SESSION D: AUTOMATIC CONTROL, IT AND ARTIFICIAL INTELLIGENCE

Minimum positioning-time for mechatronic systems

Vojislav Filipović[1], Vladimir Đorđević[1*]

[1] University of Kragujevac, Faculty of Mechanical and Civil Engineering in Kraljevo, Department for Automatic Control and Fluid Technique, Kraljevo, Serbia

ARTICLE INFO

* Correspondence: djordjevic.v@mfkv.kg.ac.rs

DOI: https://doi.org/10.46793/ET26.D06F **PAGES:** D49 – D55

ABSTRACT

The paper considers a method for transferring the state of a dynamic system to a specified state for minimum positioning time (positioning servo systems, manipulators, walking robots). The system under consideration is described by a sampled-data model based on a first-order hold element. It is a first-designed feed-forward controller using a special case of an optimal linear system based on a quadratic performance index. When the final state is fixed (final-state control), the result is open-loop control. Owing to the fact that all mechanical systems have resonant modes at high frequencies, the output of a given feed-forward controller must be converted to a smooth reference signal. For that, we use a lowpass filter implemented as an integrator. In the real system, various disturbance sources (disturbances, parameter uncertainty) exist. For such systems, a two-degree-of-freedom control strategy is proposed. In that case, in the paper, a feedback controller is designed using the superstability concept, and the solution is based on linear programming. The main contributions of the paper are: (i) design of a feedforward controller for sampled-data systems with a first-order hold element, based on optimal control theory; (ii) design of a low-pass filter for the above system; (iii) design of two-degree-of-freedom control systems where the feedback controller is based on the super-stability theory of systems.

KEYWORDS

Sampled-data systems, First-order hold, Final-state control, Mechanical resonance, Two-degree-of-freedom control, Superstability.

1. INTRODUCTION

The term mechatronics introduced by Tetsuro Mori, an engineer of the Japanese company Yaskawa Electric Corporation in 1969. Mechatronics is the synergy of mechanical engineering, electrical engineering, control theory and computer sciences. The purpose of this interdisciplinary engineering field is to control complex systems by providing hardware and software solution. Examples of such systems can be found in different industrial areas ranging from aerospace to automobile industries [1]. The mechatronics is a hot research topic with its rapid development along with artificial intelligence, internet of thing, digital twin, machine learning and deep learning [2-3]. Application of iterative learning control (ILC) for the high-precision engineering application is presented in [4].

In this paper we consider problem of transferring the state of dynamic system to a specified state within a specified time in the field of servo systems, manipulators and walking robots. The solution based on mathematical model of hand motions which are observed when one holds an object and transfers it to a specified position [5-6]. For solution of this problem two methods are known: (i) one is fixed-final state optimal control [7]; (ii) other is deadbeat control [8-9]. In this paper will be used (i) strategy and by using that strategy it is designed feedforward controller. Strategy (i) give open loop optimal control. In the reference [10] proposed final-state control using compensation input. The proposed method includes solution for fixed final state optimal control.

Final state control (FSC) is a method to obtain the feedforward input that drives an initial state to a final state in Nh sampling time (h is a sampling time) where $N \geq n$ (n is a order of the considered system). Problem belongs to a classical optimal control problem with two boundary conditions [7]. That is methodology for a design of feedforward controller. The output of the controller can excite the high frequency resonant modes of the mechanical system. In order to avoid mechanical resonance, in references [11-12], it is introduced integrator after the feedforward signal. This way, one obtains reference trajectory that balance the trade-off between minimum positioning time and smoothness of the control input. Since the minimum jerk input is smooth, it is difficult to excite the high frequency resonant modes of the mechanical systems. In this paper we introduce digital lowpass filter based on the first-order hold element.

Due to the presence of various sources of uncertainty (unmodeled dynamics, disturbances), a two-degree-of-freedom (TDOF) control strategy [13] is introduced. Here we consider design of feedback controller based on concept of super-stability which is introduced in [14]. Superstable systems is narrowed class in comparison to linear time invariant systems. Many hard problems such as static output stabilization, fixed-order controller design, uniform rejection of exogenous disturbances, etc., admit simple solutions provided that the stability requirement for the closed-loop system is replaced with superstability [15-16]. All the mentioned problems can be solved using linear programming techniques [17]. This approach has also drawbacks: (i) a controllable linear system is not necessarily superstabilizable using state feedback; (ii) in optimal control one can get upper bound for performance index and the controller is suboptimal.

