



Faculty of Engineering
University of Kragujevac



Ministry of Science, Technological
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**11th International Congress
Motor Vehicles & Motors 2026
ECOLOGY -
VEHICLE AND ROAD SAFETY
- EFFICIENCY
Proceedings**



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FOREWORD

For 47 years, a traditional international gathering - originally known as a Symposium and, since 2006, as the "Motor Vehicles and Motors" Congress - has been held biennially at the Faculty of Engineering (formerly the Faculty of Mechanical Engineering) of the University of Kragujevac. This year, the 11th International Congress "Motor Vehicles and Motors" took place in Kragujevac on October 8th–9th, 2026, under the traditional motto: "Ecology – Vehicle and Road Safety – Efficiency".

A large number of authors from both the country and abroad responded to the organizers' invitation, resulting in numerous high-quality submissions across the event's core areas: "Power Train Technology," "Vehicle Design and Manufacturing," "Vehicle Dynamics and Intelligent Control Systems," "Driver/Vehicle Interface, Information Systems, and Assistance Systems," and "Transport Challenges in Emerging Economies." Additionally, two topics were officially introduced for the first time: "Artificial Intelligence in Automotive Technology" and "Autonomous Vehicles".

A total of 63 papers were accepted for presentation following a rigorous peer-review process. The presentations were organized into sessions, including a newly introduced poster session dedicated to students and young researchers.

The Congress organizers extend their special gratitude to the distinguished scientists in the fields of motor vehicles and transportation who agreed to deliver three inspiring introductory lectures addressing the heart of contemporary issues in the sector: prof. dr Ralph Putz (Landshut University of Applied Sciences, Germany), prof. dr Aleksandar Stevanović (University of Pittsburgh, USA), and prof. dr. Isak Karabegović (Academy of Sciences and Arts of Bosnia and Herzegovina, Bosnia and Herzegovina).

We extend our sincere gratitude to the esteemed members of the Congress Scientific Committee—who also served as reviewers for the papers submitted for presentation—for their significant contribution to the overall quality of the accepted papers.

The organization of the Congress was once again supported by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia, the Faculty of Engineering of the University of Kragujevac, and the International journal "Mobility & Vehicle Mechanics" and we sincerely thank them for their support.

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PREDGOVOR

Već 47 godina, svake druge godine, na Fakultetu inženjerskih nauka (nekada Mašinskom fakultetu) Univerziteta u Kragujevcu održava se tradicionalan međunarodni skup, nekada nazivan Simpozijum, a od 2006. godine - Kongres „Motorna vozila i motori“. Ovogodišnji, 11. Međunarodni kongres „Motorna vozila i motori“ održan je u Kragujevcu od 8. do 9. oktobra 2026. godine, uz tradicionalni moto „Ekologija – Vozilo i bezbednost na putu – Efikasnost“.

Na poziv organizatora odazvao se veliki broj autora iz zemlje i inostranstva, što je donelo veliki broj prijavljenih kvalitetnih radova iz glavnih oblasti skupa: „Tehnologija pogonskog sistema“, „Dizajn i proizvodnja vozila“, „Dinamika vozila i inteligentni upravljački sistemi“, „Interfejs vozač/vozilo, informacioni sistemi i sistemi za pomoć“, „Transportni izazovi ekonomija u usponu“, uz prvi put zvanično promovisane teme – „Veštačka inteligencija u automobilskoj tehnologiji“ i „Autonomna vozila“.

Ukupan broj od 63 radova je prihvaćen za prezentaciju posle rigoroznog procesa recenziranja. Rad je organizovan po sekcijama, uz prvi put promovisanu poster sekciju namenjenu za studente i mlade istraživače.

Organizatori Kongresa su posebno zahvalni istaknutim naučnicima iz oblasti motornih vozila i saobraćaja koji su pristali da održe tri inspirativna uvodna predavanja koja zadiru u srž savremenih pitanja vezana za vozila i saobraćaj, a to su: prof. dr Ralph Putz (Landshut University of Applied Sciences, Nemačka), prof. dr Aleksandar Stevanović (University of Pittsburgh, SAD) i prof. dr Isak Karabegović (Akademija nauka i umjetnosti Bosne i Hercegovine, Bosna i Hercegovina).

Veliku zahvalnost dugujemo našim dragim članovima Naučnog odbora Kongresa, ujedno i recenzentima radova prijavljenih za prezentaciju na Kongresu, koji su mnogo doprineli ukupnom kvalitetu konačno prihvaćenih radova.

Organizaciju Kongresa i ovog puta su podržali: Ministarstvo nauke, tehnološkog razvoja i inovacija Republike Srbije, Fakultet inženjerskih nauka Univeziteta u Kragujevcu i Međunarodni časopis „Mobility & Vehicle Mechanics“, na čemu im se iskreno zahvaljujemo.

Naučni odbor i Organizacioni odbor Kongresa MVM2026

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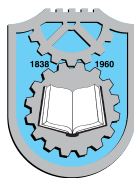
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DEVELOPMENT OF AN EXPERIMENTAL FIVE-AXIS CNC SYSTEM USING STEPPER MOTORS

ABSTRACT: This paper presents an experimental five-axis CNC milling machine designed for the research, development, and testing of multi-axis CNC systems. NEMA 17 stepper motors were used as the drive system due to their simple implementation, low cost, and sufficient positioning accuracy for the development and testing of multi-axis CNC systems. The proposed system is not intended for demanding industrial operating conditions, but rather for creating a functional platform that enables the investigation of multi-axis motion control, axis synchronization, and the evaluation of different control strategies and toolpath generation methods. Special attention is given to the potential application of this concept in education, laboratory research, and the development of future CNC systems with larger dimensions and higher power capacities. The developed system also enables the analysis of fundamental kinematic and control parameters important for improving multi-axis machining and optimizing CNC system performance. The proposed model provides a better understanding of the operation of five-axis CNC machines, as

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well as practical verification of design and control solutions before their implementation in more complex industrial systems.

KEYWORDS: multi-axis milling machine, stepper motors, CNC control, experimental system, industrial implementation

RAZVOJ EKSPERIMENTALNOG PETOOSNOG CNC SISTEMA KORIŠĆENJEM STEP MOTORA

REZIME: U radu je prikazana eksperimentalna petoosna CNC glodalica namenjena istraživanju, razvoju i testiranju višeosnih CNC sistema. Kao pogonski sistem korišćeni su NEMA 17 step motori koji su izabrani zbog jednostavne implementacije, pristupačne cene i dovoljne preciznosti za razvoj i testiranje višeosnih CNC sistema. Ovakav sistem nije namenjen za zahtevne industrijske uslove rada, već za formiranje funkcionalne platforme koja omogućava istraživanje upravljanja višeosnim kretanjem, sinhronizacije osa i ispitivanje različitih strategija upravljanja i putanja alata. Posebna pažnja posvećena je mogućnosti primene ovakvog koncepta u edukaciji, laboratorijskim istraživanjima i razvoju budućih CNC sistema većih dimenzija i snage. Razvijeni sistem omogućava i analizu osnovnih kinematskih i upravljačkih parametara koji su značajni za unapređenje višeosne obrade i optimizaciju rada CNC sistema. Razvijeni model omogućava bolje razumevanje rada petoosnih CNC mašina, kao i praktičnu proveru konstrukcionih i upravljačkih rešenja pre njihove implementacije na složenijim industrijskim sistemima.

