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CONDITIONAL SPECTRAL ANALYSIS OF VEHICLE VIBRATION DATA

ABSTRACT: Conditional spectral analysis is an advanced statistical and engineering method that has application in many areas of science and engineering. Its ability to e.g. isolate specific influences, remove background noise, and identifies hidden patterns makes it very useful in vehicle dynamics analysis. In this research, an example of application of conditional spectral analysis to determine the contributions of vertical wheels' vibrations to vibrations at the specific point on the car body was presented. For this purpose, an experiment was designed in order to measure vertical accelerations at the centres of all four wheels of a passenger car and at front-left shock absorbers connecting point with the car body. A multiple input – single output (MISO) model with four inputs and one output was created and appropriate conditional spectral analysis was done. The obtained results in the form of optimal frequency response functions and partial coherence function confirm the importance of decoupling the inter-connections between inputs in order to determine the exact contribution of each of them to the output. In this way, clear guidelines for vehicle dynamics optimization may be obtained.

KEYWORDS: conditional spectral analysis, vehicle, vibration, MISO model

USLOVNA SPEKTRALNA ANALIZA PODATAKA O OSCILOVANJU VOZILA

REZIME: Uslovna spektralna analiza je napredna statistička i inženjerska metoda koja se primenjuje u mnogim oblastima nauke i tehnike. Njena sposobnost da, na primer, izoluje određene uticaje, ukloni pozadinski šum i identifikuje skrivene obrasce čini je veoma korisnom u analizi dinamike vozila. U ovom istraživanju prikazan je primer

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primene uslovne spektralne analize za određivanje doprinosa vertikalnih vibracija točkova vibracijama na određenoj tački karoserije automobila. U tu svrhu osmišljen je eksperiment radi merenja vertikalnih ubrzanja u centrima sva četiri točka putničkog automobila, kao i na mestu spoja prednjeg levog amortizera sa karoserijom. Formiran je model sa više ulaza i jednim izlazom (MISO) i sprovedena je odgovarajuća uslovna spektralna analiza. Dobijeni rezultati, u vidu optimalnih funkcija frekventnog odziva i funkcije parcijalne koherencije, potvrđuju značaj razdvajanja međusobnih veza između ulaza kako bi se utvrdio tačan doprinos svakog od njih izlazu. Na taj način se mogu dobiti jasne smernice za optimizaciju dinamike vozila.

KLJUČNE REČI: uslovna spektralna analiza, vozilo, vibracije, model “više ulaza-jedan izlaz”

INTRODUCTION

Methodology of conditioned spectral analysis (CSA), although originating from seismology, have found its application in many other branches of science. It is the advanced statistical and engineering method that enables the analysis of frequency characteristics of signals and time series. Unlike classical spectral analysis, which views the system as a whole, conditional spectral analysis isolates specific influences, removes background noise, and identifies hidden patterns (hidden frequencies) depending on the data context.

In vehicle dynamics, CSA represents a powerful mathematical and experimental method used for system identification and vibration analysis under the influence of multiple interconnected inputs. It enables separation (decoupling) of excitation sources. This is crucial, because modern vehicles are extremely complex mechanical systems in which the mix of vibrations occurs that originates from: engine, gearbox, tires, steering forces, aerodynamic drag and road micro-roughness. Thus, some of the key applications of CSA in automotive industry and vehicle testing include: separation of noise and vibration sources (Noise, Vibration, and Harshness - NVH analysis), assessment of comfort and impact on the driver (ride comfort) and dynamic tracking of rotating components (order tracking).

In NVH analysis, CSA may determine which amount of car body vibrations originate directly from the engine operation and which amount comes from the tire-road contact at different speeds. It can analyse multiple vibration sources as in Chen et al. (2018), locate the specific noise source contribution as in Leclerc et al. (2005) or identify vibro-acoustical sources of noise emitted inside of vehicle compartment as in Hosseini Fouladi et al. (2009). CSA is used in vehicle ergonomics to determine how human body reacts to specific vibrations in Demić et al. (2002) and Demić & Lukić (2009), which aids the development of better suspension systems and seats as explained in Qui & Griffin (2004).

Analysis of influence of road roughness on stochastic loads of steering system tie-rod during straight line drive on different roads was conducted using CSA in Miloradović & Glišović (2004). It was concluded that the type of the road has influence on the intensity of power spectrum of tie-rod's normal load – worse roads induce higher normal loads during drive with the same constant speed. Also, they reduce the intensity of optimal function of frequency response of the channel “vertical acceleration at the centre of the wheel – tie rod's normal load”. Another example of the application of CSA in identification of vehicle system dynamics from the aspect of interaction between the steering and suspension systems of a passenger car can be found in Miloradović et al. (2022). As a result of the spectral analysis, the nonparametric model was identified in the form of discrete values of frequency characteristics.

In this research, CSA was used to determine the influence of the vertical vibrations of all four wheels of the passenger car on vertical vibration at the chosen point of car body – the shock absorbers connection point to the car body. Source data originate from vibration measurements on vehicle driving along different straight roads with different constant speeds. Since the vibrations were measured during straight-line driving at constant speeds, vibration data are ideal for CSA as they fall into the class of stationary stochastic processes. This means that the statistical properties of the signal are stable over time, which drastically simplifies the task of identifying the structural resonances of the vehicle body and the transfer paths from the excitation source.

EXPERIMENTAL SETUP

For measurement purposes, four vertical acceleration sensors HBM B12/200 were placed at the centres of all four wheels (inputs) and one identical sensor at the front left shock absorbers connecting point to the car body (output), Figure 1. The inputs generate the combination of deterministic excitation frequencies (imbalance of tires, bearings,...) and stochastic “noise” (road micro-structure). The “output” (shock absorber connection point) filters those excitations through suspension dynamics. This approach is crucial because the shock absorber mounting point is subjected to the direct transmission of high-frequency forces from the wheel to the chassis, which directly affects material fatigue, acoustic comfort and the longevity of the vehicle structure.