Next important fact in mechatronics systems is precision. Manufacturing machines (for example, manufacturing machines as used in lithographic integrated circuit (IC) production) and scientific instruments (highly accurate scientific instruments including microscopy, increasingly large telescopes, computerized tomography scanners) have a key role in our society and the role of these mechatronic systems will increase in the future. Typical requirements of accuracy for motion positioning systems are in the micrometre or even nanometre range [18-21]. The important ingredient of Industry 5.0 will be collaboration human and robot [22].

The main contributions of the paper are: (i) design of feedforward controller for sampled-data systems based on first-order hold element; (ii) design of digital lowpass filter using first-order hold; (iii) design of two-degree-of-freedom control systems where feedback controller is designed using superstability theory of systems.

2. PROBLEM FORMULATION

Suppose that single-input single-output (SISO) continuous-time model of the plant P_c in state-space form is

$$\dot{x}(t) = A_c x(t) + b_c r(t) \tag{1}$$

$$y(t) = c_c^T x(t) \tag{2}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^1$, $y(t) \in \mathbb{R}^1$ are state, input and output of the system respectively. Corresponding matrix $A_c \in \mathbb{R}^{n\times n}$ and vectors $b_c \in \mathbb{R}^n$ and $c_c \in \mathbb{R}^n$. From the model (1) and (2) we will derive the discrete-time model P_d using first –order hold (FOH) method. So is given a more accurate discrete approximation of the continuous-time model by linearly extrapolating from current and previous input sequence elements [23]

$$u(t) = u(kh) + \frac{u(kh) - u((k-1)h)}{h}(t - kh) \tag{3}$$

where h is sampling period. The discrete-time sequence $y(kh)$ generates the sampler S_h. The FOH dynamic is presented in the Figure 1.

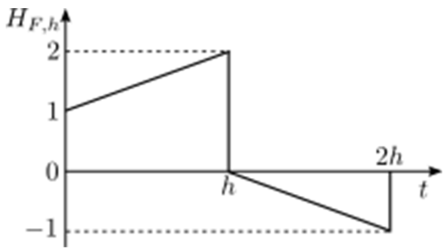

Figure 1: Impulse response of FOH

Sampled-data model has a form as in the Figure 2.

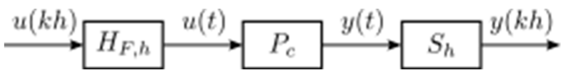

Figure 2: Sampled-data model

The discrete-time model based on the FOH has the following form [23]

$$\begin{bmatrix} x((k+1)h) \\ u(kh) \end{bmatrix} = \begin{bmatrix} A & b_1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(kh) \\ u((k-1)h) \end{bmatrix} + \begin{bmatrix} b_2 \\ 1 \end{bmatrix} u(kh) \tag{4}$$

$$y(kh) = \begin{bmatrix} c_c & 0 \end{bmatrix} \begin{bmatrix} x(kh) \\ u((k-1)h) \end{bmatrix} \tag{5}$$

where

$$A = e^{A_c h} \tag{6}$$

$$b_1 = \int_0^h \left(\frac{\eta}{h} - 1 \right) e^{A_c \eta} b_c d\eta \tag{7}$$

$$b_2 = \int_0^h \left(2 - \frac{\eta}{h} \right) e^{A_c \eta} b_c d\eta \tag{8}$$

Remark 1. Reference [24] considers FOH for multivariable systems and reference [25] describes generalized hold element.

3. DESIGN OF FEEDFORWARD CONTROLLER

Final state control (FSC) is a method for control signal design that drives an initial state to a final state in N sampling steps [10]. That is a special case of optimal control of linear systems based on a quadratic performance index. The solution is an open-loop control. In what follows we consider criterion for minimizing control energy. The cost function is

$$J(N) = \sum_{k=0}^{N-1} q_k u^2(kh) \tag{9}$$

where $N \geq n+1$, $n = \dim x(\cdot)$, and q_k, $k = 0,1,\ldots,N-1$ is a scalar weighting factor.

Now we will formulate Theorem where is described: (i) the control signal always exists; (ii) the analytical expression for feedforward control signal is given.

