Decision-Making Algorithm for Intuitionistic Fuzzy Soft Sets

Nemanja Vučićević[1], Nenad Stojanović[1], Marina Svičević[1]

[1]*Faculty of Science, University of Kragujevac, Kragujevac, Serbia, nemanja.vucicevic@pmf.kg.ac.rs, 0000-0002-4903-7280, nenad.stojanovic@pmf.kg.ac.rs, 0000-0003-3837-4043, marina.svicevic@pmf.kg.ac.rs, 0000-0003-2791-3849,*

Corresponding author: *nemanja.vucicevic@pmf.kg.ac.rs*

Abstract

This paper introduces a new numerical characteristic for intuitionistic fuzzy soft sets (IFSSs) based on the singular values of the associated membership and non-membership matrices. Using this quantity, we propose a decision-making algorithm for ranking alternatives in uncertain environments. The approach is illustrated by a case study, validated through effectiveness criteria, and complemented by a brief complexity analysis. The results indicate that the method provides a simple and informative framework for decision making with IFSSs.

Keywords: Intuitionistic fuzzy soft set, energy, singular values, decision making.

1. Introduction

Soft sets, fuzzy sets, and intuitionistic fuzzy sets ([1]) provide important tools for modeling uncertainty. Their hybridization leads to intuitionistic fuzzy soft sets (IFSSs), [2], which are suitable for decision-making problems involving several parameters and uncertain evaluations.

Many existing IFSS decision-making procedures are based on local score functions, similarity measures, or aggregation operators. In this paper, we introduce a different global numerical characteristic, called the energy of an IFSS. This characteristic is defined through the singular values of two matrices representing the membership and non-membership structure of the system ([3]). Based on it, we formulate a simple ranking procedure and illustrate its use on a practical example.

2. Materials and Methods

Definition 2.1. Let $U = \{x_1, x_2, \dots, x_n\}$ be a universe of discourse, E a set of parameters, and $A \subseteq E$. An *intuitionistic fuzzy soft set* (IFSS) over U with respect to A is a pair (F, A), where $F : A \to IF(U)$, and for each $e \in A$,

$$F(e) = \{(x, \mu_{F(e)}(x), \nu_{F(e)}(x)) \mid x \in U\}, \text{ with } 0 \leq \mu_{F(e)}(x) + \nu_{F(e)}(x) \leq 1 \text{ for all } x \in U.$$

Thus, for every parameter $e \in A$, the value $\mu_{F(e)}(x)$ describes the degree of membership of x, while $\nu_{F(e)}(x)$ describes the degree of non-membership of x.

Definition 2.2. Let (F, A) be an IFSS over $U = \{x_1, \dots, x_n\}$, where $A = \{e_1, \dots, e_m\}$. The *membership matrix* and the *non-membership matrix* of (F, A) are defined by

$$\Psi^{+}_{(F,A)} = \left[\mu_{F(e_j)(x_i)}\right]_{n\times m}, \Psi^{-}_{(F,A)} = \left[\nu_{F(e_j)(x_i)}\right]_{n\times m}.$$

These two matrices encode the complete numerical structure of the IFSS.

Definition 2.3. Let $\sigma_1{}^{+} \geq \sigma_2{}^{+} \geq \cdots \geq \sigma_n{}^{+} \geq 0$ be the singular values of $\Psi^{+}_{(F,A)}$, and let $\sigma_1{}^{-} \geq \sigma_2{}^{-} \geq \cdots \geq \sigma_n{}^{-} \geq 0$ be the singular values of $\Psi^{-}_{(F,A)}$. The *energy* of the IFSS (F, A) is defined by

$$\mathcal{E}_{(F,A)} = \sum_{i=1}^{n} (\sigma_i{}^{+} - \sigma_i{}^{-}).$$

This quantity measures the global balance between membership and non-membership information in the observed IFSS.

3. Decision-making algorithm and an estimate of the energy

The proposed decision-making rule is based on the contribution of each alternative to the total system.