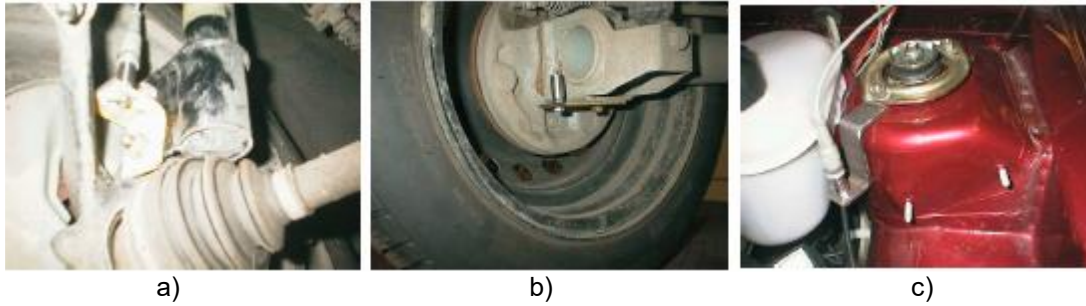


Figure 1 Placement of vertical acceleration sensors: a) front wheels, b) rear wheels, c) shock absorbers connection point to car body (Miloradović et al., 2022)

The coupling of experimental and theoretical vehicle dynamics, or the interface between vehicle dynamics, road and environment, manifests itself at the stage of recording vertical accelerations at the wheel centres, Figure 2. From reference point B, Figure 2 a), located at the centre of the front-left wheel, connections extend to the centres of the other wheels (points C, D, and E), to the road roughness at the tire-ground contact patch (point O), and to a selected point on the vehicle car body, specifically point A, where the left front shock absorber connects to the vehicle car body. Figure 2 b) presents the corresponding model for nonparametric identification of the influence of wheel vibrations on the vibration of the car body with four coupled inputs (vertical accelerations at the centres of the four wheels) and one output (vertical acceleration at the shock absorbers connecting point to car body).

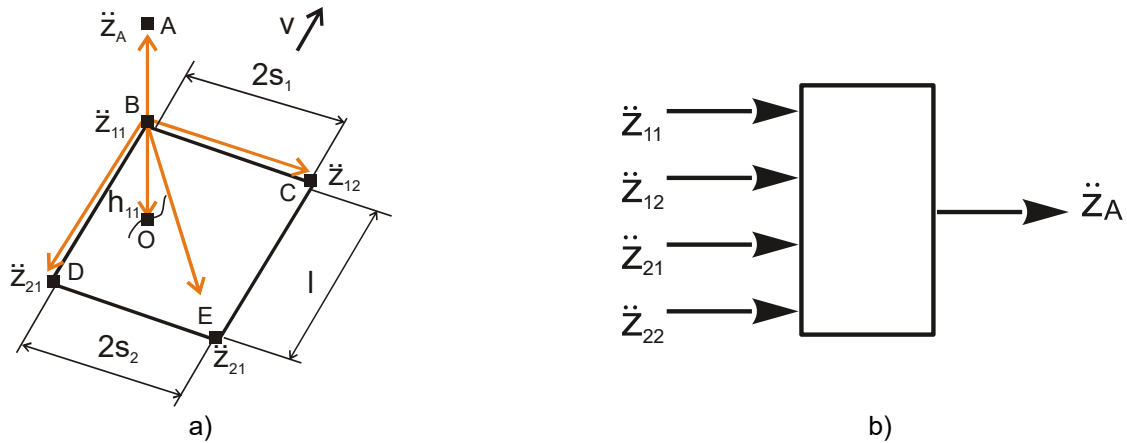


Figure 2 Interface between vehicle dynamics and road excitations in time domain: a) at the level of acquired vertical accelerations at the centres of the wheels (B, S, D, E) and at the shock absorbers connecting point to car body (A), b) model used for nonparametric identification

Vertical acceleration at the centre of the front-left wheel, $\ddot{z}_{11}$, is coupled with the other state variables (vertical accelerations at the front-right wheel, $\ddot{z}_{12}$, the rear-left wheel, $\ddot{z}_{21}$, rear right wheel, $\ddot{z}_{22}$, or at the shock absorber connecting point, $\ddot{z}_A$) and with input from the road (road roughness height h_{11}), serving as a starting point for tracking mechanical and functional couplings from a performance perspective. Vehicle speed, v , as a motion regime parameter, along with the vehicle's geometric characteristics (wheelbase, l , front axle track, $2s_1$, and rear axle track, $2s_2$) determine the road's influence on vehicle vibration and define the vehicle's excitation characteristics.

If the vehicle is designed with identical track widths across both axles ($2s_1 = 2s_2$), then the front and rear wheels on the same side of the vehicle experience the same excitation from road roughness, phase-shifted by a time interval Δt that corresponds to the time it takes the vehicle, moving at speed v , to cover a distance l equal to the wheelbase:

$\Delta t = \frac{l}{v}$. If the vehicle has different track widths on its axles ($2s_1 \neq 2s_2$), then each wheel goes over different road roughness profile. Stronger coupling between the vibrations of wheel centres on the same axle may occur if a dependent suspension system is used on that axle.

The data acquired during the straight-line highway drive of the vehicle with constant speed of $v = 70$ km/h were analysed in this paper. The chosen sample rate was 100 Hz. It makes the Nyquist frequency to be 50 Hz, as the maximum frequency that can be sensed by the sensors without aliasing. In automotive industry, the frequency range from 0 to 50 Hz is extremely critical because it covers the primary ride comfort and vehicle macro-dynamics. At this frequency range, the following key phenomena are happening:

- 1 Hz – 3 Hz: natural frequencies of the car body (bounce, pitch),
- 10 Hz – 20 Hz: natural frequencies of unsuspended masses (wheels and suspension – wheel hop) and

- 20 Hz – 50 Hz: the first structural modes of car body bending and torsion vibrations, as well as low frequency engine vibrations.

