281	1
$i = 2$	0.095	6	0.078	6	0.062	6	0.046	6	0.030	6

$i = 3$	0.172	4	0.170	4	0.167	4	0.165	4	0.162	4
$i = 4$	0.176	2	0.175	3	0.175	3	0.174	3	0.173	3
$i = 5$	0.166	5	0.161	5	0.156	5	0.151	5	0.147	5
$i = 6$	0.175	3	0.178	2	0.181	2	0.185	2	0.188	2

When the problem is considered from the perspective of each expert individually, i.e., when the criteria weights obtained for each expert are taken into account separately, alternative $i = 1$ remains ranked first in all scenarios. Likewise, alternative $i = 2$ consistently remains in the last position. However, the ranking of the remaining alternatives varies to a considerable extent, which is a consequence of the differences in the criteria weights. Figure 5 presents the ranking of the alternatives for $\alpha = 0.5$ when the aggregated criteria weights and the criteria weights obtained from each individual expert are considered.

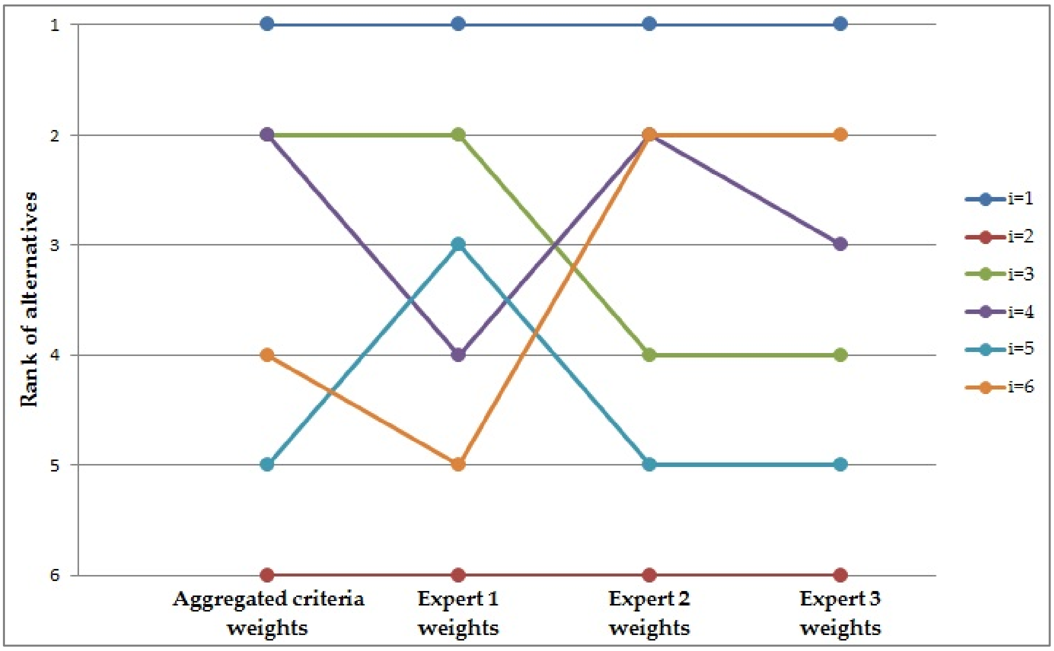

Figure 5. Alternative rankings under different criteria weights ($\alpha = 0.5$).

Figure 5 presents the compromise ranking for $\alpha = 0.5$. Two clear trends can be observed. Alternative $i = 1$ is consistently identified as the best solution, while alternative $i = 2$ always remains in the last position. The remaining alternatives may change their positions within the ranking, between the third, fourth, and fifth places. The greatest influence on these changes is exerted by the criteria weights, i.e., by the perspective of the expert assessing the relative importance of the criteria.

4.4. Multi-Method Comparison and Cross-Verification of Results

In order to validate the proposed IPF-AHP-based SREM framework by examining the stability of the obtained priorities of the process improvement measures, this section presents a comparative analysis of the employed SREM with three widely accepted MCDM methods: the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [61], the Additive Ratio Assessment (ARAS) [62], and the Simple Additive Weighting (SAW) [61] method.

