

From Point-Valued to Interval-Valued Bipolar Neutrosophic Soft Sets: The Impact of Uncertainty Representation on Decision-Making

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Decision-making under uncertainty depends not only on what an expert believes, but on how precisely that belief can be stated. Bipolar neutrosophic soft sets (BNSSs) describe every alternative–attribute pair by three positive and three negative membership degrees, each of them a single real number; the model therefore presupposes that the expert is able to commit to an exact value. Interval-valued bipolar neutrosophic soft sets (IVBNSSs) replace each of these six degrees by an interval, so that the width of the interval itself encodes the confidence of the assessment. Energy-based decision-making algorithms are available for both structures. Svičević et al. (2025) introduced the energy of a BNSS as a signed sum of the singular values of its six characteristic matrices, a construction analogous to graph energy and to the nuclear norm. Stojanović et al. (2026) extended the idea to IVBNSSs, where twelve infimum and supremum matrices give rise to an optimistic energy E^O , a pessimistic energy E^P , and a total energy E as their arithmetic mean. Both algorithms rank alternatives by a leave-one-out principle: the alternative whose removal leaves the system with the lowest residual energy is declared optimal.

What neither study addresses is the comparative question: given the same decision problem, what does the richer representation actually contribute? We answer it by solving the same problem under two representations. A point-valued BNSS is taken as the baseline, and each point value x is then widened into an interval whose half-width δ models the epistemic uncertainty of the expert,

$$X^\delta(e)(u) = [\max(a_x, x - \delta_e), \min(b_x, x + \delta_e)] \quad (1)$$

where $(a_x, b_x) = (0,1)$ for the positive components and $(a_x, b_x) = (-1,0)$ for the negative ones, and where δ_e may vary across attributes. Widening produces a family of IVBNSSs indexed by δ , all of which encode the same point estimates but differ in the amount of admitted imprecision. A first observation anchors the comparison: for $\delta = 0$ every interval collapses to a point, the infimum and supremum matrices coincide, and consequently

$$E^O(B_A) = E^P(B_A) = E(B_A), \text{ for } \delta = 0, \quad (2)$$

so that the IVBNSS algorithm reduces exactly to the BNSS algorithm. This was verified numerically for every alternative to within 10^{-9} , which confirms that any subsequent divergence is attributable to the interval representation alone and not to implementation differences.

The benchmark is the candidate-selection problem of Svičević et al. (2025), with five alternatives and three attributes. Our implementation reproduces the published BNSS energies $E(B_1) = 0.2566, E(B_2) = 0.5237, E(B_3) = 0.1633, E(B_4) = 0.7045$ and $E(B_5) = 1.1541$, and the ranking $u_3 > u_1 > u_2 > u_4 > u_5$. Two widening regimes were then scanned for $\delta \in [0, 0.5]$: a uniform regime, in which every entry is widened equally, and a heterogeneous regime with attribute-dependent half-widths $(\delta_1, \delta_2, \delta_3) = (0.5\delta, \delta, 2\delta)$, representing an expert who is confident about the first attribute and markedly less so about the third.

Table 1. Mean energy gap $|E^O - E^P|$ (averaged over the leave-one-out sets) and induced ranking under uniform and heterogeneous interval widening.

δ	Mean gap (U)	Ranking (uniform)	Mean gap (H)	Ranking (heterogeneous)
0.00	0.000	$u_3 > u_1 > u_2 > u_4 > u_5$	0.000	$u_3 > u_1 > u_2 > u_4 > u_5$
0.10	4.007	$u_3 > u_1 > u_2 > u_4 > u_5$	4.391	$u_3 > u_1 > u_2 > u_4 > u_5$
0.15	5.693	$u_3 > u_1 > u_2 > u_4 > u_5$	6.082	$u_3 > u_1 > u_2 > u_4 > u_5$
0.20	7.386	$u_3 > u_1 > u_2 > u_4 > u_5$	7.669	$u_1 > u_3 > u_2 > u_4 > u_5$
0.30	10.068	$u_3 > u_1 > u_2 > u_4 > u_5$	10.462	$u_1 > u_3 > u_2 > u_4 > u_5$
0.40	12.462	$u_3 > u_1 > u_4 > u_2 > u_5$	13.004	$u_2 > u_3 > u_1 > u_4 > u_5$
0.50	14.952	$u_3 > u_1 > u_2 > u_4 > u_5$	14.497	$u_2 > u_3 > u_1 > u_4 > u_5$

The two regimes behave very differently. Under uniform widening, the optimal alternative remains unchanged over the entire scanned range. The ranking is also highly stable, with only temporary changes in the ordering of the lower-ranked alternatives as the interval width increases. Under heterogeneous widening, however, the optimal alternative itself changes. The initial optimum u_3 is replaced by u_1 for intermediate levels of uncertainty and, for wider intervals, by u_2 . The dependence is not monotonic, partly because clipping at the domain boundaries progressively limits further interval expansion. In both regimes, the mean energy gap generally increases with the interval width and provides information that is unavailable in the point-valued model: it quantifies the divergence between the optimistic and pessimistic endpoint-based evaluations, independently of the ranking itself.

The comparison suggests a practical conclusion. Uniform inflation of uncertainty is close to rank-neutral, because it shifts the energy level of all alternatives in a similar way; the additional modelling effort of the interval-valued representation is then largely redundant, and the point-valued BNSS answer is robust. The interval-valued extension earns its cost precisely when uncertainty is asymmetric across attributes or alternatives, since only then does the optimistic–pessimistic decomposition redistribute weight between alternatives and change the decision. This gives a concrete, testable criterion for when the move from BNSSs to IVBNSSs is justified, rather than the customary argument that the latter is merely mathematically richer. Ongoing work addresses a robustness index derived from the critical value of δ at which the optimum first changes, and its use as a confidence indicator for the recommendation itself.

Keywords: bipolar neutrosophic soft set, interval-valued bipolar neutrosophic soft set, energy, nuclear norm, decision-making

References:

- Ali, M., Son, L. H., Deli, I., & Tien, N. D. (2017). Bipolar neutrosophic soft sets and applications in decision making. *Journal of Intelligent & Fuzzy Systems*, 33(6), 4077–4087. <https://doi.org/10.3233/JIFS-17999>
- Deli, I., Şubaş, Y., Smarandache, F., & Ali, M. (2016). Interval valued bipolar fuzzy weighted neutrosophic sets and their application. *2016 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, 2460–2467. <https://doi.org/10.1109/FUZZ-IEEE.2016.7738002>
- Stojanović, N., Svičević, M., Stojanović, T., Vučičević, N., & Andrić, F. (2026). A novel decision-making algorithm based on interval-valued bipolar neutrosophic soft sets. *Information*, 17(8), 772. <https://doi.org/10.3390/info17080772>
- Svičević, M., Vučičević, N., Andrić, F., & Stojanović, N. (2025). Analyzing decision-making processes using the energy of bipolar neutrosophic soft sets. *International Journal of Intelligent Systems*, 2025, 1820548. <https://doi.org/10.1155/int/1820548>