MISO MODEL FORMULATION

The model for nonparametric system identification presented in Figure 2 b) is the classical multiple input / single output (MISO) system in time domain. By using the Fourier transforms to obtain the spectra of its inputs and output:

- spectrum of vertical acceleration at the centre of the front left wheel, $\ddot{Z}_{11}(f) = X_1$,
- spectrum of vertical acceleration at the centre of the front right wheel, $\ddot{Z}_{12}(f) = X_2$,
- spectrum of vertical acceleration at the centre of the rear left wheel, $\ddot{Z}_{21}(f) = X_3$,
- spectrum of vertical acceleration at the centre of the rear right wheel, $\ddot{Z}_{22}(f) = X_4$,
- spectrum of vertical acceleration at the top of the front left shock absorber, $\ddot{Z}_A(f) = X_5$,

and introducing the output noise spectra, N , the model transforms to MISO coupled model in frequency domain as shown in Figure 3. Noise spectrum represents all possible deviations from the linear model due to time delays, nonlinearities, statistical errors, non-stationarity, instrument noise, etc. Each input, X_i , $i = \overline{1,4}$, affects the output X_5 through corresponding transfer channels characterized by optimum constant parameter ordinary linear frequency response functions, H_{i5} , $i = \overline{1,4}$.

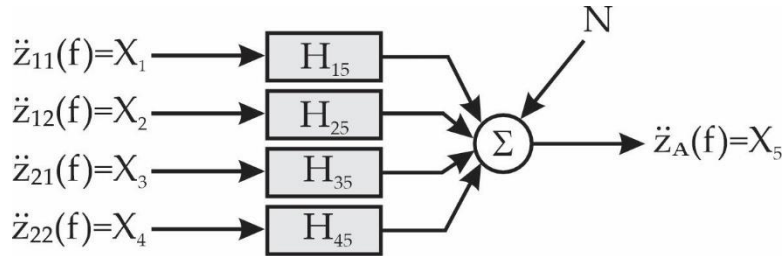


Figure 3 MISO model for coupled inputs

To begin the analysis, the spectra of the inputs and the output, X_i , $i = \overline{1,5}$, are calculated via Fourier transforms. Then, the auto-spectra, S_{ii} , $i = \overline{1,5}$, and the cross-spectra, S_{ij} , $i = \overline{1,5}$, $j = \overline{1,5}$, $i \neq j$, of the inputs and the output and S_{nn} of the output noise are calculated using the following expression, for a definite data record length of T :

$$\begin{aligned} S_{ij} &= \frac{1}{T} \cdot E[X_i^* \cdot X_j], \quad i = \overline{1,5}, \quad j = \overline{1,5}, \\ S_{nn} &= \frac{1}{T} \cdot E[N_i^* \cdot N_j], \end{aligned} \quad (1)$$

where: E is the mathematical expectation function, while X_i^* , $i = \overline{1,5}$ and N^* are complex conjugate values of spectra X_i , $i = \overline{1,5}$ and N , respectively.

In this coupled system, the movement of the front left wheel is directly influenced by forces coming not only from the road beneath it, but also from the remaining three wheels and the inertia of the body (due to pitch and roll coupling). Ordinary spectral analysis cannot separate how much each axle or side of the road individually contributes to the front left shock absorber cup vibrations. Therefore, *conditional spectral analysis* is introduced, which uses partial coherence functions to "eliminate" the influence of other wheels and isolate the pure contribution of only one specific input. In this case, it allows to identify the contribution of each individual wheel (input) to the body vibrations (output), thereby eliminating the mutual correlation that exists between the inputs due to the movement of the same vehicle on the same road.

Figure 4 shows conditioned MISO model with eliminated correlation between inputs by optimal transfer function. The original inputs were decoupled using the conditioned spectral analysis and replaced by a set of ordered conditioned inputs, $X_{i(i-1)}$, $i = \overline{1,4}$. The ordinary frequency response functions were replaced with optimal frequency response functions, L_{i5} , $i = \overline{1,4}$, of the system with conditioned inputs. They are different from the frequency response

functions of the system with arbitrary selected inputs. Spectrum of the output noise N , and spectrum of the output, X_5 , remain the same.

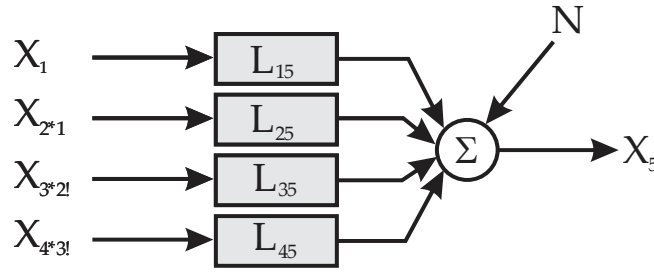


Figure 4 MISO model for conditional (decoupled) inputs

ALGORITHM FOR CONDUCTING CONDITIONAL SPECTRAL ANALYSIS

The theoretical foundations of conditional spectral analysis presented by Bendat and Piersol (2010), served as the basis for developing an algorithm for this special implementation.