The aforementioned methods were selected based on their specific mathematical approaches to the normalization and processing of input data. In this way, the results obtained using MCDM methods with different mathematical foundations were compared. The TOPSIS was selected as a representative of distance-based methods, specifically those based on the Euclidean distance from the positive and negative ideal solutions. In addition, TOPSIS is the most frequently used method in the relevant literature. The ARAS method belongs to the group of additive methods based on comparing the performance of alternatives with a predefined ideal solution. The SAW method was selected as one of the basic, simplest, and linearly based methods.

Table 7 presents the ranking coefficients and the ranking obtained using the SREM (the compromise solution for α = 0.5), as well as those obtained using the conventional versions of the TOPSIS, ARAS, and SAW methods.

Table 7. Cross-method verification of ranking coefficients and alternative priorities.

i	Ranking Coefficient	SREM ($\alpha = 0.5$)	Ranking Coefficient	TOPSIS	Ranking Coefficient	ARAS	Ranking Coefficient	SAW
$i = 1$	0.206	1	0.935	1	0.845	1	0.230	1
$i = 2$	0.069	6	0.068	6	0.332	6	0.061	6
$i = 3$	0.162	2–3	0.868	4	0.735	3	0.176	4
$i = 4$	0.162	2–3	0.916	2	0.748	2	0.184	2–3
$i = 5$	0.154	5	0.839	5	0.715	4	0.166	5
$i = 6$	0.157	4	0.873	3	0.678	5	0.184	2–3

Based on the results presented in Table 7, it can be concluded that all four considered methods produce very similar rankings. The agreement is primarily reflected in the ranking of the best alternative, $i = 1$, and the worst alternative, $i = 2$. Even alternative $i = 4$ is ranked either in second place or shares second place in all considered cases. The rankings of the remaining alternatives vary slightly, which can be seen more clearly in Figure 6.

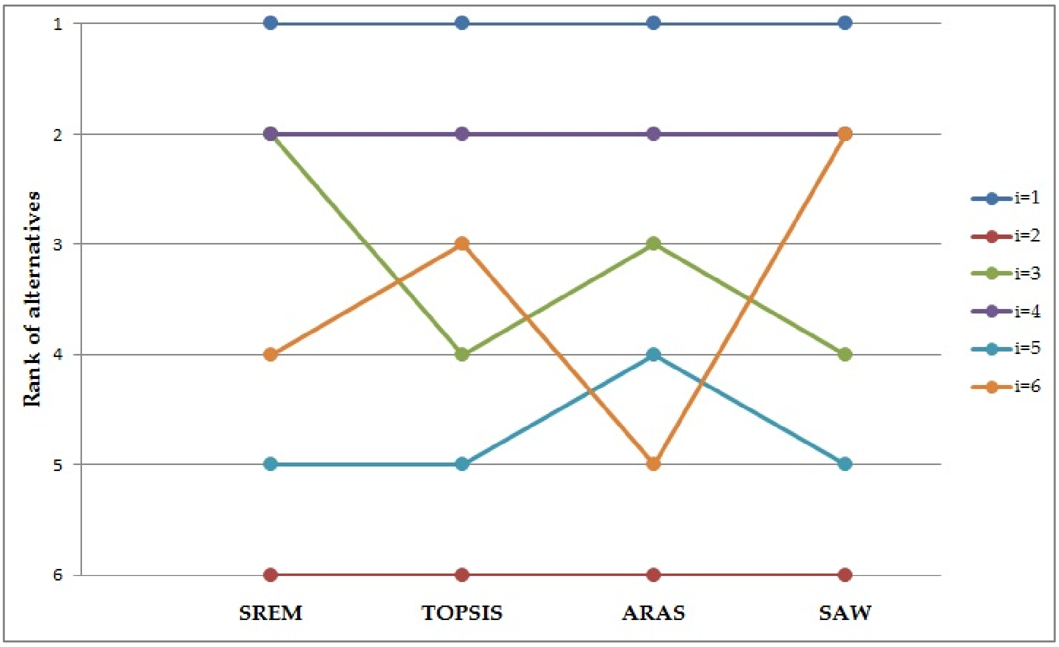

Figure 6. Comparison of alternative rankings generated by SREM, TOPSIS, ARAS, and SAW.

The differences in the rankings were caused by the very close values of the ranking coefficients for alternatives $i = 3$, $i = 4$, $i = 5$, and $i = 6$. When such small differences exist among the alternatives, the normalization procedure, as well as the mathematical logic of the method itself, plays an important role. Nevertheless, the Pearson correlation coefficients between the ranking coefficients indicate a high level of agreement among the obtained results. The Pearson correlation coefficient between the ranking coefficients obtained by the SREM and TOPSIS methods is 0.93, between SREM and ARAS it is 0.98, and between SREM and SAW it is 0.99.