Algorithm 3.1. Energy-based decision-making algorithm for IFSSs

Input: An intuitionistic fuzzy soft set (F, A) defined over the universal set $U = \{x_1, x_2, \ldots, x_n\}$.
Output: A ranking of the alternatives according to their contribution to the system, obtained by analyzing the energies of the reduced IFSSs.
Step 1. Input the intuitionistic fuzzy soft set (F, A).
Step 2. Form n new intuitionistic fuzzy soft sets, denoted by $(F, A)_i$, over $U \setminus \{x_i\}$ for each $x_i \in U$.
Step 3. For each intuitionistic fuzzy soft set from Step 2, form the corresponding membership and non-membership matrices $\Psi^{+}_{(F,A)}$ and $\Psi^{-}_{(F,A)}$.
Step 4. Determine the singular values for all matrices obtained in Step 3.
Step 5. Compute the energies $\mathcal{E}_{(F,A)_i}$ for each reduced intuitionistic fuzzy soft set $(F, A)_i$, using the singular values obtained in Step 4.
Step 6. Compare the values obtained in Step 5 and interpret the result.
The alternative whose removal gives the smallest remaining energy is the **most structurally influential**, while the alternative whose removal gives the largest remaining energy is the **least contributing** one.

The interpretation is immediate. If the energy of the remaining system is small after removing x_i, then x_i had a strong contribution to the original system. If the remaining system still has a large energy, then x_i contributed less. Therefore, $\min_i \mathcal{E}_{(F,A)_i}$ corresponds to the most contributing alternative, while $\max_i \mathcal{E}_{(F,A)_i}$ corresponds to the least contributing alternative.

Theorem 3.2. Let (F, A) be an IFSS over $U = \{x_1, \ldots, x_n\}$, where $|A| = m$. Then

$$-n\sqrt{m} \leq \mathcal{E}_{(F,A)} \leq n\sqrt{m}.$$

3. Case Study and Validation of the Energy-Based Method

We illustrate the proposed method on a team-selection problem with six candidates $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$, evaluated with respect to six parameters $A = \{e_1, e_2, e_3, e_4, e_5, e_6\}$, which represent creativity, managerial skills, intuition, research productivity, ability to work under time pressure, and knowledge. The corresponding IFSS (F, A) is given in Table 1.

Table 1. IFSS (F, A)

(F, A)	e_1	e_2	e_3	e_4	e_5	e_6
x_1	(0.8,0.1)	(0.6,0.2)	(0.4,0.4)	(0.7,0.2)	(0.6,0.1)	(0.7,0.3)
x_2	(0.6,0.2)	(0.7,0.2)	(0.4,0.5)	(0.5,0.1)	(0.6,0.2)	(0.5,0.3)
x_3	(0.7,0.3)	(0.8,0.1)	(0.6,0.3)	(0.7,0.1)	(0.2,0.5)	(0.7,0.1)
x_4	(0.8,0.1)	(0.9,0.1)	(0.6,0.2)	(0.5,0.3)	(0.7,0.2)	(0.6,0.2)
x_5	(0.5,0.4)	(0.6,0.3)	(0.7,0.2)	(0.2,0.6)	(0.7,0.1)	(0.3,0.5)
x_6	(0.6,0.1)	(0.7,0.2)	(0.5,0.3)	(0.8,0.1)	(0.6,0.3)	(0.4,0.4)

For the original system, the energy equals $\mathcal{E}_{(F,A)} = 2.306$.

Next, we remove one candidate at a time and compute the energies of the reduced systems. The obtained values are presented in Table 2.

Table 2. Energies of reduced IFSSs

	x_1	x_2	x_3	x_4	x_5	x_6
$\mathcal{E}_{(F,A)_i}$	1.883	2.230	1.986	1.950	2.380	2.045

Since $\mathcal{E}_{(F,A)_1} < \mathcal{E}_{(F,A)_4} < \mathcal{E}_{(F,A)_3} < \mathcal{E}_{(F,A)_6} < \mathcal{E}_{(F,A)_2} < \mathcal{E}_{(F,A)_5}$, the ranking of contributions is $x_1 \succ x_4 \succ x_3 \succ x_6 \succ x_2 \succ x_5$. Hence, candidate x_1 is the most influential for the structure of the system, while candidate x_5 is the least contributing one. The bar chart confirms this conclusion visually: the smallest remaining energy is obtained after removing x_1, whereas the largest remaining energy is obtained after removing x_5.