KLJUČNE REČI: višeosna glodalica, step motori, CNC upravljanje, eksperimentalni sistem, industrijska implementacija

INTRODUCTION

The development of modern material processing technologies is closely associated with advances in the control and drive systems of computer numerical control (CNC) machine tools. Five-axis CNC machines are of particular importance in this field, as they enable the relative motion between the cutting tool and the workpiece to be controlled along five axes. The number, type, and arrangement of translational and rotational axes depend on the structural and kinematic configuration of the machine. Such configurations enable simultaneous positioning and orientation of the tool relative to the workpiece, thereby facilitating the efficient machining of components with complex geometries.

Depending on the machine design, the rotary axes may be located on the worktable, incorporated into the spindle head, or distributed between the table and the spindle in a combined configuration. The selected configuration directly affects the kinematic relationships within the system, the available workspace, and the strategy used for coordinating axis motion. Regardless of the specific structural arrangement, five-axis CNC systems offer significant advantages in the machining of geometrically complex surfaces, including a reduced number of workpiece setups and repositioning operations, shorter preparation and machining times, and improved surface quality. Consequently, these systems are widely applied in industries with demanding requirements regarding machining accuracy and component geometry, including the aerospace and automotive industries, tool and die manufacturing, and the production of medical implants and other precision components (Lu, Y. et al. 2023, Zhang, H. et al. 2024).

The increasing geometrical complexity of automotive components has created a growing demand for machining strategies capable of combining manufacturing flexibility with accurate control of complex tool - workpiece motion. A representative example is the machining of spiral bevel gears used in automotive transmission systems. Escudero, G. et al. (2022) experimentally investigated their manufacture using five axis double flank CNC machining with custom shaped tools. The proposed approach demonstrated the potential of five-axis machining for the efficient and accurate manufacture of complex gear geometries. The application of five-axis CNC machining in the automotive industry is not limited to transmission components, but also encompasses complex engine components, such as automobile cylinder blocks (Yan, J. 2022). Beyond these applications, the need for coordinated multi-axis motion is also evident in crankshaft manufacturing, where eccentric geometrical features impose additional requirements on tool and workpiece motion. In this context, Onodera, S. et al. (2025) developed a feature - based simultaneous four - axis machining approach for eccentric crankshaft features. These developments illustrate a broader transition in automotive manufacturing from conventional machining strategies towards increasingly flexible multi - axis solutions for complex components.

Future developments in multi - axis machining for automotive applications are also closely related to advances in intelligent process planning and digital manufacturing. In the field of crankshaft production, Amouzgar, K. et al. (2025) proposed a smart process planning framework based on CAD feature recognition, machining simulation and multi - objective optimization. Their study also considered the further use of artificial intelligence and adaptive process control in machining systems. These developments indicate that the future potential of multi - axis machining in the automotive industry will increasingly be associated with the integration of machining processes with intelligent and adaptive process - control strategies.

The realization of such advanced machining capabilities imposes significant requirements on machine tool drive and control systems. Industrial five axis CNC systems are therefore predominantly based on servo drives with position feedback provided by incremental or absolute encoders, together with digital current, velocity, and position controllers commonly arranged in a cascaded control structure. Such systems rely on computationally capable CNC controllers to perform tool path interpolation and real - time coordination of multiple machine axes (Rao, S.S. et al. 2026). Although these solutions provide high positioning accuracy and repeatability, as well as favourable dynamic performance, their implementation requires substantial investment in drive components, measurement systems, and control electronics. This may limit their suitability for educational applications, laboratory research, and the development of experimental machine concepts during the early stages of design.

In this context, increasing attention has been devoted to the development of low-cost functional platforms based on stepper motors for investigating and testing multi-axis motion-control concepts. Stepper motors are commonly operated in open-loop control systems, in which the rotor position is not measured directly but is inferred from the number of commanded input pulses. Although open-loop stepper drives cannot generally provide the same level of accuracy, reliability, and dynamic performance as closed-loop servo systems, their simple implementation, relatively low cost, uncomplicated control electronics, and satisfactory positioning resolution, particularly when microstepping is applied, make them suitable for experimental and educational CNC platforms (Vhu, C.-H. et al. 2022, Rao, S.S. et al. 2026).

A fundamental limitation of open - loop stepper - motor systems is the possibility of step loss when the torque required to execute the commanded motion exceeds the torque available from the motor at a given operating speed. Since no direct position feedback is available, the resulting positioning error cannot be detected or corrected during operation. Consequently, reliable application of stepper motors, particularly under variable loading conditions, requires an assessment of the dynamic feasibility of the prescribed motion. Such an assessment should determine whether the available motor torque is sufficient to satisfy the requirements imposed by the prescribed trajectory while accounting for motor speed, acceleration, inertia of the moving components, resistive effects, and external loads (Rao, S.S. et al. 2026).

Studies of multi-axis machining systems have largely focused on kinematic modelling and motion simulation in CAD/CAM environments in order to verify axis coordination, available ranges of motion, and tool-path continuity prior to physical implementation (Bošković, J. et al. 2026, Lu, Z. et al. 2023). Although such analyses are essential during the initial stages of machine development, kinematic verification alone does not provide information about the ability of the selected drive system to execute the prescribed motion under actual dynamic requirements. In the authors' previous research, a three-dimensional model of a five-axis CNC milling machine comprising three translational axes (X, Y, and Z) and two rotational axes (A and C) was developed. Kinematic simulation was performed in CATIA V5 R21, while the resulting position, velocity, and acceleration data for the individual axes were analysed in MATLAB environment. The results confirmed the functionality of the developed kinematic model and the feasibility of coordinated five-axis motion. However, the previous analysis did not address the dynamic feasibility of the prescribed trajectories with respect to the torque capability of the selected NEMA 17 stepper motors. Moreover, the influence of machining loads, particularly cutting forces, on the ability of the drive system to execute the required motion was not considered (Bošković, J. et al. 2026).

The dynamic requirements imposed on the drive system are also closely related to the characteristics of the prescribed tool path. In five-axis machining, the tool path is frequently approximated by a sequence of short linear segments, which may introduce discontinuities in tangent direction and curvature at segment junctions. Since five-axis motion requires simultaneous coordination of the Tool Centre Point (TCP) and Tool Orientation Vector (TOV), such discontinuities may produce abrupt changes in the velocity and acceleration demands of individual machine axes and consequently affect their dynamic behaviour. Zhang, H. et al. (2024) proposed a path-smoothing method based on sliding convolution windows, in which the TCP and TOV signals are processed while preserving their mutual synchronization. Their error-prediction model enables the deviation of the smoothed path to be controlled within prescribed tolerances through iterative adjustment of the convolution-window parameters (Zhang, H. et al. 2024).

Another important issue is the stability of kinematic mapping, particularly in the vicinity of singular configurations, where the inverse kinematic solution may become indeterminate or numerically unstable. This may result in abrupt changes in the commanded positions of the rotary axes and, consequently, increased dynamic demands on the drive system. Lu, Z. et al. (2023) developed a closed-loop inverse-kinematics approach based on a damped pseudoinverse of the Jacobian matrix and compensation of accumulated errors. The proposed method improves numerical stability

near singular configurations and reduces deviations between the commanded and realised trajectories (Lu, Z. et al. 2023).

In addition to tool-path continuity and kinematic stability, five-axis machining accuracy is influenced by geometric deviations between the nominal and machined surfaces. This issue is particularly relevant in flank milling of complex surfaces, where machining errors depend on both the tool trajectory and orientation and on the geometry of the cutting tool itself. Chu, C. et al. (2022) developed a computational method for the simultaneous optimisation of tool paths and tool geometry using continuous - function representations, with the aim of reducing overall geometric deviations and improving the contact conditions between the cutting tool and the machined surface. Although these approaches primarily address geometric and kinematic aspects of five - axis machining, they also demonstrate that the commanded motion may impose rapidly varying requirements on individual machine axes and therefore highlight the importance of considering the corresponding drive-system limitations (Chu, C. et al. 2022).