Based on the above, it can be concluded that the Pearson correlation coefficients indicate an extremely high level of agreement between the employed SREM and the conventional and well-established MCDM approaches. Even the slight variations and differences in the rankings of the alternatives are not a direct consequence of model instability but are exclusively caused by the very similar performance of the alternatives. In general, the analysis demonstrated that the obtained results are reliable and that the final prioritization of the process improvement measures is mathematically well-founded and sufficiently stable for decision-making in a real industrial system.

5. Discussion

Based on consultations with the company experts, it was concluded that the obtained ranking is fully expected from the perspective of resource optimization, since the proposed methodology prioritizes measures with the fastest return on investment, i.e., so-called quick-win solutions. The measure of additional die leading ($i = 1$) justifiably occupies the first position in the ranking because it requires by far the least time for development and manufacturing and has the lowest cost, which, within the mathematical model, compensates for its more modest technical performance. On the other hand, CNC machining of the cavity and gutter ($i = 2$) is expectedly ranked last because its very high cost and implementation time represent a substantial investment, causing it to be placed at the bottom of the ranking according to the mathematical model.

However, when viewed from the perspective of engineering reasoning and risk management in the considered production facility, the ranking reveals a potential limitation of the mathematical model, as the measure of integral die leading ($i = 5$) has a high potential for risk reduction while remaining economically very affordable. In fact, this limitation arises from the subjectivity of the expert assessments, since this measure is ranked in the penultimate position according to the two most important criteria (based on the aggregated criteria weights). Nevertheless, when only the criteria weights obtained from the assessments of Expert 1 are considered, this measure consistently occupies the third position in the ranking. This further confirms that the criteria weights had a significant influence on the ranking of this measure. A very similar situation can be observed for all other measures, except for $i = 1$ and $i = 2$, as their positions in the ranking largely depend on the criteria weights, which can be clearly observed in Figure 5.

Therefore, it can be concluded that the obtained ranking is logical if the company's objective is the rapid and reliable stabilization of the process at minimal cost. If the objective is long-term reliability and tool safety, the weights of the input criteria should be adjusted in favor of technical benefits, which would substantially alter the ranking structure.

From a methodological perspective, it was demonstrated that the SREM is a very suitable choice for problems that need to be analyzed from different perspectives. Although the ranking of the alternatives did not undergo substantial changes as the value of parameter α varied from 0 to 1, i.e., the alternatives most often shifted by only one position in the ranking, there was one case in which this change was more pronounced. Specifically, when the ranking was performed using only the criteria weights obtained from Expert 2 (Table 5), alternative $i = 6$ moved from the fourth position at $\alpha = 0$, through the third position at $\alpha = 0.25$ and 0.5, and to the second position at $\alpha = 0.75$ and 1. This indicates that, when the differences between alternatives are very small, the SREM can serve as a highly useful tool, providing decision-makers with the opportunity to evaluate the potential of alternatives in a more systematic and comprehensive manner.

6. Conclusions

https://doi.org/10.3390/jmmp10070250

Within this study, a hybrid, mathematically based, IPF-AHP-based SREM framework was developed for the prioritization of process improvement measures that were previously identified as a result of the implementation of a PFMEA in the considered forging production facility. In the considered case study, a total of six process improvement measures were analyzed and evaluated with respect to seven criteria, which can be classified into three categories, namely technical, economic, and operational criteria.

The results of the study showed that the additional die-leading measure represents the best solution when all of these aspects are considered simultaneously. On the other hand, CNC machining of the cavity and gutter stands out as the least cost-effective measure, which resulted in its placement at the last position in the ranking under all considered scenarios.

The sensitivity analysis showed that the ranking of alternatives may vary depending on the value of the transformation parameter α. However, these variations did not affect the most important measure and its ranking position. Furthermore, the sensitivity analysis revealed that criteria weights may influence the ranking of alternatives, particularly those that exhibit strong advantages with respect to one group of criteria while having significantly unfavorable values with respect to the remaining criteria. Based on the obtained results, it can be concluded that the SREM is an appropriate tool when the objective is to observe the considered problem from multiple decision-making perspectives. In this way, decision-makers and experts are provided with additional information regarding the stability of the obtained solution.