Effectiveness Test

In order to provide additional validation of the proposed energy-based method, we briefly examine the three effectiveness criteria introduced by Wang and Triantaphyllou ([4]). The considered example satisfies all three criteria.

Step 1: Verification of Criterion 1.

Criterion 1 requires that a mild deterioration of a non-optimal alternative should not change the optimal decision, nor disturb the relative order of the unaffected alternatives. Starting from the ranking $x_1 \succ x_4 \succ x_3 \succ x_6 \succ x_2 \succ x_5$, we consider the non-optimal alternative x_4 and replace its value under parameter e_1 by the corresponding inferior value of the worst-ranked alternative x_5. After applying the proposed algorithm again, the obtained ranking is $x_1 \succ x_3 \succ x_4 \succ x_6 \succ x_2 \succ x_5$. Thus, the best alternative remains unchanged, and the order of the unaffected alternatives is preserved. Therefore, Criterion 1 is satisfied.

Step 2: Verification of Criterion 2.

Criterion 2 requires consistency on subproblems. We consider the following subproblems with three alternatives each: $\{x_4, x_5, x_6\}, \{x_2, x_3, x_4\}, \{x_3, x_4, x_5\}, \{x_1, x_2, x_3\}, \{x_3, x_5, x_6\}, \{x_2, x_3, x_5\}, \{x_2, x_3, x_6\}$, and $\{x_1, x_2, x_4\}$. Applying the energy-based method to these subproblems yields the rankings $x_4 \succ x_6 \succ x_5, x_4 \succ x_3 \succ x_2$, $x_4 \succ x_3 \succ x_5$, $x_1 \succ x_3 \succ x_2$, $x_3 \succ x_6 \succ x_5$, $x_3 \succ x_2 \succ x_5$, $x_3 \succ x_6 \succ x_2$, and $x_1 \succ x_4 \succ x_2$. Each of these partial rankings is consistent with the overall order $x_1 \succ x_4 \succ x_3 \succ x_6 \succ x_2 \succ x_5$. Hence, Criterion 2 is satisfied.

Step 3: Verification of Criterion 3.

Criterion 3 requires that the aggregation of consistent subproblem rankings should reproduce the overall ranking. Since the rankings obtained in Step 2 are mutually compatible and induce the same global order $x_1 \succ$

$x_4 \succ x_3 \succ x_6 \succ x_2 \succ x_5$, Criterion 3 is also satisfied. Therefore, the proposed energy-based method passes all three effectiveness criteria.

Computational Complexity

We briefly comment on the computational complexity of the proposed algorithm. Let the IFSS contain n alternatives and m parameters. For each alternative xi, the algorithm forms a reduced IFSS $(F, A)_i$, whose membership and non-membership matrices have dimension $(n-1) \times m$. The dominant computational cost lies in computing the singular values of these matrices.

The singular value decomposition of a matrix of size $(n-1) \times m$ requires $O((n-1) \cdot m \cdot \min(n-1, m))$ operations. Since this computation is performed for all n reduced systems, the total complexity of the method is $O(n(n-1)m \cdot \min(n-1, m))$. Therefore, the proposed procedure has polynomial complexity and is computationally feasible for decision-making problems of moderate size. In practical implementations, the cost is dominated by the singular value computations, while the construction of the reduced IFSSs and their associated matrices is comparatively negligible.

5. Conclusions

In this paper, we introduced a new approach to decision making in the framework of intuitionistic fuzzy soft sets. The proposed method is based on a matrix representation of the observed system and provides a global criterion for ranking alternatives. In addition, the obtained estimate gives a general bound for the introduced numerical characteristic, while the illustrative example shows that the method is easy to apply and yields a clear ranking of alternatives. These results indicate that the proposed approach can serve as a useful tool for decision-making problems under uncertainty.

Acknowledgements

The authors are supported by the Ministry of Science, Technological Development and Innovation, Republic of Serbia, Contract No. 451-03-34/2026-03/200122.

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