Theorem. Let us suppose that for system (4) and criterion (9) is satisfied

1. $\left(\begin{bmatrix} A & b_1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} b_2 \\ 1 \end{bmatrix} \right)$ is completely controllable,
2. The matrix

$$Q(N) = \begin{bmatrix} q_0 & & & \mathbf{0} \\ & q_1 & & \\ & & \ddots & \\ \mathbf{0} & & & q_{N-1} \end{bmatrix} > 0$$

3. $N \geq n+1$
4. Lagrange multiplier $\lambda(N) \in \mathbb{R}^{n+1}$.

Then

A) Control input $U(N)$ always exists where $U^T(N) = \left[u(0), u(h), \ldots, u((N-1)h) \right]$.

B) The optimal feedforward input $U(N)$ is given with the next relation

$$U(N) = Q^{-1}(N) W^T(N) \left(W(N) Q^{-1}(N) W^T(N) \right)^{-1} \left(\begin{bmatrix} x(Nh) \\ u((N-1)h) \end{bmatrix} - \begin{bmatrix} A & b_1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(0) \\ u(-h) \end{bmatrix} \right)$$

where

$$W(N) = \left[\begin{bmatrix} A & b_1 \\ 0 & 0 \end{bmatrix}^{N-1} \begin{bmatrix} b_2 \\ 1 \end{bmatrix}, \ \begin{bmatrix} A & b_1 \\ 0 & 0 \end{bmatrix}^{N-2} \begin{bmatrix} b_2 \\ 1 \end{bmatrix}, \ \ldots, \ \begin{bmatrix} b_2 \\ 1 \end{bmatrix} \right].$$

Proof: Omitted.

On the figure 3, the feedforward controller is presented

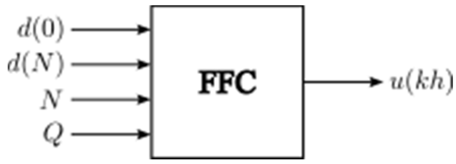

Figure 3: Feedforward controller (FFC)

where $d(0)=\left[x(0), \quad u(-h)^T\right]$ and $d(N)=\left[x(Nh), \quad u((N-1)h)\right]^T$.

For realization of optimal feedforward controller it is necessary to use initial state of the system $x(0)$, final state of the system $d(Nh)$, sampling steps N and weighting matrix Q. The control signal $u(kh)$ can be nonsmooth and can excites mechanical resonant modes. To prevent excitation of the high frequency resonant modes which result in residual vibration it is possible to introduce digital lowpass filter. So is given trade-off between minimum positioning time and smoothness of the control signal. We use lowpass filter in the form of integrator with the transfer function $G_F(s)=1/s$. We need next lemma.

Lemma. In the continuous time the transfer function of filter is $G_F(s)=1/s$. The digital lowpass filter, based on first-order-hold element, has a form

$$u_{ff}(kh)=u_{ff}((k-1)h)+\frac{h}{2}u((k-1)h)+\frac{h}{2}u(kh)$$

where $u(\cdot)$, $u_{ff}(\cdot)$ and h are input of the filter, output of the filter and sampling time respectively.

Proof: Omitted.

The structure of feedforward control system is presented in the figure 4.

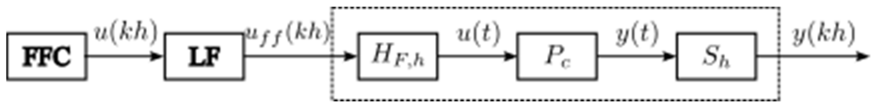

Figure 4: Feedforward control systems: FFC – feedforward controller; LF lowpass filter; SDS – sampled data system

4. TWO-DEGREE-OF-FREEDOM CONTROL SYSTEM

Due to presence of various disturbance sources (unmodeled dynamics, disturbance) it is necessary to introduce feedback controller and so we will give two-degree-of –freedom control (TDOF) systems. We will present design of static output feedback controller. That problem is very difficult and first result presented in reference [27], but the problem is still remains open [28]. Here we use methodology of superstability introduced in [15] and [16]. Superstability is a sufficient condition for stability and is formulated using elements of matrix for closed loop systems. In the classical stability theory stability is formulated in terms of the eigenvalues of matrix of closed loop systems.

The problems with standard stability of control systems are [15]

a) stability is an asymptotic property and at the initial time instants large deviation in the system trajectory can occur,

b) in the parameter space the set of stable systems and the set of stabilizing controllers are nonconvex.