From a practical perspective, it has been demonstrated that PFMEA can be effectively enhanced through the application of MCDM approaches and that the priority of the proposed process improvement measures can be evaluated from aspects other than the quality perspective required by PFMEA. Through the proposed framework, the research objective was achieved, namely to provide decision-makers in the company with support in the decision-making process, enabling them to establish a balance between implementation complexity, economic feasibility, and the time required for implementation.

The main limitation of the proposed approach is that the case study was conducted in only one company and involved consultations with only three domain experts. Future research directions will be focused on the application of the proposed approach in other relevant industries, including the use of different MCDM approaches, particularly objective methods for determining criteria weights.

Author Contributions: Conceptualization, N.K. (Nikola Kastratović) and N.K. (Nikola Komatina); methodology, N.K. (Nikola Komatina); validation, D.A., formal analysis, D.A.; investigation, M.D.; resources, N.K. (Nikola Kastratović); data curation, D.A.; writing—original draft preparation, N.K. (Nikola Kastratović) and N.K. (Nikola Komatina); writing—review and editing, D.M. and M.D.; visualization, D.A. and M.D.; supervision, D.M. and N.K. (Nikola Komatina). All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

AHP	Analytic Hierarchy Process
ARAS	Additive Ratio Assessment

AP	Action Priority
CNC	Computer Numerical Control
CR	Consistency Ratio
FMEA	Failure Mode and Effects Analysis
IFSs	Intuitionistic Fuzzy Sets
IPF-AHP	Analytic Hierarchy Process extended through the application of IVPFNs
IVPFNs	Interval-Valued Pythagorean Fuzzy Numbers
MCDM	Multi-Criteria Decision-Making
PFMEA	Process Failure Mode and Effects Analysis
PFSs	Pythagorean Fuzzy Sets
SAW	Simple Additive Weighting
RPN	Risk Priority Number
SREM	Square-Root-Based Evaluation Method
TOPSIS	Technique for Order Preference by Similarity to Ideal Solution

References

1. Rathi, M.G.; Jakhade, N.A. An Overview of Forging Processes with Their Defects. *Int. J. Sci. Res. Publ.* **2014**, *4*, 1–7.
2. Hawryluk, M.; Rychlik, M.; Więcław, M.; Jabłoński, P. Analysis of the Industrial Process of Producing a Hub Forging Used in Motorcar Power Transmission Systems—A Case Study. *J. Manuf. Mater. Process.* **2021**, *5*, 32. https://doi.org/10.3390/jmmp5020032.
3. Hawryluk, M. Review of Selected Methods of Increasing the Life of Forging Tools in Hot Die Forging Processes. *Arch. Civ. Mech. Eng.* **2016**, *16*, 845–866. https://doi.org/10.1016/j.acme.2016.06.001.
4. Karastojković, Z. Temperature as a Criterion for Classification of Cold, Warm and Hot Cracking. *Struct. Integr. Life* **2024**, *24*, 9–16.
5. Ratković, N.; Arsić, D.; Nikolić, R.R.; Delić, M.; Jovanović Pešić, Ž.; Mandić, V.; Pastorková, J. Experimental and Numerical Analysis of Rotary Friction Welding for Al-Cu Joints: Effects of Friction Time on Plastic Deformation and Joint Integrity. *Materials* **2025**, *18*, 1932. https://doi.org/10.3390/ma18091932.
6. Behrens, B.-A.; Brunotte, K.; Hübner, S.; Wester, H.; Müller, F.; Müller, P.; Wälder, J.; Matthias, T. Further Development of Wear Calculation and Wear Reduction in Cold Forging Processes. *J. Manuf. Mater. Process.* **2021**, *5*, 36. https://doi.org/10.3390/jmmp5020036.
7. Sunday, J.D.; Murillo-Marrodán, A.; García, E.; Slater, C. Investigation of Die Design Influence on Required Forces in Near-Solidus Forging of Complex Components. *J. Manuf. Mater. Process.* **2025**, *9*, 245. https://doi.org/10.3390/jmmp9070245.
8. Beker, I.; Lazarević, M.; Orošnjak, M. mSIPOC—Work Design Procedure for Complex Industrial Production Process. *Teh. Vjesn.—Tech. Gaz.* **2025**, *32*, 756–765. https://doi.org/10.17559/TV-20240718001857.
9. Komatina, N.; Petrović, N.; Pamučar, D.; Simić, V.; Marinković, D. Multi-Criteria Ranking of Industrial Presses with Respect to Operational Performance Using the Square-Root Based Evaluation Method (SREM). *Facta Univ. Ser. Mech. Eng.* **2026**, *online first*. https://doi.org/10.22190/FUME260305009K.
10. Erdebilli, B.; Yilmaz, İ.; Aksoy, T.; Hacıoglu, U.; Yüksel, S.; Dinçer, H. An Interval-Valued Pythagorean Fuzzy AHP and COPRAS Hybrid Methods for the Supplier Selection Problem. *Int. J. Comput. Intell. Syst.* **2023**, *16*, 124. https://doi.org/10.1007/s44196-023-00297-4.
11. Lazic, V.; Cukic, R.; Aleksandrovic, S.; Milosavljevic, D.; Arsic, D.; Nedeljkovic, B.; Djordjevic, M. Techno-Economic Justification of Reparatory Hard Facing of Various Working Parts of Mechanical Systems. *Tribol. Ind.* **2014**, *36*, 287.
12. Komatina, N.; Marinković, D. An Objective Multi-Attribute Decision-Making Framework for Identifying Critical Phases in Automotive Component Manufacturing. *J. Eng. Manag. Syst. Eng.* **2025**, *5*, 1–9. https://doi.org/10.56578/jemse050101.
13. Akram, M.; Luqman, A.; Alcantud, J.C.R. An Integrated ELECTRE-I Approach for Risk Evaluation with Hesitant Pythagorean Fuzzy Information. *Expert Syst. Appl.* **2022**, *200*, 116945. https://doi.org/10.1016/j.eswa.2022.116945.
14. Almeraz-Durán, S.; Pérez-Domínguez, L.A.; Luviano-Cruz, D.; Hernández Hernández, J.I.; Romero López, R.; Valle-Rosales, D.J. A Proposed Framework for Developing FMEA Method Using Pythagorean Fuzzy CODAS. *Symmetry* **2021**, *13*, 2236. https://doi.org/10.3390/sym13122236.
15. Li, H.; Lv, L.; Li, F.; Wang, L.; Xia, Q. A Novel Approach to Emergency Risk Assessment Using FMEA with Extended MULTIMOORA Method under Interval-Valued Pythagorean Fuzzy Environment. *Int. J. Intell. Comput. Cybern.* **2020**, *13*, 41–65. https://doi.org/10.1108/IJICC-08-2019-0091.