Related studies in multi - axis additive manufacturing further emphasise the importance of appropriate drive selection and coordinated axis motion. Sarwar, S. (2024) investigated motion planning and coordination for a mobile five-axis 3D-printing system, including the control of additional rotary axes, inverse - kinematics solutions, and collision avoidance (Sarwar, S. 2024). Liu, S. (2024) presented a 3D printer incorporating a rotary platform and stepper-motor drives, considering transmission ratios, operating speeds, and system torque requirements. Although these studies concern additive rather than subtractive manufacturing, they support the broader requirement that drive limitations must be considered when assessing the reliable execution of coordinated multi-axis motion (Liu, S. 2024).

Taken together, the available literature demonstrates substantial progress in tool - path accuracy, inverse - kinematics stability, and multi - axis motion coordination. However, comparatively less attention has been devoted to the dynamic feasibility of five - axis motion implemented using stepper - motor drives, particularly from the perspective of the torque available under varying operating loads. This issue becomes especially relevant in low - cost experimental CNC systems, where open - loop operation makes the available torque margin directly related to the risk of step loss and, consequently, to the reliability of trajectory execution.

The need for reliable kinematic and dynamic modelling is further reinforced by the long-term objective of developing a Digital Twin (DT) of the experimental five - axis CNC system. A DT can be regarded as a dynamic virtual representation of a physical system in which appropriate models are combined with data acquired from the physical counterpart to support state monitoring, analysis, and prediction of system behaviour. In CNC machine tools, the reliability of such a representation depends strongly on the fidelity of the underlying kinematic and dynamic models used to describe the position, velocity, acceleration, and loading of the individual machine axes (Rao, S.S. et al. 2026).

At the current stage of development, a virtual model of the experimental five - axis CNC milling machine has been established and kinematically verified. In parallel, NEMA 17 stepper motors intended to drive the translational and rotational axes have been selected and experimentally tested, while the mechanical integration of the complete machine is still in progress. At this stage, dynamic analysis provides an essential link between the existing kinematic model and the subsequent integration of the virtual and physical systems. In particular, it enables the capability of the selected motors to execute the prescribed trajectories to be evaluated with respect to their torque limitations before completion of the mechanical integration. The resulting analysis also provides a basis for further refinement of the dynamic model and its subsequent integration with data acquired from the physical system.

Accordingly, the aim of this study is to evaluate the dynamic feasibility of the previously defined five - axis CNC configuration, with particular emphasis on the ability of NEMA 17 stepper motors to execute the prescribed trajectories under different loading conditions. Two operating regimes are considered. The first represents motion without machining, in which the required drive torque is determined by the inertia of the moving components and the assumed resistive loads. The second additionally accounts for machining loads by incorporating cutting forces during milling, estimated using the Kienzle cutting - force model. In this way, the previously developed kinematic model is extended to include a dynamic feasibility assessment, providing a more realistic evaluation of the proposed five - axis CNC configuration and establishing a basis for the subsequent development of its DT.

MATERIALS & METHODS

The dynamic feasibility analysis of the five - axis motion was carried out through a sequence of interconnected steps. After defining the drive - system parameters and the mechanical characteristics of the individual axes, a synthetic test trajectory was established. The corresponding axis velocities and accelerations were determined analytically and subsequently converted to the motor side of the drive system. Based on the relationship between the required and available motor torque, a torque reserve indicator was defined as a criterion for the preliminary assessment of dynamic feasibility. The analysis was performed for motion without external process loading and, for the translational axes, for a reference loading scenario based on a simplified Kienzle cutting - force model (Figure 1).

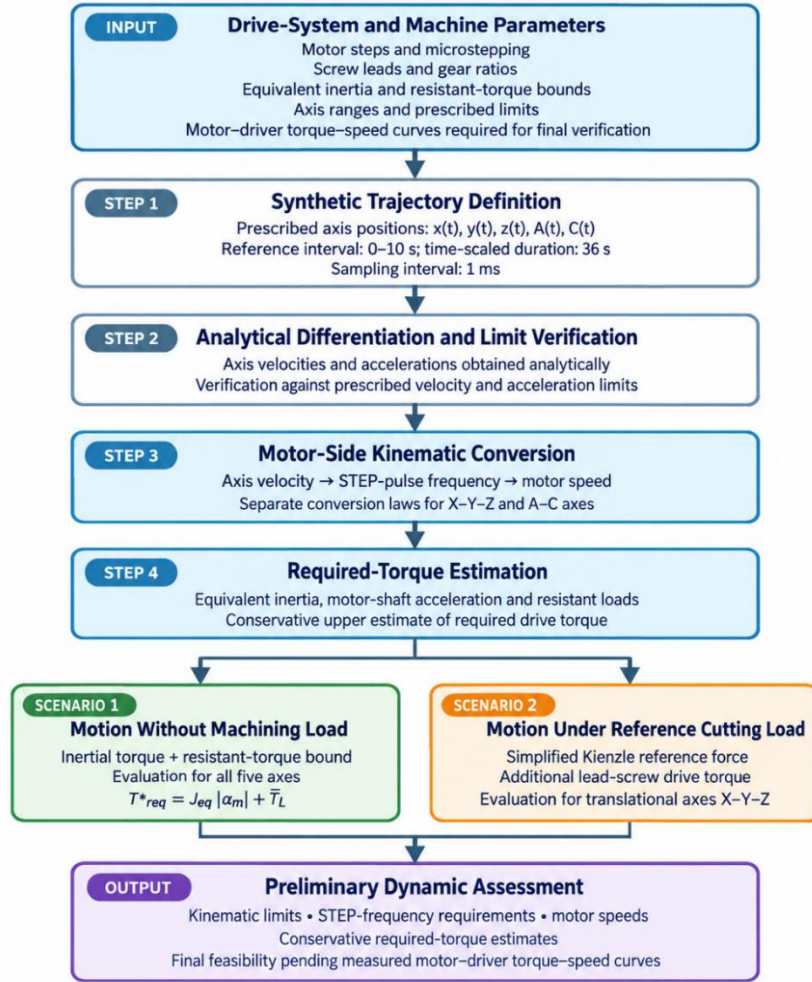


Figure 1 Algorithm flow for assessing the dynamic feasibility of five-axis CNC motion

For the preliminary dynamic assessment, harmonic and constant - velocity reference profiles were used to provide controlled and analytically differentiable excitation of the machine axes. Sinusoidal and shifted - cosine position profiles are suitable for this purpose because they provide smooth variations of position with explicitly defined velocity and acceleration characteristics (Hafez I. & Dhauadi R. 2025). Accordingly, harmonic profiles were assigned to the X, Z, and A axes, while linear profiles were assigned to the Y and C axes to represent constant - velocity motion. The profile amplitudes were selected according to the available axis ranges, whereas the frequency parameters were selected according to the adopted kinematic limits of the experimental CNC system.

The original motion laws were defined over a reference time interval of 10s, after which a common time - scaling factor of $s_t=3.6$. was applied to all five axes. In the time - scaled analysis, each prescribed position profile was evaluated using the normalized time argument t/s_t , increasing the total motion duration from 10 s to 36 s. This transformation preserved the geometric shape of the prescribed trajectory and the relative synchronization of the axes, while reducing the corresponding velocities and accelerations by factors of s_t and s_t^2 , respectively. All numerical results presented below therefore refer to the time - scaled position, velocity, and acceleration profiles. The numerical analysis was performed using a time step of $\Delta t=1$ ms.