First, using an iterative process, the optimal frequency response functions of the system with four conditional (decoupled) inputs, L_{ij} , $i, j = \overline{1,4}$, and the conditional power cross-spectra, $S_{i,j,r!}$, $r = \overline{1,3}$, are calculated alternately, in the following order:

$$\begin{aligned}
 L_{1j} &= \frac{S_{1j}}{S_{11}}, & j &= \overline{2,5} \\
 S_{ij,1!} &= S_{ij} - L_{1j} S_{i1}, & i &= \overline{2,5}, \quad j = \overline{i,5} \\
 L_{2j} &= \frac{S_{2j,1!}}{S_{22,1!}}, & j &= \overline{3,5} \\
 S_{ij,2!} &= S_{ij,1!} - L_{2j} S_{i2,1!}, & i &= \overline{3,5}, \quad j = \overline{i,5} \\
 L_{3j} &= \frac{S_{3j,2!}}{S_{33,2!}}, & j &= \overline{4,5} \\
 S_{ij,3!} &= S_{ij,2!} - L_{3j} S_{i3,2!}, & i &= \overline{4,5}, \quad j = \overline{i,5} \\
 L_{45} &= \frac{S_{45,3!}}{S_{44,3!}}, \\
 S_{nn} &= S_{55,3!} - L_{45} S_{54,3!}
 \end{aligned} \tag{2}$$

The conditionally uncoupled system inputs are obtained based on the following iterative procedure:

$$\begin{aligned}
 X_{2,1!} &= X_2 - \frac{S_{12}}{S_{11}} X_1 \\
 X_{3,2!} &= X_3 - \frac{S_{13}}{S_{11}} X_1 - \frac{S_{23,1!}}{S_{22,1!}} X_{2,1!} \\
 X_{4,3!} &= X_4 - \frac{S_{14}}{S_{11}} X_1 - \frac{S_{24,1!}}{S_{22,1!}} X_{2,1!} - \frac{S_{34,2!}}{S_{33,2!}} X_{3,2!} \\
 N = X_{5,4!} &= X_5 - \frac{S_{15}}{S_{11}} X_1 - \frac{S_{25,1!}}{S_{22,1!}} X_{2,1!} - \frac{S_{35,2!}}{S_{33,2!}} X_{3,2!} - \frac{S_{45,3!}}{S_{44,3!}} X_{4,3!} .
 \end{aligned} \tag{3}$$

The frequency response functions of the original system with coupled inputs, H_{i5} , $i = 4,3,2,1$, can be determined (in reverse order) from the previously calculated optimal frequency response functions of the system with conditional inputs, L_{ij} , $i = \overline{1,4}$, $j = \overline{i,5}$, according to the following iterative procedure:

$$\begin{aligned}
H_{45} &= L_{45} \\
H_{35} &= L_{35} - L_{34}H_{45} \\
H_{25} &= L_{25} - L_{23}H_{35} - L_{24}H_{45} \\
H_{15} &= L_{15} - L_{12}H_{25} - L_{13}H_{35} - L_{14}H_{45}.
\end{aligned} \tag{4}$$

Partial coherence functions between conditional inputs and output are calculated based on the expressions:

$$\begin{aligned}
\gamma_{15}^2 &= \frac{|S_{15}|^2}{S_{11}S_{55}} \\
\gamma_{25:1!}^2 &= \frac{|S_{25:1!}|^2}{S_{22:1!}S_{55:1!}} \\
\gamma_{35:2!}^2 &= \frac{|S_{35:2!}|^2}{S_{33:2!}S_{55:2!}} \\
\gamma_{45:3!}^2 &= \frac{|S_{45:3!}|^2}{S_{44:3!}S_{55:3!}}
\end{aligned} \tag{5}$$

The multiple coherence function of the four-input, single-output system is calculated using:

$$\gamma_{5:4!}^2 = \frac{S_{55} - S_{nn}}{S_{55}} = \frac{S_{55} - S_{55:4!}}{S_{55}} \tag{6}$$

When the inputs are uncorrelated (as is the case with models with conditional inputs), the following expression also holds for the multiple coherence function:

$$\gamma_{5:4!}^2 = \sum_{i=1}^4 \gamma_{i5}^2 \tag{7}$$

Based on the presented algorithm, corresponding programs were written in the MATLAB environment and appropriate diagrams needed for the vibration analysis were generated.

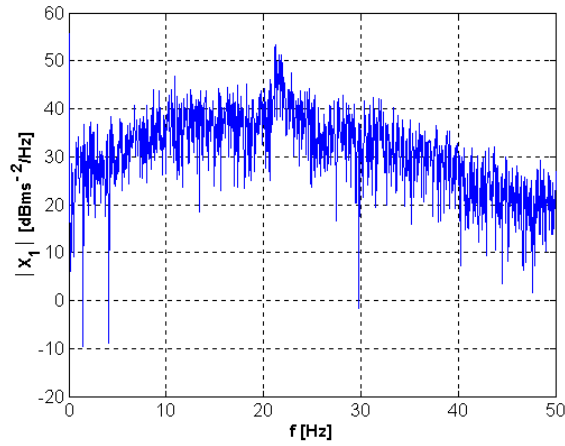
RESULTS AND DISCUSSION

Figure 5 shows the amplitude spectra of vertical accelerations at the centres of all four wheels and at the shock absorber-to-body connection point measured during straight line drive of the vehicle with constant speed of 70 km/h. The red curves represent vertical accelerations in the coupled state (prior to decoupling), while the blue curves show the conditioned (or decoupled) spectra following the elimination of cross-coupling effects from the excitations. The amplitude spectra are displayed on a dB scale.

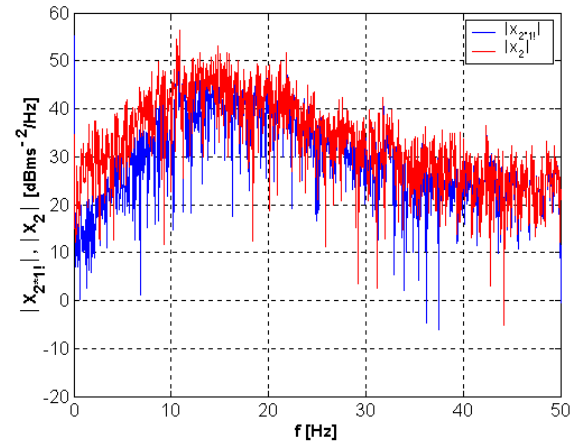
From diagrams in Figure 5 b), c) and d) it can be seen that the decoupling effect is most pronounced in the frequency range 0 – 10 Hz. At some points, the blue curves are 10 dB to 15 dB lower than the red ones. This demonstrates that vibrations originating from body motion (pitch and roll coupling) or from the other wheels have been successfully eliminated from the signal.