16. Ilbahar, E.; Karaşan, A.; Cebi, S.; Kahraman, C. A Novel Approach to Risk Assessment for Occupational Health and Safety Using Pythagorean Fuzzy AHP & Fuzzy Inference System. *Saf. Sci.* **2018**, *103*, 124–136. https://doi.org/10.1016/j.ssci.2017.10.025.
17. Akram, M.; Luqman, A.; Alcantud, J.C.R. Risk Evaluation in Failure Modes and Effects Analysis: Hybrid TOPSIS and ELECTRE I Solutions with Pythagorean Fuzzy Information. *Neural Comput. Applic* **2021**, *33*, 5675–5703. https://doi.org/10.1007/s00521-020-05350-3.
18. AIAG&VDA. *Failure Mode and Effects Analysis—FMEA Handbook: Design FMEA, Process FMEA, Supplemental FMEA for Monitoring & System Response*; Automotive Industry Action Group: Southfild, MI, USA, 2019.
19. Yucesan, M.; Kahraman, G. Risk Evaluation and Prevention in Hydropower Plant Operations: A Model Based on Pythagorean Fuzzy AHP. *Energy Policy* **2019**, *126*, 343–351. https://doi.org/10.1016/j.enpol.2018.11.039.
20. Božanić, D.; Puška, A.; Tešić, D.; Štilić, A.; Ullah, K.; Muhsen, Y.R.; Hezam, I.M. Fuzzy AHP—Fuzzy MABAC Model for Ranking a Combined Construction Machine—Backhoe Loader. *Facta Univ. Ser. Mech. Eng.* **2025**, *23*, 605–625. https://doi.org/10.22190/FUME250801030B.
21. Deepak, D.; Dwivedi, S.; Farwaha, H.S.; Kumar, R.; Stević, Ž.; Chandra, M.; Kumar, R.; Agrawal, A.P.; John, V. A Decision-Oriented Framework for Risk-Based Maintenance Planning in High-Performance Mechanical Systems Using Entropy-Integrated FMEA–MCDM Approaches. *J. Eng. Manag. Syst. Eng.* **2026**, *5*, 137–155. https://doi.org/10.56578/jemse050202.
22. Stevic, Z.; Ulutas, A.; Topal, A.; Marinkovic, D.; Cavoski, S. A New Objective Method for Determining Criteria Weights in MCDM Models—LOGSTA. *Int. J. Simul. Model.* **2025**, *24*, 589–600. https://doi.org/10.2507/IJSIMM24-4-738.
23. Więckowski, J.; Sałabun, W. Making More Informed Decisions in Multi-Criteria Problems under Uncertain Conditions of Criteria Weights: Drone Selection Study Case. *Facta Univ. Ser. Mech. Eng.* **2025**, *online first*. https://doi.org/10.22190/FUME240930019W.
24. Ricoy-Casas, R.M.; Lago Cabo, J.M.; Vila Biglieri, J.E.; Bozanic, D. Economics and Multi-Criteria Decision Support for Sustainable Cobalt Production. *Int. J. Econ. Sci.* **2026**, *15*, 137–157. https://doi.org/10.31181/ijes1512026237.
25. Tadić, D.; Lukić, J.; Komatina, N.; Marinković, D.; Pamučar, D. A Fuzzy Decision-Making Approach to Electric Vehicle Evaluation and Ranking. *Teh. Vjesn./Tech. Gaz.* **2025**, *32*, 1066–1075. https://doi.org/10.17559/TV-20250223002410.
26. Ervural, B.C. Evaluating the Sustainable Development Performance of G20 Economies Using an Integrated Decision-Making Framework. *J. Eng. Manag. Syst. Eng.* **2026**, *5*, 63–84. https://doi.org/10.56578/jemse050105.
27. Kushwaha, D.K.; Panchal, D.; Sachdeva, A. A Modified FMEA Approach Based Integrated Decision Framework for Overcoming the Problems of Sudden Failure and Accidental Hazards in Turbine and Alternator Unit. *Facta Univ. Ser. Mech. Eng.* **2025**, *23*, 459–478. https://doi.org/10.22190/FUME221126010K.
28. Bošković, S.; Jovčić, S.; Simic, V.; Švadlenka, L.; Dobrodolac, M.; Bacanin, N. A New Criteria Importance Assessment (CIMAS) Method in Multi-Criteria Group Decision-Making: Criteria Evaluation for Supplier Selection. *Facta Univ. Ser. Mech. Eng.* **2025**, *23*, 335–349. https://doi.org/10.22190/FUME230730050B.
29. Garg, H. A Novel Accuracy Function under Interval-Valued Pythagorean Fuzzy Environment for Solving Multicriteria Decision Making Problem. *J. Intell. Fuzzy Syst.* **2016**, *31*, 529–540. https://doi.org/10.3233/IFS-162165.
30. Yu, Z.; Wang, D.; Kassim, A.M.; Zhu, S.; Liu, Z. An Extended Interval-Valued Pythagorean Fuzzy VIKOR Approach With Logarithmic Distance Measures for Service Quality Evaluation of University Library. *J. Math.* **2025**, *2025*, 9966857. https://doi.org/10.1155/jom/9966857.
31. Zadeh, L.A. The Concept of a Linguistic Variable and Its Application to Approximate Reasoning—I. *Inf. Sci.* **1975**, *8*, 199–249. https://doi.org/10.1016/0020-0255(75)90036-5.
32. Kannan, J.; Jayakumar, V.; Pethaperumal, M. Integrating Pythagorean Fuzzy Soft Sets with SWARA and WASPAS for Enhanced MCDM in Education 5.0: A Novel Approach for next Generation Engineering Solutions. *Eng. Rev.* **2025**, *45*, 29–51. https://doi.org/10.30765/er.2582.
33. Ayyildiz, E.; Taskin Gumus, A. Interval-Valued Pythagorean Fuzzy AHP Method-Based Supply Chain Performance Evaluation by a New Extension of SCOR Model: SCOR 4.0. *Complex Intell. Syst.* **2021**, *7*, 559–576. https://doi.org/10.1007/s40747-020-00221-9.
34. Puška, A.; Marinković, D.; Nedeljković, M.; Peci, A. Evaluation of Tractors Based on Sustainability Criteria Using Multi-Criteria Decision-Making Methods. *Eng. Rev.* **2025**, *45*, 131–145. https://doi.org/10.30765/er.2706.
35. Rani, P.; Mishra, A.R.; Alshamrani, A.M.; Alrasheedi, A.F.; Atanasković, P.; Simic, V. Intuitionistic Fuzzy Distance Measure-Based Approach for Adopting Blockchain Technology in the Logistics Industry. *Facta Univ. Ser. Mech. Eng.* **2024,** *23*, 555–578. https://doi.org/10.22190/FUME240820051R.

