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A Critical Analysis of the PISA Mathematics Tasks

Olivera Marković[1], Milenko Pikula[2] and Marina Zubac[3]
[1]*University of Kragujevac, Faculty of Education in Užice*
[2]*University of East Sarajevo, Faculty of Philosophy*
[3]*University of Mostar, Faculty of Natural Sciences and Mathematics, and Educational Sciences*

Abstract

In this paper, the authors deal with a critical analysis of some of the problems associated with PISA testing in mathematics. First of all, the social, economic and political circumstances of the launch of the PISA project are pointed out. This is followed by an analysis of what the authors consider as very important concepts for understanding PISA philosophy, such as applicable knowledge and problem solving in a real-life situation. We compared two different ways of defining these concepts and showed what the consequences of learning and teaching mathematics are in each of the cases. The authors present the results of the research that was conducted with mathematics teachers, as well as teachers of other subjects. The research dealt with the determination of the importance and role of the mathematical tasks of the PISA test for students' further mathematical education. The results of the research have shown that there is a significant statistical difference between the assessment provided by the teachers who do and those who do not teach mathematics. The paper also analysed some specific tasks that the students were solving in PISA tests. The authors point out some deficiencies and inaccuracies observed in the mathematics tasks on PISA tests, as well as the weaknesses of PISA testing itself.

Key words: *functional knowledge; mathematical literacy; real-life situation; teaching mathematics.*

Introduction

The first international PISA testing of fifteen-year-old students was organized in 2000 by the OECD. Since then, it has been implemented every third year, now in over 70 countries, in order to *evaluate reading, mathematical and natural-scientific literacy*.

In each test cycle, two-thirds of the questions relate to one of the listed domains, so that these major domains alternate. In 2018, the PISA project *has come of age* and entered *adulthood*, but can we say that it has got over the teething troubles they have encountered from the onset? Have all the doubts and ambiguities about the motive for launching and financing PISA testing by the OECD, an organization devoted to world economic development, been clarified? Are the concepts such as *functional knowledge, knowledge applicable in everyday life, mathematical and scientific literacy*, etc. clearer to us today than they were when, with great publicity, they entered the educational discourse of all participants in the world education process? Why is PISA testing in some countries, for example in Serbia, considered a first-rate political event, and moving up several places in ranking is the front page news, while in other countries, such as the United States, it is relatively unnoticed, according to Hopmann (2008). Why do educational policy-makers in many countries consider it imperative that the school curriculum literally adapts to the requirements of PISA tests, despite the high number of objections and denials by the expert and scientific public? (Hopmann, 2008).

If we consider the nature of education, its importance and significance, we will surely agree with the statement, which is equally valid for Plato's *State* as well as for any modern state, that education cannot be viewed as isolated from the most important flows of a country's life, its aspirations and values (Serder & Ideland, 2016). Therefore, it cannot be excluded from politics either if politics expresses concern for the vital needs of a community. However, some questions arise, such as those referring to how we understand the nature of education, where we see its place in society, and how we determine its function in the state. In other words, in what direction we think education policy should go, to what extent it is the expression of the organic and essential needs of the society from which it originates, and to what extent it is the expression of the power struggle, as Foucault (2004) defined it. In this case, we see that a political organization such as the OECD, whose slogan is "For a Better World Economy", is interested in a project which is purely academic in its most important features. However, the OECD and its members did not start the PISA project because they were interested in basic research in the field of education or learning theory. This organization decided to invest in PISA testing because education is crucial to the economy, as Sjöberg (2007) claims. Thus, a seemingly strong link which is established between education and economy becomes the basic axiom and the starting point. PISA starts from the basic premise that economic wealth is in a positive correlation with the knowledge and competences of students, whose success, on the other hand, depends on certain characteristics of schools. This is followed by the conclusion that the introduction of desirable skills and knowledge increases the wealth of the country. However, both the starting premise and the conclusions derived from it are largely simplified, according to Fernández-Cano (2016). Or, as Hopmann (2008, p. 439) says, PISA "relies on 'strong assumptions' (Fertig, 2004) based on poor data referring to daily wisdom ("Education is important, isn't it?"," School structures make a difference, right?"), but it does not offer almost any empirical or historical research to substantiate its implicit causality. However, this did not prevent PISA researchers or the public from behaving as if causative relationships were indeed given." It is precisely this apparent safety, corroborated by tables and ranking lists, that has led to the belief that the relationship between pupils, school, and country results is self-evident, and that the results of PISA testing are an undeniable diagnosis that is never questioned just like we do not question the validity of a test as a precise instrument by which we obtain relevant information.