The reference position profiles of the translational and rotational axes were defined by:

$$x(t) = \frac{H_x}{2} \sin(\Omega_x t), \quad (1)$$

$$y(t) = H_y - v_{y0} t, \quad (2)$$

$$z(t) = \frac{H_z}{2} [1 - \cos(\Omega_z t)], \quad (3)$$

$$A(t) = A_{\max} \sin(\Omega_A t), \quad (4)$$

$$C(t) = \omega_{c0} t, \quad (5)$$

where the adopted trajectory parameters are $H_x = 90 \text{ mm}, H_y = 100 \text{ mm}, H_z = 70 \text{ mm}, A_{x\max} = 20^\circ, \Omega_x = 0.4 \text{ s}^{-1}, \Omega_z = \Omega_A = 0.3 \text{ s}^{-1}, v_{y0} = 8 \frac{\text{mm}}{\text{s}}, \omega_{yC0} = 36^\circ/\text{s}$.

For the X axis, the coordinate origin was placed at the midpoint of the available travel range, resulting in the admissible interval $-\frac{H_x}{2} \leq x(t) \leq \frac{H_x}{2}$. The prescribed motion therefore represents a synthetic test trajectory designed to simultaneously evaluate the kinematic and drive requirements of all five axes. It does not represent a specific machining tool path, nor is it intended to cover the entire available workspace of the machine. The common time scaling was introduced to reduce the required linear velocities to within the adopted limit of 5 mm/s, while preserving both the geometric shape of the trajectory and the relative temporal synchronization of the axes.

The velocity and acceleration of each axis were determined analytically as the first- and second - time derivatives of the corresponding generalized coordinate:

$$v_q(t) = \frac{dq(t)}{dt}, \quad (6)$$

$$a_q(t) = \frac{dv_q(t)}{dt} = \frac{d^2q(t)}{dt^2}, \quad (7)$$

here $q(t)$ denotes the corresponding translational or rotational coordinate. Since the prescribed position profiles were available in analytical form, the corresponding velocity and acceleration profiles were obtained by direct analytical differentiation.

Based on Eqs. (1) - (3), the velocity and acceleration expressions for the translational axes are given by Eqs. (8) - (12), whereas differentiation of Eqs. (4) and (5) yields the corresponding expressions for the rotational axes, given by:

$$v_x(t) = \frac{H_x \Omega_x}{2} \cos(\Omega_x t), \quad (8)$$

$$a_x(t) = -\frac{H_x \Omega_x^2}{2} \sin(\Omega_x t), \quad (9)$$

$$v_y(t) = -v_{y0}, a_y(t) = 0, \quad (10)$$

$$v_z(t) = \frac{H_z \Omega_z}{2} \sin(\Omega_z t), \quad (11)$$

$$a_z(t) = \frac{H_z \Omega_z^2}{2} \cos(\Omega_z t). \quad (12)$$

$$\dot{A}(t) = A_{\max} \Omega_A \cos(\Omega_A t), \quad (13)$$

$$\ddot{A}(t) = -A_{\max} \Omega_A^2 \sin(\Omega_A t), \quad (14)$$

$$\dot{C}(t) = \omega_{C0}, \ddot{C}(t) = 0. \quad (15)$$

For the adopted time-scaling factor $s_t=3.6$, the maximum absolute velocities of the translational axes are $vX_{peak} = 5 \frac{\text{mm}}{\text{s}}, vY_{peak} = 2,22 \frac{\text{mm}}{\text{s}}, vZ_{peak} = 2,92 \frac{\text{mm}}{\text{s}}$. The corresponding maximum absolute accelerations are $aX_{peak} = 0,56 \frac{\text{mm}}{\text{s}^2}, aY_{peak} = 0, aZ_{peak} = 0,24 \frac{\text{mm}}{\text{s}^2}$. These values were obtained from the time - scaled profiles, for which velocities and accelerations are reduced by factors of s_t and s_t^2 , respectively. All three translational axes therefore satisfy the velocity and acceleration limits of 5mm/s and 100 mm/s² adopted for the present preliminary numerical analysis. These limits represent numerical assessment criteria rather than experimentally determined performance limits of the physical prototype.

No separate velocity or acceleration limits were prescribed for the rotational axes A and C . Therefore, their calculated values were used only to determine the drive - related quantities and were not assessed against specific limiting criteria. Since some axes exhibit non - zero velocities at the boundaries of the analysed interval, the prescribed trajectory represents a segment of established motion rather than a complete motion cycle. Start up and stopping transients were not included, as their analysis would require separately defined acceleration and deceleration profiles and the corresponding transient drive - torque assessment.

For the translational axes X , Y , and Z , the stepper - motor shaft was assumed to be directly coupled to the lead screw. A screw lead of $p_j= 1.25 \text{ mm/rev}$ was adopted for all three axes.

For a stepper – motor - driven axis, the motor position is determined by the number of input pulses, whereas its rotational speed is determined by the STEP-pulse frequency. The total number of pulses required for one motor revolution depends on the number of full motor steps per revolution and the selected microstepping factor (Hughes, A. & Drury, B. 2019; Campbell, A.A. et al., 2014). Accordingly, the required STEP - pulse frequency for translational axis j , $j \in \{X, Y, Z\}$, is determined from the magnitude of the corresponding linear velocity as follows:

$$f_j(t) = \frac{N m_{mic}}{p_j} |v_j(t)|, j \in \{X, Y, Z\}, \quad (16)$$

where N is the number of full motor steps per revolution, m_{mic} is the microstepping factor, and p_j is the screw lead.

For the rotational axes, the axis angular velocity is converted to the corresponding motor - shaft velocity using the transmission ratio. The required STEP - pulse frequency is therefore:

$$f_j(t) = \frac{N m_{mic} i_j}{360} |\dot{\theta}_j(t)|, j \in \{A, C\}, \quad (17)$$

where $\theta_A = A$, $\theta_C = C$, and the transmission ratio is defined as $i_j = \frac{\omega_{m,j}}{\omega_{ose,j}}$. The STEP - pulse frequency is treated as a non - negative quantity, while the direction of motion is determined separately from the sign of the axis velocity and the adopted axis orientation. A limiting STEP - pulse frequency of $f_{lim} = 20000\text{Hz}$ was adopted for each axis as a numerical control - system requirement rather than as an experimentally verified hardware limit. Accordingly, the required STEP - pulse frequency was assessed using the condition $f_j(t) \leq f_{j,max}$.

Starting from the lead - screw kinematic relationship between angular and linear displacement given by Iqbal. H. et al. (2022), the corresponding motor angular velocity and angular acceleration for each translational axis were obtained by differentiation with respect to time:

$$\omega_{m,j}(t) = \frac{2\pi}{p_j} v_j(t) \text{ i } \alpha_{m,j}(t) = \frac{2\pi}{p_j} a_j(t), j \in \{X, Y, Z\}. \quad (18)$$

For the rotational axes, the corresponding relationships are:

$$\omega_{m,j}(t) = \frac{\pi}{180} i_j \dot{\theta}_j(t) \text{ i } \alpha_{m,j}(t) = \frac{\pi}{180} i_j \ddot{\theta}_j(t), j \in \{A, C\}. \quad (19)$$

Since each STEP - pulse produces one commanded motor step or microstep and the pulse rate determines the motor rotational speed, as described by Campbell, A.A. et al. (2014), the magnitude of the motor angular velocity can alternatively be determined from the STEP - pulse frequency:

$$|\omega_{m,j}(t)| = \frac{2\pi f_j(t)}{N m_{mic}}. \quad (20)$$

The number of full motor steps per revolution and the microstepping factor determine the required STEP-pulse frequency but do not alter the mechanical relationship between axis displacement and motor-shaft rotation.