36. Razak, S.A.; Rodzi, Z.M.; Al-Sharqi, F.; Ramli, N. Revolutionizing Decision-Making in E-Commerce and IT Procurement: An IVPNS-COBRA Linguistic Variable Framework for Enhanced Multi-Criteria Analysis. *Int. J. Econ. Sci.* **2025**, *14*, 1–31. https://doi.org/10.31181/ijes1412025176.
37. Atanassov, K.T. Intuitionistic Fuzzy Sets. In *Intuitionistic Fuzzy Sets*; Studies in Fuzziness and Soft Computing; Physica-Verlag HD: Heidelberg, Germany, 1999; Volume 35, pp. 1–137; ISBN 978-3-7908-2463-6.
38. Atanassov, K.T. *Intuitionistic Fuzzy Logics*; Studies in Fuzziness and Soft Computing; Springer International Publishing: Cham, Switzerland, 2017; Volume 351; ISBN 978-3-319-48952-0.
39. Karasan, A.; Ilbahar, E.; Kahraman, C. A Novel Pythagorean Fuzzy AHP and Its Application to Landfill Site Selection Problem. *Soft Comput.* **2019**, *23*, 10953–10968. https://doi.org/10.1007/s00500-018-3649-0.
40. Mandal, S.; Gazi, K.H.; Giri, B.C.; Salahshour, S.; Mondal, S.P. An Interval-Valued Pythagorean Trapezoidal Fuzzy Multi-Criteria Decision Making Technique for Psychiatric Disorder Diagnosis. *RAIRO-Oper. Res.* **2025**, *59*, 3851–3889. https://doi.org/10.1051/ro/2025147.
41. Yager, R.R. Pythagorean Membership Grades in Multicriteria Decision Making. *IEEE Trans. Fuzzy Syst.* **2014**, *22*, 958–965. https://doi.org/10.1109/TFUZZ.2013.2278989.
42. Yager, R.R. Properties and Applications of Pythagorean Fuzzy Sets. In *Imprecision and Uncertainty in Information Representation and Processing*; Angelov, P., Sotirov, S., Eds.; Studies in Fuzziness and Soft Computing; Springer International Publishing: Cham, Switzerland, 2016; Volume 332, pp. 119–136; ISBN 978-3-319-26301-4.
43. Cingöz, K. Evaluating Supplier Selection Criteria with Interval Pythagorean Fuzzy AHP. *OPUS Toplum Araştırmaları Derg.* **2025**, *22*, 769–778. https://doi.org/10.26466/opusjsr.1710659.
44. Kose, Y.; Civan, H.N.; Ayyildiz, E.; Cevikcan, E. An Interval Valued Pythagorean Fuzzy AHP–TOPSIS Integrated Model for Ergonomic Assessment of Setup Process under SMED. *Sustainability* **2022**, *14*, 13804. https://doi.org/10.3390/su142113804.
45. Sumrit, D. Evaluating Readiness Degree for Industrial Internet of Things Adoption in Manufacturing Enterprises under Interval-Valued Pythagorean Fuzzy Approach. *Prod. Manuf. Res.* **2022**, *10*, 226–256. https://doi.org/10.1080/21693277.2022.2064931.
46. Peng, X.; Yang, Y. Fundamental Properties of Interval-Valued Pythagorean Fuzzy Aggregation Operators: Interval-Valued Pythagorean Fuzzy Aggregation Operations. *Int. J. Intell. Syst.* **2016**, *31*, 444–487. https://doi.org/10.1002/int.21790.