However, we note that even before PISA tests, some scientists challenged standard achievement tests. End-of-the-year knowledge tests in the US school system were particularly criticised (Popham, 2003). At the end of the 20th century, these tests gained in importance and became a measure of the success of the education system, schools and teachers. Schools whose students were less likely to pass the tests had their funding reduced, teachers had their salaries reduced, and parents took children out of such schools, says Popham (2003). Since the test results were directly proportional to the school rating and its material situation, as well as that of the teachers, teachers focused their efforts on training students to do the final tests. Materials that were not included in the tests were treated differently and superficially, says Popham (2003). Thus, in US schools, the final test was put to the forefront, not the processing and essential knowledge of students, which was challenged by many scientists. Since the mid-twentieth century, specialised companies have been working on the tests, earning themselves huge profits. Let us add that, according to Sjöberg (2007), the total cost of PISA and TIMSS testing in 2006 was over $100 million for all countries together.

Therefore, this project should also be considered in this wider context because it has not been conducted by impartial researchers in the field of education, but by experts gathered around a neoliberal economic and political paradigm aimed at developing skills and competencies that promote economic and political goals of the OECD. We believe that the interest in these economic and political goals is perfectly natural, as is the legitimate right of every government to seek and obtain relevant information to guide its policy and establish economic development priorities. One of the priorities is definitely the desire of the government of a particular country to ensure the efficient functioning of the education system, which is completely understandable, especially when considering that mass education is extremely expensive (Popham, 2003). This is the basic fact that the authors of the PISA project have not tried to conceal, but it is very often forgotten in public and political discussions about the results and significance of PISA testing. However, a large number of authors dealing with the concept, scope and results of PISA testing believe that the idea of the PISA project cannot be understood at all beyond the issues on the agenda of the OECD, and beyond the context of economic development and competition at the global level of market economy, as Sjöberg (2007) says. We should also mention Hopmann's study (2008), which provides an excellent overview of philosophical and political theories that try to understand PISA testing in the context of political and economic relations in the world.

Nevertheless, despite all the research, we have still not obtained clear and definite answers on whether the PISA project has really improved the economies of the participating countries, in which segments and to what extent. For example, can a clear causal link between the performance in PISA tests and economic development of the state be established? However, while we are waiting for these economic indicators, we have received a lot of feedback from the expert and scientific public on a series of problems caused by PISA testing philosophy within school systems, especially in countries such as Serbia, which clearly understand the PISA project as the most important value and the key political framework for the development of its education system. "We are moving into a cycle of reforms and it is exactly what PISA measures. Our previous education system has been reduced to transferring content to students, and the new concept is based on planning and applying the acquired knowledge," said the Minister of Education Šarčević (2018). Advisor to the Minister, Gordana Kosanović, talking about the new cycle of PISA testing said that "PISA testing is an indicator of the quality of the education system, and that besides testing reading, mathematical, scientific and financial literacy, students also fill in a questionnaire, whose analysis provides the participating countries with a number of useful data they can use to make educational policies - from what influences pupils' achievements, what the motivation of students is, whether they are satisfied with the work of their teachers, and what the school ethos is" (Kosanović, 2018).

There is one important feature of the PISA project concept which is very often forgotten in the discussions - PISA testing, unlike TIMSS, is not intended to evaluate school knowledge, but to evaluate what is called functional, applicable knowledge that is not firmly linked to national and school curricula. On the contrary, it is emphasized, as Bodin (2005) says, that it is the knowledge which is largely non-school, but a result of personal experience gained in the family, socializing with peers or living in a wider cultural environment. Nevertheless, despite all these remarks, the public and politicians attribute all the test results, both good and bad, only to the school, expecting the school to do everything it can in order to earn their country a higher rank, as we have already seen in previous statements. Highly ranked countries, such as Finland, are becoming a model, and their schools are studied as a model in other countries (Bodin, 2005). This, among other things, confirms that the basic intention of PISA testing is ignored, and that is to measure the knowledge which is the result of economic, cultural, historical and political factors shaping the lives of young people in one country, with the role of school knowledge and institutions in the country unjustifiably emphasized. Simply put, it is claimed that PISA tests evaluate the applicability of a portion of the fifteen-year-olds' knowledge that they acquire mainly through out-of-school experiences, and only to a small extent in school. After that, it is declared that the school is responsible for the test results, good or bad, although on the basis of the above-mentioned characteristics of the PISA tests, it cannot be. Requests are then made to change the school curriculum in such a way that the respondents could achieve the best results in the next examination by doing tests that do not, as the authors of the test claim themselves, assess school knowledge. It could be said that the source of the greatest number of problems concerning the understanding of the PISA project and its philosophy, as well as its place in the education systems of the participating countries is in this circle of inconsistency and contradiction.