The main resonant wheel-hop domain (10 Hz – 20 Hz) is the most energy-intensive zone on the diagrams, where peaks reach absolute maximums of approximately 52 dB to 60 dB at frequencies between 11 Hz and 15 Hz. This corresponds to the natural frequency of the unsprung mass (the wheel). Following decoupling (blue curve), a general drop in energy level of several dB is observed, and the spectrum is "cleared" of extreme parasitic spikes.

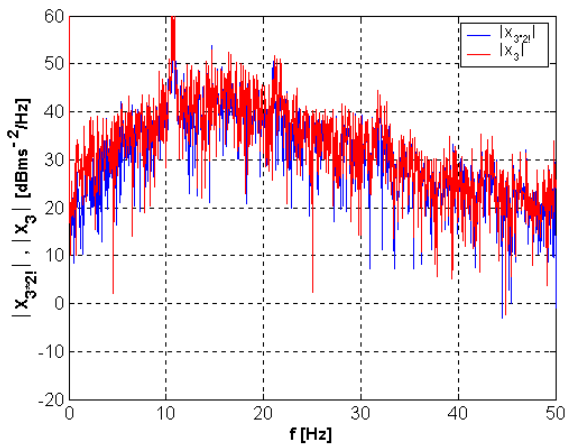
In frequency range above 20 Hz, acceleration intensity gradually decreases toward values of 20 dB – 30 dB. In this domain, the curves converge significantly and intertwine. This indicates that high-frequency behaviour is dominated by the local dynamics of the wheel itself and road roughness, over which the global decoupling of the suspension systems no longer exerts a dominant influence.



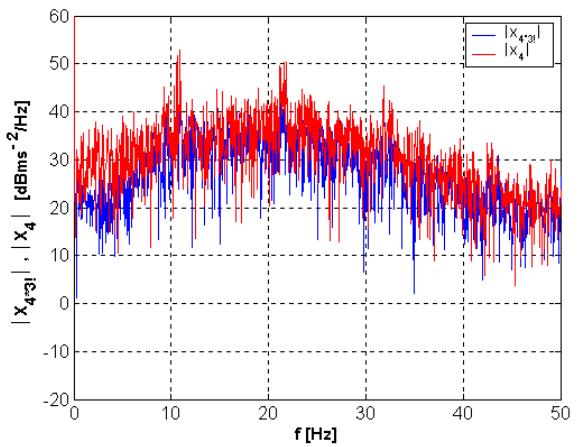
a) Amplitude spectrum of vertical acceleration at the centre of front left wheel



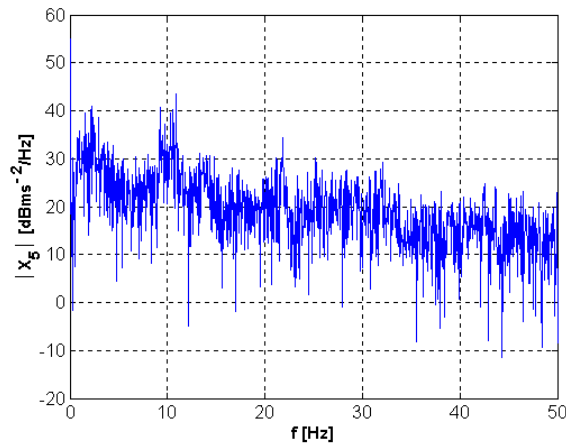
b) Amplitude spectrum of vertical acceleration at the centre of front right wheel after and before decoupling



c) Amplitude spectrum of vertical acceleration at the centre of rear left wheel after and before decoupling



d) Amplitude spectrum of vertical acceleration at the centre of rear right wheel after and before decoupling



e) Amplitude spectrum of vertical acceleration at the shock absorber-to-body connection point

Figure 5 Amplitude spectra of vertical accelerations at the centres of four wheels and the shock absorber-to-body connection point for straight line drive with constant speed of $v = 70$ km/h

Figure 6 shows the amplitude-frequency characteristics (AFC) of the frequency response functions for vertical acceleration transfer channels from all four wheels to the shock absorber-to-body connection point, for straight-line drive at a constant speed of $v = 70$ km/h and for the original (coupled) inputs. Given that the study involves experimental measurements on an actual vehicle, the displayed diagrams reflect all the imperfections and complexities of the real-world system - aspects often overlooked by classical theoretical studies. The pronounced oscillations, "noise," and sharp drops in the diagram are direct consequences of the actual measurement conditions.