The superstable systems are narrower class of systems and can overcome above difficulties. In the frame of superstable systems it is possible to solve static output stabilization, simultaneous stabilization, robust stabilization etc.

The superstability has some drawbacks

c) it is not possible to guarantee this property for an arbitrary controlled systems,

d) the obtained controllers are suboptimal.

The similar problems, under the name of diagonal stability, is considered in [29] and [30].

Let us consider the system (4). The matrix

$$H=\begin{bmatrix} A & b_1 \\ 0 & 0 \end{bmatrix}, \qquad H=\left[\left(h_{ij}\right)\right]\in \mathbb{R}^{(n+1)\times(n+1)} \tag{10}$$

is discrete superstable if

$$\alpha = \|H\|_1 = \max_{1\le i\le n+1}\left(\sum_{j=1}^{k+1}\left|h_{ij}\right|\right) < 1 \tag{11}$$

and $1-\alpha$ is degree of superstability of matrix H.

The static output feedback has form

$$u(kh) = \beta y(kh) \tag{12}$$

From relation (4), (5), (10) and (11) it follows that

$$\begin{bmatrix} x((k+1)h) \\ u(kh) \end{bmatrix} = \left(\begin{bmatrix} A & b_1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} b_2 \\ 1 \end{bmatrix}\begin{bmatrix} c_c & 0 \end{bmatrix}\beta\right)\begin{bmatrix} x(kh) \\ u((k-1)h) \end{bmatrix} = (H+T\beta)\begin{bmatrix} x(kh) \\ u((k-1)h) \end{bmatrix} \tag{13}$$

where $T = \left[\left(t_{ji}\right)\right] \in \mathbb{R}^{(n+1)\times(n+1)}$ and

$$T = \begin{bmatrix} b_2 \\ 1 \end{bmatrix}\begin{bmatrix} c_c & 0 \end{bmatrix}$$

The matrix

$$M = H + T\beta, \qquad \beta \in \mathbb{R}^1 \tag{14}$$

is the matrix for closed-loop system and β is a scalar gain of the static output controller. Condition for superstability of matrix M is

$$\sum_{j=1}^{n+1}\left|m_{ij}(\beta)\right| < 1, \qquad i = 1,2,\ldots,n+1 \tag{15}$$

where analytical expression for $m_{ij}(\cdot)$, based on relation (14), has a form

$$m_{ij}(\beta) = h_{ij} + t_{ij}\beta \tag{16}$$

In order to find scalar gain of controller one must to solve next linear programming problem

$$\begin{aligned} &\min \alpha \\ &\sum_j n_{ij} \le \alpha, \qquad i = 1,2,\ldots,n+1 \\ &-n_{ij} \le m_{ij}(\beta) \le n_{ij}, \qquad i,j = 1,2,\ldots,n+1, \qquad i \ne j \end{aligned} \tag{17}$$

In relation (17) variables are α, β and $N = \left[\left(n_{ij}\right)\right] \in \mathbb{R}^{(n+1)\times(n+1)}$.

The interpretation of solution of linear programming problem, relation (17), is next: Assume that α, β are solution of problem (17). Then

A) if $\alpha < 1$ the controllers $u(kh) = \beta y(kh)$ ensures superstability of matrix M,

B) if $\alpha \ge 1$ the matrix M is not superstable for all β.

For the TDOF control systems it is necessary to find discrete transfer function for state space model (4) and (5). The transfer function is [23]

$$P_d(z) = \begin{bmatrix} c & 0 \end{bmatrix}\left(zI - \begin{bmatrix} A & b_1 \\ 0 & 0 \end{bmatrix}\right)^{-1}\begin{bmatrix} b_2 \\ 1 \end{bmatrix} \tag{18}$$

From the last relation it is easy to find function $P_d\left(q^{-1}\right)$ based on pulse transfer operator. Now the block diagram for TDOF control system is given in the figure 5.