We will here outline two problems that make up the imagined coordinate system of the PISA project: defining the terms *applicable knowledge* and *real-life situation*. These two concepts, in many aspects, reflect most of the numerous problems mentioned here. Let us look at this brief historical reminder and the accompanying analysis of the problems that arise when trying to understand these concepts.

Many ancient nations, such as the Babylonians, the Chinese and the Egyptians, already two thousand years B.C. knew that the triangle with the sides of 3, 4, 5 was a right triangle, and they used this knowledge in building very complex buildings that we still admire today. However, Pythagoras was the first to prove this practical rule in theory, thus enabling a potentially infinite number of conclusions that had never been observed by anyone and would never be observed in the future. Moreover, it is traditionally considered that Pythagoras School is responsible for introducing the concept of *proof* in mathematics. However, this practical benefit that theory brings from the standpoint of common sense logic is a complete paradox because common sense brings practical benefit only to the knowledge of individual cases. Jan Lukašjevič, speaking about creative elements in science, on the contrary, writes: "Today scientists are more inclined to see practical values in generality. General statements, by defining the conditions under which phenomena occur, make it possible for us to predict the future, provoke useful phenomena and prevent those harmful ones from happening." (Lukašjevič, 1997, p. 78). We can say that today's mathematics is only a natural result of the development of mathematics, which was made science by Pythagoras, who separated it from trade affairs. We should remember that the word *arithmetic* itself first referred to the skill of using numbers which had no practical use value. According to Aristoxen's testimony, before Pythagoras people had used numbers exclusively in trade affairs: "The science of numbers seems to have been improved and honoured by Pythagoras most of all: he took it out of the sales department and compared all things with numbers" (Diels, 1983, p. 398). In this way, the philosophy and mathematics of ancient Greece, in terms of their concept teaching as a skill of distinguishing between the substantial and the ephemeral, and the proof as an argument set the foundations of European science and culture. According to Lukašjevič (1997), the more this difference between the relevant and the irrelevant is established at a general level, potentially the more practical it is, because it can be applied to a greater number of cases. When it comes to the applicability of such knowledge, we can safely argue that there is no reason, coming either from logic or from life, that one who has truly understood some regularity at the general level would not be able to apply it to an actual case in everyday life. When it comes to mathematics, this means that the primary objective of teaching is to help students build coherent and structured knowledge that is yet to become the *organon* (tool) for successfully solving problems in real life, including the problems which do not yet exist and which are yet to appear, as demonstrated in the study *The concept of functional knowledge* by Erić (2011).

However, what appears in the PISA project as an issue is that the problems of applying mathematics, learning and teaching mathematics are not approached from the point of view of the mathematician-practitioner who is experienced in teaching, but rather from the point of view of the so-called *common sense*. Bodin (2005) testifies to the strong resistance of teachers to PISA tasks because they feel they are not compatible with the curricula. Simply put, the questions are not mathematical. Such resistance of mathematics teachers has also been confirmed by the present research. In fact, the problem appears at two levels - one problem is at the level of the test itself, where we check how *common sense* deals with extremely simplified "mathematical problems" in a real-life situation, and another problem arises at the level of the concept of the test where this same *common sense* gives us its own understanding and view of mathematics and mathematics education by advising us what we should change in school so that students can be more successful in real-life situations. We have already mentioned this problem. Here we define *common sense* as a *general ability*, as has already been done by other authors. Wuttke (2007) writes that, for example, 75% (Greece) and 92% (Holland) of the total variance of the respondents' competencies can be attributed only to one component, which the members of the PISA project decisively reject, and even mock. The ideological and strategic reasons for this conflict are obvious: when it is established that PISA measures a single general factor, general ability, it is difficult not to link this factor to general intelligence research that the PISA project members perceive as a threat since, of course, no government would spend millions to get notified about students' intelligence. We must admit that we are inclined to support Wuttke's (2007) observations, and we believe that our research has shown this because the level of mathematical knowledge needed to solve the tasks in the PISA test is extremely low.