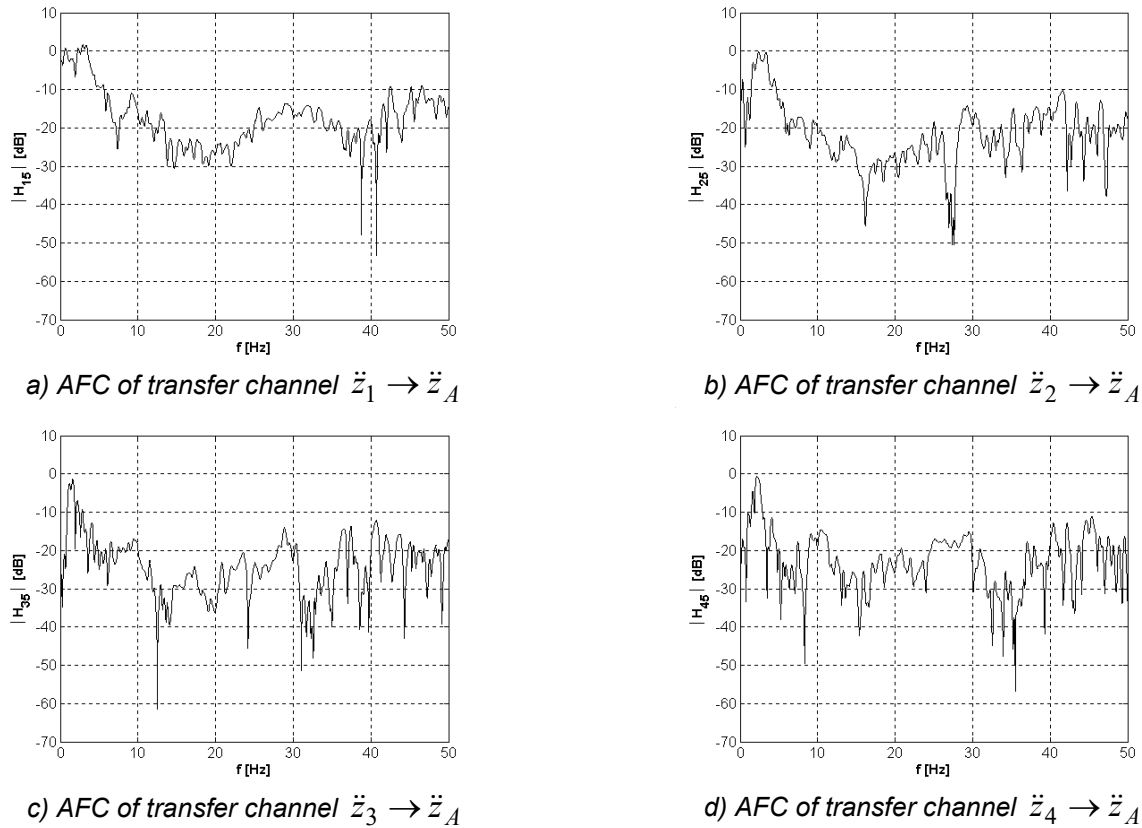


Figure 6 Amplitude-frequency characteristics (AFC) of ordinary frequency response functions for straight-line drive at constant speed $v = 70$ km/h for the original inputs

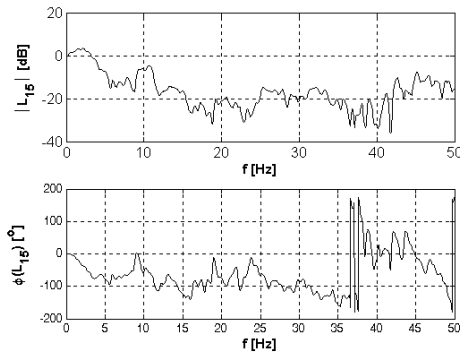
Analysis of characteristic frequency zones on the diagrams in Figure 6 shows the following:

- The amplitude reaches a maximum in the range of vehicle body resonance. Road excitation in this range is transmitted to the vehicle body at a 1:1 ratio. Distinct peaks are observed around 2 Hz, representing the resonance of the sprung mass (car body).
- In the range of wheel resonance (10 Hz to 20 Hz), the curves show a drop in the overall level to between -20 dB and -30 dB. A local maximum is visible around 10 Hz, resulting from the resonance of the unsprung mass (the wheel hop).
- In the range of structural vibrations (above 20 Hz) the diagrams display complex waveforms with pronounced fluctuations. Dynamics at these frequencies are determined not only by spring stiffness but also by the elasticity of the vehicle body structure itself. Some extremely sharp drops in amplitude (anti-resonances) are observed that indicate that complete cancellation of dynamic forces occurs at the shock absorber mounting point (structural zeros). At these frequencies, vibrational energy is transferred in another direction (e.g., transverse or longitudinal motion), or phase cancellation of the waves occurs between the point of excitation (tire contact patch) and the point of measurement (accelerometer on the shock absorber mount). In the higher ranges, the amplitude rises again. This indicates the presence of high-frequency structural modes of the chassis or suspension components.

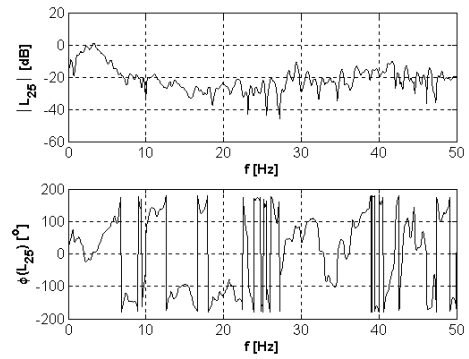
Figure 7 shows amplitude frequency characteristics (AFC) and phase frequency characteristics (FFC) of optimal frequency response functions of the model of straight-line drive at constant speed $v = 70$ km/h with decoupled inputs.

The example analysis of AFC in Figure 7 a) of frequency response functions of the transfer path between the vertical acceleration at the front left wheel centre and the shock absorber-to-body connection point shows a clear peak slightly above 0 dB that represents the resonance of the vehicle body (sprung mass), where the system amplifies vertical vibrations originating from the road. Second resonance around 10 Hz to 12 Hz represents a secondary local peak that corresponds to the resonance of the wheel and tire (wheel hop), characterized by pronounced vibrations of the unsprung mass. In frequency domain above 20 Hz, the amplitude drops, indicating that the suspension successfully dampens high frequencies. Pronounced fluctuations at higher frequencies indicate the presence of additional vibration sub-modes, such as vibrations of the seat, engine, or elastic chassis mounts.