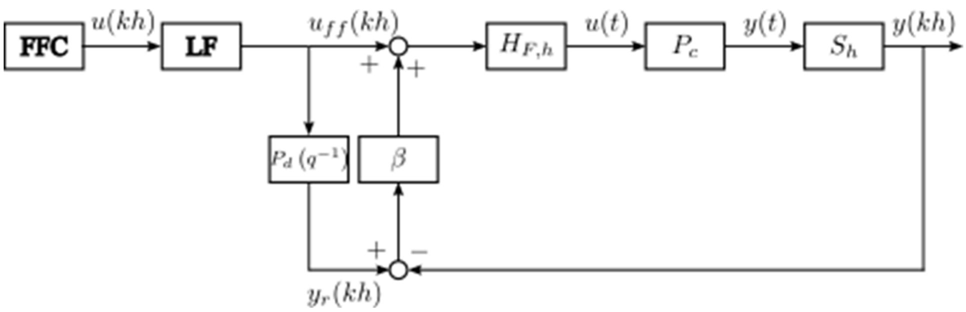

Figure 5: Two-degree-of-freedom control systems with feedforward input based on final state control theory

In the proposed control strategy, the feedback controller works only when output $y(kh)$ of the actual plant and the output of the model $y_r(kh)$ differ.

5. CONCLUSIONS

In the paper, the design of a controller for mechatronic systems is considered that minimises positioning time. We consider sampled-data system in which the implemented first-order hold elements. First, feedforward controller is designed. The design is based on optimal control theory with fixed final state of the system. To avoid mechanical resonance, after the feedforward controller, the lowpass digital filter is used. In the real systems exists various sources of uncertainty (disturbance, unmodeled dynamics). Their compensation can be performed with two-degree-of-freedom control systems. The feedback control is designed using superstability concept in control theory. Further investigations will be dedicated to extend results of the paper to multivariable systems, and design of feedback controller in the form of fixed order controllers.

ACKNOWLEDGEMENTS

This study was supported by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia from the grant 451-03-34/2026-03/200108.

REFERENCES

[1] A. Kumar, P. Kumar, S. Rathee,, and B. Kumar (Eds.). “Mechatronics: concepts, tools, applications, and new trends”, CRC Press, Boca Raton (USA), (2025)

[2] P. Naveenkumar, V. Sharma, G. Devarajan, A. Abdullah and A.A. Elngar, “Handbook of AI-based Mechatronics Systems and Smart Solutions in Industrial Automation”, CRC Press, , Boca Raton (USA), (2026)

[3] A. Azizi (Ed.), “Intelligent Mechatronic Control Systems”, Springer, (Singapore), (2026)

[4] D. Kostić, M. Matijević, V. Filipović and S. Ćuković, “Implementation of iterative learning control in a walking beam motion system with a flexible coupled robotic mechanism”. Advances in Mechanical Engineering, Vol. 17(8), https://doi.org/10.1177/16878132251369355, (2025)

[5] T. Totani and S Miyakawa, "A mathematical model of hand transfer motion for application to manipulator control.", Trasactions of the ASME, Vol. 102, pp. 152-157, https://doi.org/10.1115/1.3139655, (1981)

[6] J. Heo et al., “Hand Tracking: Survey”, International Journal of Control, Automation, and Systems, Vol. 22(6), pp. 1761-1778, https://doi.org/10.1007/s12555-024-0298-1, (2024)

[7] F.L. Lewis, V. Draguna, and V.L. Syrmos, “Optimal control”, John Wiley & Sons, New Jersey (USA), (2012)

[8] N. Touati and I. Saidi, "Dahlin deadbeat internal model control for discrete MIMO systems", International journal of electrical and computer engineering systems, Vol. 15(6), pp. 483-490, https://doi.org/10.32985/ijeces.15.6.2, (2024)

[9] H. Ren, C. Gan, C. Zhang, H. Wang, S. Wang, J. Sun, and R. Qu, "Iterative Learning Compensation for Parameter Robustness Enhancement of Deadbeat Current Control in FMDRM Drives", IEEE Transactions on Industrial Electronics, Vol. 72(11), pp. 11048-11058, https://doi.org/10.1109/TIE.2025.3557995, (2025)

[10] T. Totani, and H. Nishimura, "Final-state control using compensation input", Transactions of the Society of Instrument and Control Engineers, Vol. 30(3), pp. 253-260, https://doi.org/10.9746/sicetr1965.30.253, (1994)

[11] Y. Mizoshita, S. Hasegawa and K Takaishi, "Vibration minimized access control for disk drives", IEEE Transactions on Magnetics, Vol. 32(3), pp. 1793-1798, https://doi.org/10.1109/20.492867, (1996)