47. Lestyánszka Škůrková, K.; Fidlerová, H.; Niciejewska, M.; Idzikowski, A. Quality Improvement of the Forging Process Using Pareto Analysis and 8D Methodology in Automotive Manufacturing: A Case Study. *Standards* **2023**, *3*, 84–94. https://doi.org/10.3390/standards3010008.
48. Kahraman, C. A Brief Literature Review for Fuzzy AHP. *Int. J. Anal. Hierarchy Process.* **2018**, *10*, 293-297. https://doi.org/10.13033/ijahp.v10i2.599.
49. Ohta, R.; Salomon, V.A.P.; Silva, M.B. Classical, Fuzzy, Hesitant Fuzzy and Intuitionistic Fuzzy Analytic Hierarchy Processes Applied to Industrial Maintenance Management. *J. Intell. Fuzzy Syst.* **2020**, *38*, 601–608. https://doi.org/10.3233/JIFS-179433.
50. Kahraman, C.; Öztayşi, B.; Uçal Sarı, İ.; Turanoğlu, E. Fuzzy Analytic Hierarchy Process with Interval Type-2 Fuzzy Sets. *Knowl.-Based Syst.* **2014**, *59*, 48–57. https://doi.org/10.1016/j.knosys.2014.02.001.
51. Tadić, D.; Marinković, D.; Komatina, N. A Novel IPF-AHP-FMEA Approach for Prioritizing Failure Modes in the Automotive Industry. *Proc. Inst. Mech. Eng. Part O J. Risk Reliab.* **2026**, *240*, 46–63. https://doi.org/10.1177/1748006X251361955.
52. Vahabzadeh, S.; Haghshenas, S.S.; Ghoushchi, S.J.; Guido, G.; Simic, V.; Marinkovic, D. A New Framework for Risk Assessment of Road Transportation of Hazardous Substances. *Facta Univ. Ser. Mech. Eng.* **2025**, *23*, 479–509. https://doi.org/10.22190/FUME240801007V.
53. Liu, H.C. FMEA Using Uncertainty Theories and MCDM Methods. In *FMEA Using Uncertainty Theories and MCDM Methods*; Springer Singapore: Singapore, 2016; pp. 13–27; ISBN 978-981-10-1465-9.
54. Lo, H.W.; Shiue, W.; Liou, J.J.H.; Tzeng, G.-H. A Hybrid MCDM-Based FMEA Model for Identification of Critical Failure Modes in Manufacturing. *Soft Comput.* **2020**, *24*, 15733–15745. https://doi.org/10.1007/s00500-020-04903-x.
55. Aleksić, A.; Tadić, D.; Komatina, N.; Nestić, S. Failure Mode and Effects Analysis Integrated with Multi-Attribute Decision-Making Methods Under Uncertainty: A Systematic Literature Review. *Mathematics* **2025**, *13*, 2216. https://doi.org/10.3390/math13132216.
56. Komatina, N.; Marinković, D. Optimization of PFMEA Team Composition in the Automotive Industry Using the IPF-RADAR Approach. *Algorithms* **2025**, *18*, 342. https://doi.org/10.3390/a18060342.
57. Zhang, X.; Xu, Z. Extension of TOPSIS to Multiple Criteria Decision Making with Pythagorean Fuzzy Sets: Extension of TOPSIS to Multiple Criteria Decision Making. *Int. J. Intell. Syst.* **2014**, *29*, 1061–1078. https://doi.org/10.1002/int.21676.