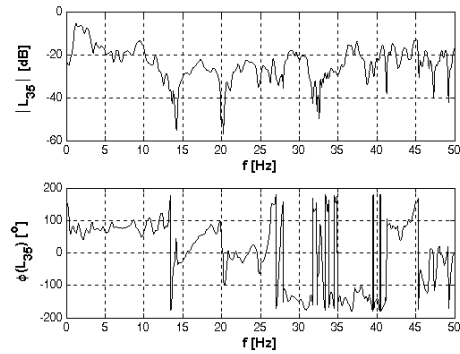
Analysis of FFC in Figure 7 a) shows that the phase starts at 0° and decreases toward -100° and -150° as frequency rises, indicating a natural time lag (delay) in the vehicle mass response relative to road excitation. Phase jumps are seen as abrupt vertical transitions from -180° to $+180^\circ$ that represent so-called „phase wrapping“. This is a computational/graphical effect occurring in software (such as MATLAB) when the phase crosses the $\pm 180^\circ$ boundary. It also indicates the system's high sensitivity and complexity at these frequencies.



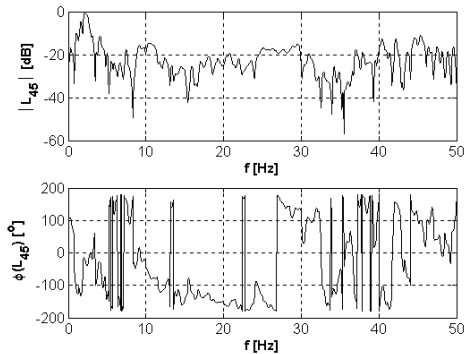
a) AFC and FFC of channel $X_1 \rightarrow X_5$



b) AFC and FFC of channel $X_{2*1} \rightarrow X_5$



c) AFC and FFC of channel $X_{3*2} \rightarrow X_5$



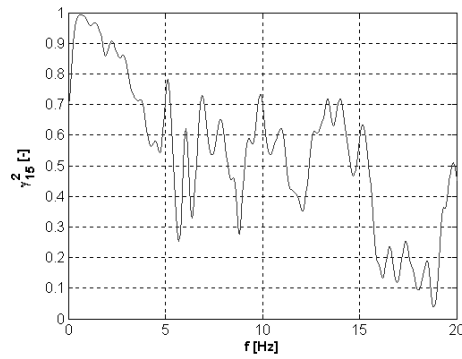
d) AFC and FFC of channel $X_{4*3} \rightarrow X_5$

Figure 7 AFC and FFC of frequency response functions for the model of straight-line drive at constant speed $v = 70$ km/h with decoupled inputs

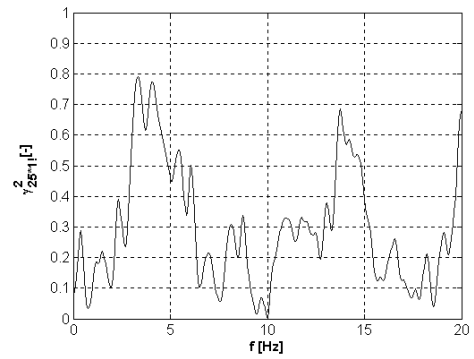
Analysis of partial coherence functions is an advanced method in signal processing and vibration analysis that is used to determine the linear dependence between two signals after mathematically removing the influence of other measured sources from input and output signals. In this case, „other sources“ are: the influence of the remaining three wheels, the engine, aerodynamic noise, etc.

Figure 8 shows partial coherence functions for a model of straight-line drive at constant speed of $v = 70$ km/h. Based on the provided diagrams, a detailed analysis of the wheel–shock absorber transmission system's behaviour can be performed across several characteristic frequency zones in the 0 to 20 Hz range. The example presented in diagram in Figure 8 a) shows the following:

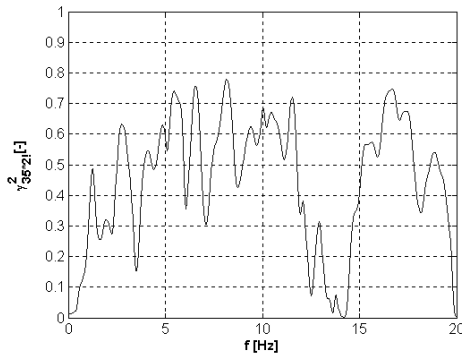
- Extremely high linearity is observed in low frequency zone (0 – 3 Hz), with coherence values between 0.85 and 1.0 and maximum at 0.5 Hz – 1 Hz. This range is dominated by basic car body resonances (bounce and pitch). The car body directly follows macro-movements of the wheel, with minimal influence of nonlinearities.
- In frequency zone between 3 Hz and 5 Hz, a drop in coherence values to 0.55 is evident. This means that the system begins to lose its strictly linear coupling. Frequently, it is the zone in which the first effects of shock absorber damping and suspension elasto-kinematics occur.
- Pronounced vibrations and nonlinearity occur in the frequency range between 5 Hz and 15 Hz. The partial coherence values fluctuate between 0.25 to 0.70 with the occurrence of local peaks and sharp falls. This range encompasses the frequency of the unsprung mass (so-called "wheel hop"), typically between 10 Hz and 15 Hz). The fluctuations indicate the pronounced nonlinear effects (asymmetry of shock absorber damping - difference between the compression and expansion strokes, dry friction in gaskets and nonlinear behaviour of rubber silent blocks).
- The loss of linear coupling and isolation induces the dramatic drop of coherence values in the frequency range between 15 Hz and 19.5 Hz. At these frequencies, vertical wheel acceleration at shock absorber connecting point is no longer linearly coupled with the acceleration at the wheel centre. Also, low coherence indicates that local structural resonances begin to dominate at the shock absorber connecting point.
- Above 19.5 Hz, there is a renewed growth in coherence values towards 0.5, which hints at the entrance to the new zone of structural couplings, or resonance above 20 Hz.