[12] T. Yamaguchi, M. Hirata and C. Pang, (Eds), “High-speed precision motion control”, CRC press, Boca Raton (USA), (2017)

[13] L. Kevizky and C. Bányász, “Two-degree-of-freedom control systems: The Youla parameterization approach”, Academc Press, London (UK), (2015)

[14] B.T. Polyak and M. E. Halpern, "Optimal design for discrete-time linear systems via new performance index", International Journal of Adaptive Control and Signal Processing Vol 15(2), pp. 129-152, DOI: 0.1002/acs.647, (2001)

[15] B.T. Polyak and P.S. Shcherbakov, "Superstable linear control systems. I. Analysis", Automation and Remote Control, Vol. 63(8), pp. 1239-1254, https://doi.org/10.1023/A:1019823208592, (2002)

[16] B.T. Polyak and P.S. Shcherbakov, "Superstable linear control systems. II. Design", Automation and Remote Control, Vol. 63(11) pp. 1745-1763, https://doi.org/10.1023/A:1020999113912, (2002)

[17] M.A. Dahleh and I. J. Diaz-Bobillo, “Control of uncertain systems: a linear programming approach”, Prentice-Hall, NEW Jersy (USA), (1994)

[18] S. Devasia, E. Eleftheriou and S.O.R. Moheimani, "A Survey of Control Issues in Nanopositioning", IEEE Transactions on Control Systems Technology, Vol. 15(5), pp. 802-823, https://doi.org/10.1109/TCST.2007.903345, (2007)

[19] M. Hirata and T. Kidokoro. "Servo performance enhancement of motion system via a quantization error estimation method—Introduction to nanoscale servo control", IEEE Transactions on Industrial Electronics Vol. 56(10), pp. 3817-3824, https://doi.org/10.1109/TIE.2009.2015751, (2009)

[20] C.X. Li, , G.Y. Gu, M.J. Yang and L.M. Zhu, "High-speed tracking of a nanopositioning stage using modified repetitive control", IEEE Transactions on Automation Science and Engineering, Vol. 14(3), pp. 1467-1477, https://doi.org/10.1109/TASE.2015.2428437, (2015)

[21] J. Ling, Z. Feng, L. Chen, Y. Zhu, and Y. Pan, "Neural network-based iterative learning control of a piezo-driven nanopositioning stage", Precision Engineering, Vol. 81, pp. 112-123, https://doi.org/10.1016/j.precisioneng.2023.02.006, (2023)

[22] U.E. Ogenyi, J. Liu, C. Yang, Z. Ju and H. Liu, "Physical human–robot collaboration: Robotic systems, learning methods, collaborative strategies, sensors, and actuators", IEEE Transactions on Cybernetics, Vol 51(4), pp. 1888-1901, https://doi.org/10.1109/TCYB.2019.2947532, (2019)

[23] J.I. Yuz, and C.G. Graham, "Sampled-data models for linear and nonlinear systems", Springer, London (UK), (2014)

[24] V. Filipović, M. Matijević and S. Ćuković, "Design of LQ regulators with prescribed degree of stability for dual-rate systems based on reinforcement learning", Measurement and Control, Vol. 58(7), pp. 857-869, https://doi.org/10.1177/00202940241287638, (2025)

[25] V. Filipovic, M. Matijevic, and D. Kostić. "Digital preview controller design using reinforcement learning", International Congress Motor Vehicles & Motors 2024, 10 October – 11 October, 2024, Kragujevac (Serbia), (2024)

[26] B. Wittenmark, K.E. Årzén, and K.J. Åström, "Computer control: An overview", IFAC Professional Brief, (2002)

[27] W.S. Levine and M. Athans, "On the determination of the optimal constant output feedback gains for linear multivariable systems", IEEE Transactions on Automatic control, Vol. 15(1), pp. 44-48, https://doi.org/10.1109/TAC.1970.1099363, (1970)

[28] V.D. Blondel, E.D. Sontag, M. Vidyasagar, and J.C. Willems (eds.), "Open problems in mathematical systems and control theory", Springer, London (UK), 2012

[29] D.D. Šiljak, "Large-scale dynamic systems: stability and structure", North-Holland, New York (USA), (1978)

[30] E. Kaszkurewicz and A. Bhaya, "Matrix diagonal stability in systems and computation", Birkhäuser, Boston (USA), 2012.