58. Zeng, S.; Chen, J.; Li, X. A Hybrid Method for Pythagorean Fuzzy Multiple-Criteria Decision Making. *Int. J. Inf. Technol. Decis. Mak.* **2016**, *15*, 403–422. https://doi.org/10.1142/S0219622016500012.
59. Milošević, M.R.; Milošević, D.M.; Stević, D.M.; Kovačević, M. Interval Valued Pythagorean Fuzzy AHP Integrated Model in a Smartness Assessment Framework of Buildings. *Axioms* **2023**, *12*, 286. https://doi.org/10.3390/axioms12030286.
60. Saaty, T.L. Decision-Making with the AHP: Why Is the Principal Eigenvector Necessary. *Eur. J. Oper. Res.* **2003**, *145*, 85–91. https://doi.org/10.1016/S0377-2217(02)00227-8.
61. Hwang, C.L.; Yoon, K. Methods for Multiple Attribute Decision Making. In *Multiple Attribute Decision Making: Methods and Applications a State-of-the-Art Survey*; Springer: Berlin/Heidelberg, Germany, 1981; pp. 58–191.
62. Zavadskas, E.K.; Turskis, Z. A New Additive Ratio Assessment (ARAS) Method in Multicriteria Decision-making. *Technol. Econ. Dev. Econ.* **2010**, *16*, 159–172. https://doi.org/10.3846/tede.2010.10.