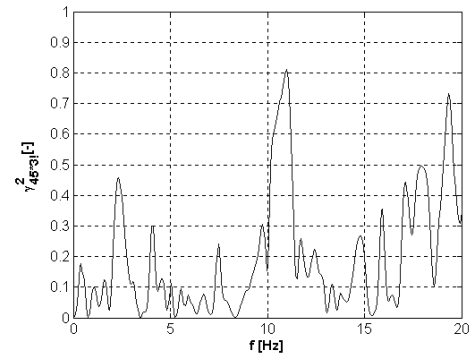
a/ Transfer channel $X_1 \rightarrow X_5$



c/ Transfer channel $X_{2*1} \rightarrow X_5$



e/ Transfer channel $X_{3*2} \rightarrow X_5$



g/ Transfer channel $X_{4*3} \rightarrow X_5$

Figure 8 Partial coherence functions for a model of straight-line drive at constant speed of $v = 70$ km/h

A comparison between partial coherence plots in diagrams shown in Figure 8 a) (path between front left wheel vertical acceleration and front left shock absorber vertical acceleration) and d) (path between rear right wheel vertical acceleration and front left shock absorber vertical acceleration), clearly reveals that the front-left wheel housing is subjected to two completely different types of excitation: at low frequencies, it is driven by the local (left) wheel, whereas at frequencies of 11 Hz and 19.3 Hz, it is dominated by energy transmitted from the diagonally opposite (rear-right) wheel through the vehicle body structure.

The frequency of 11 Hz corresponds to the resonance of the unsprung mass. With a rear solid axle of the experimental vehicle, this resonance does not occur solely in the vertical plane; instead, it triggers a "trampoline effect" (bouncing combined with rotation), causing the axle to oscillate laterally and vertically around its centre of mass. When the rear right wheel enters resonance at 11 Hz, the solid beam acts as a lever, transferring force to the left side of the axle. In the absence of a rear stabilizer bar to dampen this tilting motion, the combined force (vertical force plus torsional moment) is transmitted to the chassis via the trailing arms and axle mounting points. This energy wave travels through the floor and strikes the front-left suspension, which is coupled to the front axle via the front stabilizer bar. The result is a cross-coherence peak exceeding 0.8, indicating that the front-left suspension is subjected to mechanical stress caused by the resonance of the rear solid axle.

Figure 9 presents the multiple coherence function during straight-line drive at constant speed of $v = 70$ km/h.

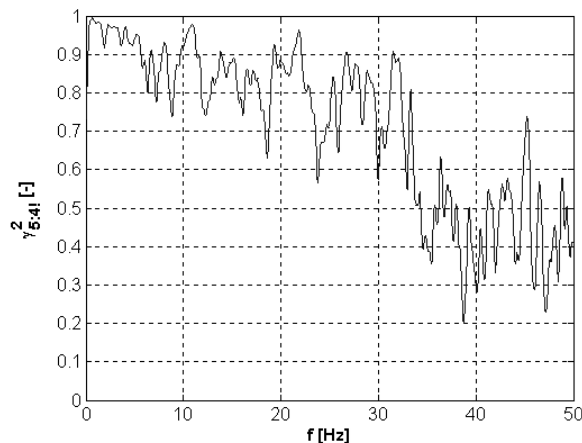


Figure 9 Multiple coherence function for the output during straight-line drive at constant speed of $v = 70$ km/h

The multiple coherence function values confirm a strong linear dependence between the four inputs (vertical accelerations at wheels) and the point on the vehicle body (output – vertical acceleration at the front left shock absorber connecting point) in the range up to 33 Hz, followed by a sharp drop and coherence dispersion at higher frequencies. Multiple coherence is extremely high (between 0.85 and 0.98) in the frequency range between 0 and 10 Hz. Almost all of the vibration at the shock absorber mount is a direct, linear consequence of excitation from the wheels. The system is nearly perfectly linear here. Coherence remains at a high level (mostly between 0.65 and 0.95) in the range between 10 Hz and 33 Hz, but with noticeable drops. The unsprung mass resonance occurs around 10 Hz to 15 Hz. The system continues to predominantly transmit energy from the wheels to the body. The drops at 19 Hz and 24 Hz may indicate local nonlinearities or the brief interference of another vibration source. In the range above 33 Hz, multiple coherence drops sharply and fluctuates between 0.2 and 0.7. Vibrations at the shock absorber mount at frequencies above 33 Hz can no longer be explained solely by the vertical movement of the wheels.

CONCLUSIONS

The application of the conditional spectral analysis (CSA) to calculate frequency response functions is the most methodologically correct approach when dealing with experimental measurements on roads with multiple coupled inputs. If a simple (classical) frequency response function between the front-left shock absorber vertical acceleration and the front-left wheel vertical acceleration is calculated, the cross-coupling from e.g. front right wheel would introduce a systematic error (bias). CSA successfully solves this problem.

Decoupling done by application of CSA gave excellent results in reducing the transmitted energy at low and medium frequencies (0 – 25 Hz). This confirmed that the pure contribution of the primary excitation of the front left wheel was successfully isolated, which in practice provides a more accurate identification of suspension parameters and a better analysis of tire-road contact. The experiment confirmed that the connection between the wheels at the selected vehicle speed is highly linearly dependent at low frequencies (up to 10 Hz), partially dependent in the wheel resonance zone (10 Hz – 25 Hz), and completely independent at high frequencies (above 25 Hz). The CSA successfully isolated the own dynamics of the front left wheel, cleaning the signal from the influence of the opposite side of the vehicle.

Based on the CSA and the conditional spectra, clear guidelines for vehicle optimization may be obtained. For example, the manner in which the stiffness of the shock absorber cup should be adjusted can be obtained. If the CSA shows high coherence at frequencies above 30 Hz, the rubber element at the shock absorber - body connection point must be softer to absorb the structure-borne noise. In view of damping optimisation, CSA allows the isolation of "false" peaks in the spectrum that originate from the subsequent sway of the rear of the vehicle, guiding engineers to the exact frequencies at which the damper itself effectively cancels the ground energy.

Key advantages of this analysis in NVH (Noise, Vibration, and Harshness) area may be seen in performing structural stiffness isolation. CSA allows engineers to see whether the transmission of vibrations from the left side or from the rear axle is due to the road being similar (excitation correlation) or because the body is too flexible (mechanical transmission).

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