The available torque of a stepper - motor drive decreases with increasing operating speed and must therefore be evaluated using the torque - speed characteristic of the selected motor - driver combination. The holding torque specifies the motor capability at standstill and cannot, by itself, be treated as the torque available during motion (Campbell A. A. et al., 2014; Hughes A. and Drury B. 2019). Accordingly, the available motor torque is represented as:

$$T_{available,j}(t) = T_{lim,j}(|\omega_{m,j}(t)|), \quad (21)$$

where $T_{lim,j}$ pred denotes the limiting torque that the drive can deliver at the corresponding motor speed under steady - state operating conditions.

Following the motor - selection procedure for multi - axis CNC drives presented by Iqbal H. et al. (2022), the dynamic quantities and mechanical loads were referred to the motor shaft. The required motor torque was therefore expressed as the sum of the inertial and non-inertial load contributions:

$$T_{req,j}(t) = J_{eq,j} \alpha_{m,j}(t) + T_{L,j}(t), \quad (22)$$

where $J_{eq,j}$ is the total equivalent moment of inertia referred to the motor shaft and $T_{L,j}(t)$ pot is the signed torque required to overcome non-inertial loads. The signs of all contributions are defined relative to the adopted positive direction of motor rotation.

The equivalent inertia includes the motor - rotor inertia and the inertial contributions of the driven components referred to the motor shaft. For the translational axes, the translating masses are converted into an equivalent rotational inertia through the lead - screw kinematic relationship, whereas for the rotational axes the corresponding transmission ratio is taken into account.

When the instantaneous variation of the non - inertial load torque is not known, its magnitude may be represented by an adopted upper bound:

$$|T_{L,j}(t)| \leq \bar{T}_{L,j}, \quad (23)$$

Applying the triangle inequality to Eq. (22) then gives:

$$|T_{\text{req},j}(t)| \leq J_{\text{eq},j} |\alpha_{m,j}(t)| + \bar{T}_{L,j}. \quad (24)$$

Accordingly, the following conservative estimate of the required motor torque is adopted:

$$T_{\text{req},j}^*(t) = J_{\text{eq},j} |\alpha_{m,j}(t)| + \bar{T}_{L,j}. \quad (25)$$

Equation (25) does not represent the exact instantaneous drive torque. Instead, it provides a conservative upper estimate based on the adopted bound of the non - inertial loads. This formulation enables a preliminary assessment of the drive requirements even when the detailed time-dependent behaviour of the individual load components has not yet been identified.

To define a reference machining-load scenario, the tangential cutting-force component was estimated using the empirical Kienzle–Victor force model, which relates the cutting force to the undeformed chip width and thickness through the specific cutting-force coefficient and the corresponding material exponent (Klippel H. et al. 2024):

$$F_c = k_{c1.1} b h \left(\frac{h}{h_0} \right)^{-m_c}, \quad h_0 = 1 \text{ mm}, \quad (26)$$

where $k_{c1.1}$ is the specific cutting force at the reference undeformed chip thickness, $b = h = 1 \text{ mm}$ is the undeformed chip width, and h is the undeformed chip thickness, and m_c is the exponent of the model. For the reference numerical scenario, the following assumptions were adopted:

$$b_{\text{ref}} = a_p, \quad h_{\text{ref}} = f_z. \quad (27)$$

The resulting reference cutting force is calculated as:

$$F_{\text{ref}} = k_{c1.1} a_p f_z \left(\frac{f_z}{h_0} \right)^{-m_c}. \quad (28)$$

For the adopted parameter values $k_{c1.1} = 700 \frac{\text{N}}{\text{mm}^2}$, $a_p = 1 \text{ mm}$, $f_z = 0,03 \text{ mm}$, $m_c = 0,25$ the resulting force is $F_{\text{ref}} \approx 50,46 \text{ N}$.

For a force F_j acting along a translational axis, the additional motor - shaft torque required to overcome this load through a directly coupled lead screw is (Iqbal, H. et al. 2022):

$$T_{\text{cut},j} = \frac{|F_j| p_j}{2\pi \eta_{p,j} 1000}, \quad j \in \{X, Y, Z\}, \quad (29)$$

where $\eta_{p,j}$ denotes the efficiency of the corresponding transmission.

In the reference loading scenario, each translational axis is assessed separately under a load of magnitude $|F_j| = F_{\text{ref}}$. This approach enables a direct comparison of the responses of the three translational axes under the same prescribed reference load. The conservative upper estimate of the required motor torque for this scenario is therefore:

$$T_{\text{req},j}^{*,\text{ref}}(t) = J_{\text{eq},j} |\alpha_{m,j}(t)| + \bar{T}_{L,j} + T_{\text{cut},j}. \quad (30)$$

The transmission efficiency and the adopted resistive torques must be defined consistently to avoid accounting for the same losses more than once. The present machining - load assessment is restricted to the translational axes. Evaluation of the rotational axes under machining conditions would additionally require determination of the moments generated by the cutting forces about the A and C axes, based on the location of the force application point and the kinematic configuration of the machine.

A dimensionless torque feasibility ratio is defined as the ratio between the available motor torque and the adopted conservative estimate of the required motor torque:

$$\eta_{T,j}(t) = \frac{T_{\text{available},j}(t)}{T_{\text{req},j}^*(t)}. \quad (31)$$

The ratio is defined for a positive required torque; when the torque demand is zero, torque availability is assessed directly. The basic torque - feasibility criterion is:

$$T_{\text{available},j}(t) \geq T_{\text{req},j}^*(t), \text{ or equivalently } \eta_{T,j}(t) \geq 1. \quad (32)$$

For design purposes, an additional safety factor $S_j > 1$ may be introduced:

$$T_{\text{available},j}(t) \geq S_j T_{\text{req},j}^*(t), \text{ or equivalently } \eta_{T,j}(t) \geq S_j. \quad (33)$$

The appropriate safety factor should be selected according to the uncertainty of the input parameters, variability of the operating loads, and drive - system requirements.

In addition to the instantaneous torque feasibility ratio, the extent of torque - criterion violation over the analysed motion may be characterized by the percentage of the total time interval during which $\eta_{T,j}(t) < 1$:

$$P_j = \frac{100\%}{T} \int_0^T \mathbf{1}_{\{\eta_{T,j}(t) < 1\}} dt. \quad (34)$$

Here, $\mathbf{1}_{\{\eta_{T,j}(t) < 1\}}$ denotes an indicator function that takes the value 1 when the torque - feasibility criterion is not satisfied and 0 otherwise. For uniformly spaced time samples, P_j may be approximated by Eq. (35). For uniformly spaced time samples, the corresponding fraction of the analysed interval can be approximated as:

$$P_j \approx \frac{N_{\eta_{T,j} < 1}}{N_t} 100\%. \quad (35)$$

RESULTS

The numerical analysis was performed for the prescribed synthetic five-axis trajectory over a total time interval of $T = 36$ s, using a time step of $\Delta t = 1$ ms and a common time - scaling factor of $st = 3.6$. For the translational axes X, Y, and Z, a screw lead of 1.25 mm/rev was adopted.

The analysis provided the time histories of the positions, velocities, and accelerations of all five axes, together with the corresponding required STEP-pulse frequencies and motor angular velocities. Conservative upper estimates of the required drive torque were subsequently calculated. The torque requirements were first evaluated without an external machining load and then, for the translational axes, with the adopted reference machining load included.

The position profiles of the three translational axes and the two rotational axes over the analysed time interval are shown in Figure 2. The corresponding minimum and maximum axis positions are summarized in Table 1.