It is most often suggested in this concept, as it was perfectly observed by Skemp (1993), that the best way for students to learn mathematics is by relying on real-life problems that require mathematics to be solved from the very beginning. However, even very little theory will be enough to show that this is not the best way. Skemp (1993) proposes as an example a widely known common sense fact that iron sinks, which is a fact that common sense challenges daily. So, how is it possible then that there are ships made of iron that do not sink? In order to understand this, we will not need any individual cases, no experiential conclusions. We need to know a simple, but powerful theory - Archimedes' law. This is the principle that allows us to predict whether a ship made of iron or even reinforced concrete will sail or not. Using this principle, we will get farther than we could ever with common sense, says Skemp (1993). We will use this example of Skemp's to do another analysis. Let us imagine that one of our students now is given the opportunity to repair a ship since he is a shipbuilding engineer. How will he succeed in this if, during schooling, he only encountered malfunctions *different from one case to* *another*, and this particular malfunction is not on the list of cases known to him. And precisely this type of specification, practicality and learning from life is suggested as the best *methodos* (way) for teaching mathematics, as we will see in the following analyses. In all cases of PISA tests, this concrete, real-life situation is simulated by providing surplus data in a task. As Bodin (2005) correctly observes, in some cases, the main difficulty the students face is simply understanding the text that is not mathematical at all. Of course, this can also be part of the solution to the problem, but real mathematical work begins when the problem is fully understood. We can also point here to the work of Anić and Pavlović Babić (2011) who, after analysing the problems of solving tasks in a real context, give suggestions for teaching practice.

Naturally, there is also the question of why in the formulation of the task we stop only at some surplus data when in the real-life situation for which we are preparing a student, the data are infinite: the colour of the ship, the squawking of seagulls above the head, the murmur of the waves, the roar coming from the ship's restaurant and so on, *ad infinitum*. If our engineer-to-be, is not capable of abstracting everything and focusing only on the *principle* which makes the ship sail because of his education, then he can, provided the ship does not sink in the meantime, sit and analyse real-life situations for days. This ability, for which education is preparing students, i.e. considering a problem from the point of view of the essence, will recommend him for this work rather than some common sense passer-by. Erić (2011) wondered why then, during their education, students should waste time on a concrete series of variations of a single situation when the likelihood of actually encountering one of them is meaningless.

If we would like to formalize the above-mentioned and isolate the most important steps in the abstraction that relates to solving the problem, then we can say that we are actually starting from abstracting the conceptual model from the actual situation. Then, from this conceptual model, we abstract the concepts that are important for our problem and we draw a mathematical model from these concepts. When, at the model level, we solve the problem, then, taking reverse steps, we apply this solution to the real-life situation. Skemp (1993) also rightly observes that in the case of a different approach we would come to a dead-end, an impasse. If we wanted to learn without theory through concrete mathematical tasks, they would naturally have to be isolated from one another, and then students would have to learn a new method for each task because common sense does not recognize the principles that classify these tasks into categories. In this case, burdening memory with innumerable "methods" which could only apply to one case would very quickly become unbearable. And not only that, resolving real-world problems would not provide enough repetition that would fortify the newly formed concepts nor would it turn the skill of their application into a well-established routine.

Skemp (1993) proposes a method of learning that would best build abstract mathematical concepts and structured mathematical knowledge:

1. Implementation of concepts should be provided with the least amount of irrelevant data, i.e. with "low level of noise", which should be ignored when creating concepts.

2. There should be a large number of examples and concepts that are closely related in time.
3. Only one new concept should be taught at a given time.
4. In order to introduce new concepts, there must already be a corresponding scheme so that the new concept can be connected to it, and thus enable learning with relational understanding.

All these requirements are difficult or impossible to achieve with the problems of the "real world", according to Skemp (1993).