Table 1 Position ranges of the axes during the analysed trajectory

Axis	Minimum value	Maximum value
X, mm	-34.056	45
Y, mm	20	100
Z, mm	0	69.65
A, °	0	20
C, °	0	360

During the analysed motion, the X axis position varied between -34.056 mm and 45 mm, whereas the Y axis moved monotonically from 100 mm to 20 mm. The Z axis position increased from 0 to a maximum of 69.65 mm. For the rotational axes, the A axis position remained within the range from 0° to 20°, while the C axis completed the prescribed motion from 0° to 360°.

The simultaneous variation of the three translational coordinates defines the spatial trajectory shown in Figure 3. The resulting curve represents the path generated by the prescribed X, Y, and Z coordinates of the synthetic test trajectory over the 36 s interval.

The calculated velocity and acceleration values are summarized in Table 2. Among the translational axes, the highest maximum absolute velocity was obtained for the X axis (5 mm/s), followed by the Z axis (2.9167 mm/s) and the Y

axis (2.2222 mm/s). The corresponding maximum absolute accelerations were 0.5556 mm/s², 0.2431 mm/s², and 0mm/s², respectively. The zero acceleration of the Y axis is consistent with its prescribed constant - velocity motion.

For the rotational axes, the maximum angular velocity of the A axis was 1.6667 °/s, with a maximum angular acceleration of 0.1389 °/s². The C axis maintained a constant angular velocity of 10 °/s throughout the analysed interval and therefore exhibited zero angular acceleration. The maximum velocities of all three translational axes remained within the adopted limit of 5 mm/s, while their maximum accelerations were substantially below the adopted limit of 100 mm/s².

An additional characteristic of the prescribed trajectory is evident from the initial and final axis velocities listed in Table 3. Several axes exhibit non - zero velocities at one or both ends of the analysed interval. The results therefore correspond to a segment of established motion rather than to a complete motion cycle including acceleration from rest and final deceleration to rest. Start up and stopping transients are not included in the present analysis.

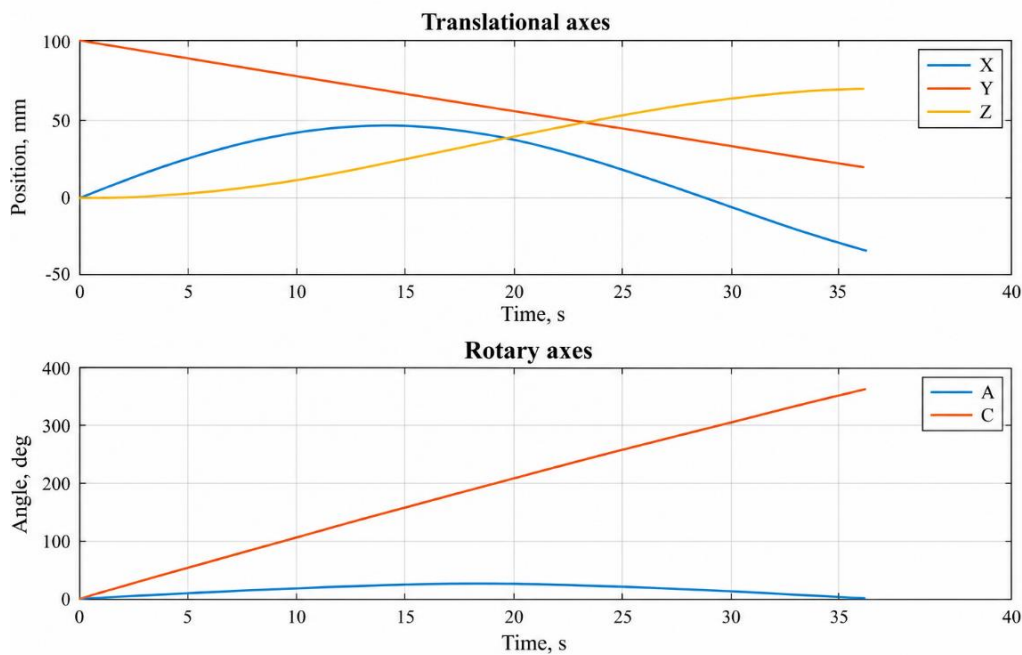


Figure 2 Position profiles of the translational and rotational axes during the analysed trajectory

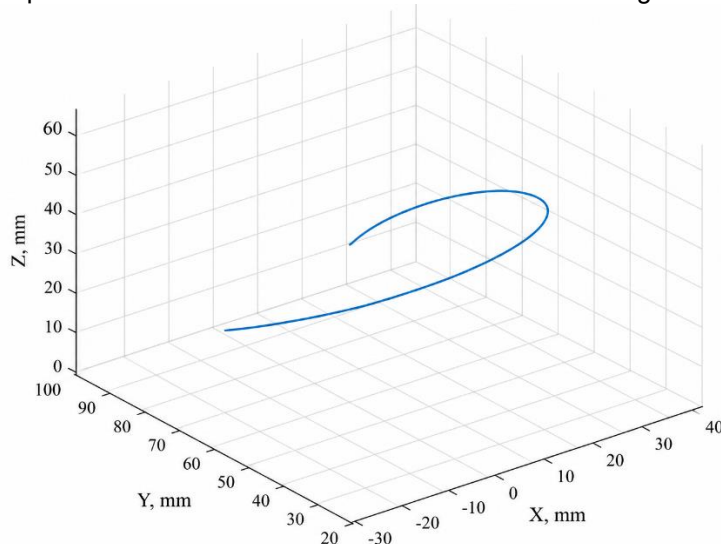


Figure 3 Spatial trajectory defined by the prescribed X – Y - Z coordinates

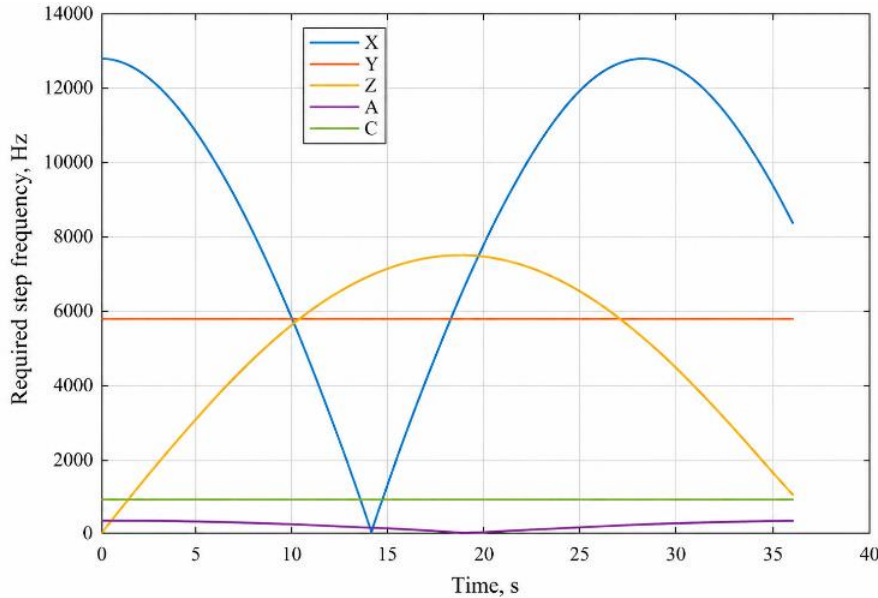
Table 2 Maximum absolute velocities and accelerations of the axes

Axis	Maximum absolute velocity	Maximum absolute acceleration
X	5 mm/s	0.5556 mm/s ²
Y	2.2222 mm/s	0 mm/s ²
Z	2.9167 mm/s	0.2431mm/s ²
A	1.6667 °/s	0.1389 °/s ²
C	10 °/s	0 °/s ²

Table 3 Initial and final axis velocities

Axis	V_0	V_T
X	5 mm/s	-3.2682 mm/s
Y	-2.2222 mm/s	-2.2222 mm/s
Z	0.0000 mm/s	0.4116 mm/s
A	1.6667 °/s	-1.6500 °/s
C	10 °/s	10 °/s

Based on the calculated axis velocities and the adopted drive - system parameters, the required STEP - pulse frequencies and corresponding motor speeds were determined. Their time histories are presented in Figure 4, while the maximum values for each axis are summarized in Table 5.

**Figure 4** Required STEP frequencies during the analyzed trajectory**Table 5** Maximum STEP-pulse frequencies and motor speeds

Axis	Maximum STEP-pulse frequency	Maximum motor speed
X	12800 Hz	240 rpm
Y	5 688.89 Hz	106.67 rpm
Z	7 466.67 Hz	140 rpm
A	296.30 Hz	5.56 rpm
C	888.89 Hz	16.67 rpm

The required STEP - pulse frequencies follow the prescribed velocity profiles and therefore differ considerably among the individual axes. The X axis imposed the highest control - frequency demand, reaching a maximum of 12.8 kHz, followed by the Z and Y axes at 7.47 kHz and 5.69 kHz, respectively. The requirements of the rotational axes were substantially lower, with maximum frequencies of 0.296 kHz for the A axis and 0.889 kHz for the C axis. The constant STEP - pulse frequency of the Y axis reflects its constant prescribed velocity. All calculated maximum STEP - pulse frequencies remained below the adopted limit of 20 kHz. The corresponding maximum motor speeds were 240 rpm for the X axis, 106.67rpm for the Y axis, and 140rpm for the Z axis. Considerably lower motor speeds were obtained for the rotational axes, with maximum values of 5.56 rpm for the A axis and 16.67 rpm for the C axis.

The dynamic requirements were subsequently evaluated using the equivalent moments of inertia, calculated motor angular accelerations, and adopted upper bounds of the non-inertial loads. Figure 5 compares the conservative upper estimates of the required motor torque for the translational axes under the two considered operating conditions: motion without the reference machining load and motion with the reference load included. The introduction of the reference load resulted in a clear increase in the required torque of all three translational axes. In contrast, the temporal variation of the required torque within each operating condition remained relatively small. This indicates that, under the adopted model assumptions, the difference between the two loading conditions has a considerably greater influence on the calculated torque demand than the inertial contribution associated with the prescribed trajectory.

Without the reference machining load, the maximum required torque was 0.020008 Nm for the X axis, 0.03 Nm for the Y axis, and 0.030005 Nm for the Z axis. For the rotational axes, the corresponding maximum values were

0.040001 Nm for the A axis and 0.015Nm for the C axis. The highest value among all five axes was therefore obtained for the A axis, as summarized in Table 6.

Application of the simplified Kienzle model yielded a reference cutting - force magnitude of $F_{ref} = 50.459$ N. For the adopted screw lead of 1.25 mm/rev and a transmission efficiency of 0.85, this reference load corresponded to an additional torque of $T_{cut} = 0.01181$ Nm for each of the three translational axes.

DISCUSSION

The obtained results show that the adopted common time scaling enables the prescribed synthetic five-axis trajectory to be executed with relatively modest kinematic demands on the translational axes. The maximum linear velocity was 5mm/s for the X axis, while the corresponding values for the Y and Z axes were 2.2222 mm/s and 2.9167 mm/s, respectively. At the same time, the maximum translational accelerations remained well below the adopted limit of 100 mm/s². This behaviour is a direct consequence of the applied time scaling, which preserves the geometric form of the prescribed trajectory and the relative synchronization of the axes while reducing the associated velocity and acceleration demands.

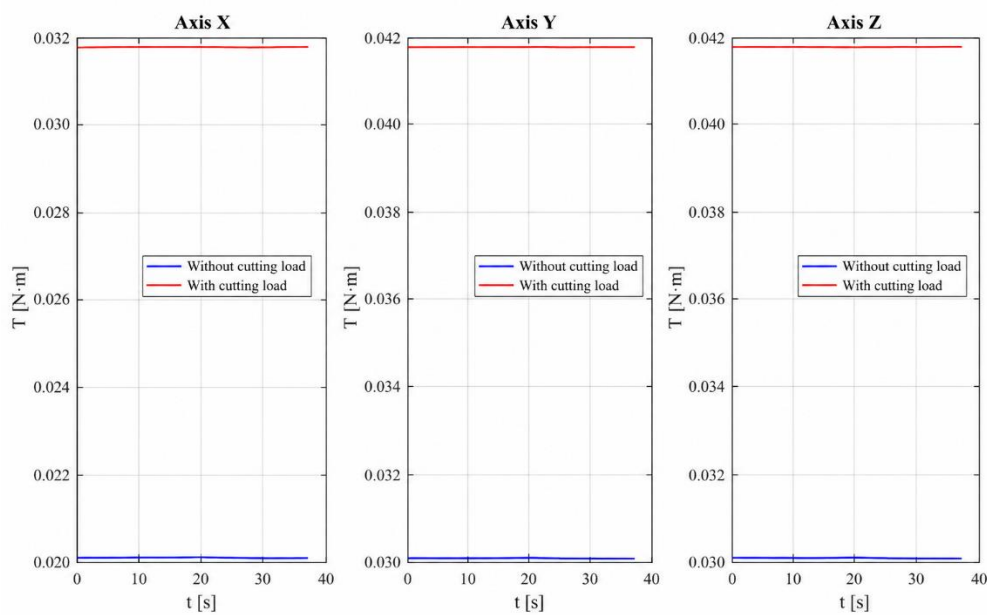


Figure 5 Conservative upper estimates of the required drive torque for the translational axes without and with the reference load

Table 6 Maximum required motor torques without the reference machining load

Axis	Maximum required torque, Nm
X	0.020008
Y	0.03
Z	0.030005
A	0.040001
C	0.015

The calculated results showed that the maximum absolute velocities and accelerations of all three translational axes remained within the limits adopted for the present preliminary analysis. However, these results apply only to the prescribed smooth trajectory segment and do not include acceleration from rest or final deceleration to rest. This distinction is important because the dynamic requirements of five-axis motion are strongly affected by trajectory continuity. Zhang, H. et al. (2024) showed that tangent and curvature discontinuities between consecutive linear tool-path segments may cause deceleration and fluctuations in axis velocities and accelerations. Consequently, the relatively low acceleration values obtained in the present study should not be interpreted as an inherent dynamic characteristic of the developed machine, but rather as a consequence of the particular trajectory and time-scaling factor adopted. Furthermore, the analysed interval does not include acceleration from rest or final deceleration to rest. Several axes exhibit non-zero velocities at the boundaries of the interval, indicating that the prescribed trajectory represents a segment of established motion. The inclusion of start-up and stopping transients could therefore result in higher acceleration levels and, consequently, greater inertial torque demands.

From the perspective of stepper-motor control, the highest required STEP-pulse frequency was obtained for the X-axis, reaching 12.8 kHz, which corresponds to 64% of the adopted numerical limit of 20 kHz. The corresponding maximum frequencies for the Y and Z axes were 5.69 and 7.47 kHz, respectively, while substantially lower values were obtained for the rotational axes. This distribution follows directly from the kinematic requirements and transmission characteristics of the individual axes, since the required STEP-pulse frequency depends on the motor speed and the number of commanded pulses per motor revolution. Within the adopted numerical framework, none of the five axes exceeded the 20 kHz criterion. However, this result should not be interpreted as experimental confirmation of the capability of a specific control system. The 20 kHz value was introduced solely as a criterion for the preliminary numerical assessment and does not represent an experimentally verified frequency limit of the actual controller–driver configuration.

Moreover, meeting the STEP - frequency requirement alone does not establish the dynamic feasibility of a stepper - motor drive. Dynamic feasibility also depends on whether the motor can provide the required torque at the corresponding operating speed. In an open - loop stepper drive, exceeding the available dynamic torque may result in loss of synchronism, skipped steps, or motor stalling. Therefore, stepper - motor operating capability should be evaluated using the pull - out torque - speed characteristic of the selected motor - driver combination, which represents the limiting load torque that can be sustained at a given steady operating speed without loss of synchronism (Hughes, A. and Drury, B. 2019; Campbell, A. A. R. et al., 2014). Since the available torque generally decreases with increasing motor speed, satisfaction of the STEP - frequency criterion cannot be treated as evidence that the required torque can also be delivered.

This distinction is essential for interpreting the present torque results. Although a nominal holding torque of 0.40Nm was adopted for the considered NEMA 17 motor, this value cannot be directly treated as the torque available during motion. Holding torque is a static characteristic specified at standstill, whereas the torque available during continuous operation depends on motor speed and on the electrical characteristics of the selected motor - driver combination. Campbell, A. A. R. et al. (2014) similarly emphasized that a stepper motor with a relatively high holding torque may nevertheless provide substantially lower torque at higher operating speeds and that the complete torque - speed characteristic should therefore be considered during motor selection. Hughes, A. and Drury, B. (2019) also identify the torque - speed relationship as a fundamental limitation of stepper - motor operation. This distinction is further reflected in the manufacturer guidance provided by Oriental Motor, which differentiates holding torque from pull - out torque and associate's operation beyond the pull - out torque - speed boundary with loss of synchronism.

This interpretation complements existing motor-selection procedures for customized multi-axis CNC systems. Iqbal, H. et al. (2022) considered motor selection for a low-cost five-axis CNC machine with an X-Y-Z-A-C configuration intended for educational and light manufacturing applications. Their procedure accounts for the mechanical configuration of the individual axes, the required drive torque, the inertia of the moving components, and the corresponding transmission arrangements. In this respect, their system is conceptually similar to the experimental platform investigated in the present study. The present analysis extends this perspective by emphasizing that, for a stepper-motor-driven system, torque feasibility cannot be established solely from the nominal or holding-torque specification. Instead, the required torque should ultimately be compared with the torque available from the selected motor–driver combination at the corresponding instantaneous operating speed.

The introduction of the reference machining load noticeably increased the torque requirements of the translational axes. The simplified Kienzle model yielded a reference cutting force of 50.45 N, corresponding to an additional motor - shaft torque of 0.01181 Nm. Consequently, the maximum required torque increased from 0.020008 Nm to 0.031818 Nm for the X axis and from approximately 0.03 Nm to 0.04181 Nm for the Y and Z axes. These increases show that the reference machining load represents a substantial part of the total torque demand. However, the Kienzle coefficients depend on the workpiece material, cutting speed, tool geometry, and other process conditions (Klippel, H. et al. 2024). Therefore, the calculated force should be interpreted as an idealized reference load rather than as a prediction of the resultant force generated during actual five - axis milling. Applying this force independently to the X, Y, and Z axes enables a controlled comparison of their drive requirements but does not represent the Cartesian decomposition of an actual cutting - force vector.

The obtained results provide a preliminary characterization of the drive - torque requirements and complement the motor - selection procedure for a comparable low - cost X-Y-Z-A-C CNC configuration presented by Iqbal, H. et al. (2022). However, they do not yet constitute final confirmation of the dynamic feasibility of the selected drives. Such confirmation requires the experimentally validated operating characteristics of the actual drive system.

Moreover, sufficient torque availability alone does not guarantee reliable stepper-motor operation. Excessive acceleration, inadequate drive conditions, and operation within resonance regions may also cause skipped steps or motor stalling (Campbell, A.A.R. et al. 2014). Experimental validation should therefore include realistic acceleration and deceleration profiles and a comparison between commanded and measured axis positions, as applied in the assessment of a stepper - driven tabletop CNC system by Rao S.S. et al. (2026). More generally, Halamka et al. (2021) demonstrated the importance of experimentally identified axis-dynamics models for the simulation and optimization of machine-drive control. Accordingly, a subsequent stage of the present research should include

experimental determination of the torque–speed characteristics of the selected NEMA 17 motor–driver combinations, implementation of realistic start-up and stopping profiles, and measurement of the resulting axis-position errors. Such validation would provide the experimental basis required for applying the proposed torque - feasibility ratio as a quantitative criterion for assessing dynamic feasibility.

These considerations are directly relevant to the planned development of a DT of the experimental machine. A representative CNC DT should extend beyond machine geometry and incorporate the kinematic, dynamic, drive, control, and process - related behaviour of the physical system. Scaglioni, B. and Ferretti, G. (2018), for example, developed a dynamic machine - tool model that integrates structural flexibility, the cutting process, transmission chains, and control systems, and validated its dynamic behaviour experimentally. Similarly, Shen, W. et al. (2020) proposed a Digital-Twin-based virtual commissioning approach that combines geometric, kinematic, and dynamic representations to support kinematic and dynamic commissioning, servo- system evaluation, and virtual machining. These studies confirm that the kinematic and torque models developed in the present analysis represent necessary components of the planned DT, while experimental identification and validation remain essential for achieving correspondence with the physical machine.

In this context, the analysis presented in this study should be regarded as an intermediate step between the previously developed and kinematically verified virtual model (Bošković, J. et al., 2026) and a future data - connected DT, rather than as a complete dynamic representation of the machine. Following the mechanical integration of the experimental system, the adopted equivalent inertias, resistive torques, transmission efficiencies, and drive characteristics should be identified or verified using measurements obtained from the physical machine. Scaglioni, B. and Ferretti, G. (2018) similarly demonstrated the importance of experimentally validated dynamic machine - tool models incorporating structural behaviour, transmission chains, control systems, and machining loads. Incorporating experimentally determined parameters and measured operating data into the present virtual model would enable its progressive refinement and improve its correspondence with the physical CNC system. Such an approach is consistent with Ward, R. et al. (2021), who integrated real - time CNC position and velocity data, model - based simulations, cutting - force estimation, and online feedback within a machining DT.

CONCLUSIONS

This study extended the previously developed kinematic model of an experimental five - axis CNC milling machine by introducing a preliminary assessment of the kinematic and dynamic requirements of the X, Y, Z, A, and C axis drives. For the analysed time - scaled synthetic trajectory, the translational axes remained within the velocity and acceleration limits adopted for the numerical analysis. The required STEP - pulse frequencies of all five axes also remained below the adopted criterion of 20 kHz, with the highest value of 12.8 kHz obtained for the X axis.

The calculated torque requirements were comparatively low for the considered motion without an external machining load. Introducing the Kienzle - based reference load increased the maximum torque requirements of the translational axes by approximately 59% for the X axis and 39.4% for the Y and Z axes. These results demonstrate that external process loading can contribute substantially to the overall drive demand and should therefore be considered together with the kinematic requirements when evaluating the suitability of the proposed stepper - motor drives. However, the adopted cutting force represents an idealized reference loading scenario and not a prediction of the resultant force generated during actual five - axis milling.

At the present stage, the results should be regarded as a preliminary assessment rather than as final confirmation of the dynamic feasibility of the selected NEMA 17 drives. Such confirmation requires experimentally determined torque - speed characteristics of the actual motor - driver combinations, identification of the relevant mechanical parameters, implementation of realistic start up and stopping profiles, and validation of the numerical model on the physical system. These steps will enable further refinement of the dynamic model and provide a basis for the subsequent development of a data - connected DT of the experimental five - axis CNC machine.

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