These or similar remarks are most often responded to by the PISA project advocates, who say that their intention is not to measure school mathematical knowledge of fifteen-year-olds because they cannot be applied in everyday life. Their goal is, as they say, to measure the knowledge that can be applied in a real-life situation. Indeed, when we look at the tasks that illustrate this concept, we see that they are mathematically extremely simplified problem situations for which the solution is very often an answer that begins with *seemingly, it appears, it seems likely* ... Logically, the question that arises is, as Bodin (2005) says, whether every question that uses numbers needs to be considered a mathematical question. And thus a circle of chaos and confusion regarding the implicit and explicit expectations of the PISA project closes. However, the essential question here is why we take testing of such week mathematics knowledge as the main parameter on the basis of which we decide on the direction for changing our work in the classroom when it comes to mathematical content. Why are we evaluating our entire education according to the results of PISA testing even though we know that school knowledge is only partially represented, only in the three areas in which we evaluate students' literacy, and even in these areas we only evaluate a small portion of the knowledge applicable in everyday life situations? We will try to get some of the possible answers to these questions through the analysis of the research we have conducted.

Research Methodology

The subject of this research are mathematical tasks that appeared in the actual as well as mock PISA tests in the period from 2000 to 2012. The research was conducted in order to answer the question how mathematics teachers, as well as other teachers working in primary and secondary schools in the Republic of Serbia, perceive the role of PISA testing in students' mathematical education. The research tasks were:

– Identification of the importance and role of mathematical tasks of the PISA test from the perspective of mathematics teachers who teach this subject in primary and secondary schools;
– Identification of the importance and role of mathematical tasks of the PISA test from the perspective of non-mathematics teachers who teach in primary and secondary schools;
– Comparison of the assessment of the significance and role of mathematical tasks of the PISA test done by the teachers who do and those who do not teach mathematics.

Research Sample

The research involved 97 mathematics teachers and 105 teachers of other subjects. Out of 105 teachers who did not teach mathematics, 44 were class teachers, 16 were Serbian language teachers, 7 were chemistry teachers, 11 were physics teachers, 7 were English language teachers, 2 were Russian language teachers, 8 were computer science teachers, 4 were history teachers, 2 were geography teachers and 4 were biology teachers. The study was conducted in the winter semester of 2017 in the schools of Zlatibor district, the city of Čačak and the city of Belgrade. Seven days before completing the questionnaire, the teachers were familiarised with the PISA brochures: *PISA Released Items - Mathematics, 2006* and *PISA Released Mathematics Items, 2013,* both in Serbian and English. They were asked to look at and examine 78 tasks from these brochures.

Instrument

For the purposes of this research, a numerical estimation scale was created. The pilot version of this scale was examined on the sample of first and second year students of the Faculty of Education in Užice. Based on their comments and responses, the final version of the scale was completed. The scale contains a list of 6 statements offered to teachers for evaluation:

- The knowledge needed to solve PISA tests in mathematics is applicable in everyday life.
- Learning through real-life situations is a good way to master mathematical content in higher grades of primary school.
- Mathematics tasks on PISA tests are useful for further mathematical education of students.
- Mathematics tasks on PISA tests are adjusted to fifteen-year-old students.
- The content of mathematics PISA tests is part of the mathematics curriculum taught in primary school.
- Percentage account and reading chart data are important mathematical competences that a fifteen-year-old student should possess.

The agreement with a certain statement was evaluated by the respondents with grades 1 to 5 (1 is the most unfavourable evaluation of a statement, and 5 is the most favourable evaluation of a statement). In the end, teachers were given the opportunity, if they wished, to provide open-type comments on the given tasks. The survey used the descriptive method, and $\chi 2$ test was performed to determine whether there was a significant difference in the perception of mathematical tasks of the PISA test by mathematics teachers and teachers of other school subjects.

Research Results

The first task of the research was to examine how mathematics teachers evaluated the tasks on the PISA test. Of the 97 interviewed teachers, 82 had already had the opportunity to get acquainted with the tasks they were given for evaluation. The data obtained by the research are shown in Table 1, indicating the individual values for each estimate and average results (M) for each of the given estimates.

The second task of the research was to determine how teachers who do not teach mathematics evaluate PISA testing tasks. Of the 105 participating teachers, 26 had previously had the opportunity to get acquainted with the tasks they were given for evaluation. The data obtained by the research are shown in Table 1.

The third task of the research was to compare the assessment of the significance and role of the mathematical tasks on the PISA test done by the teachers who do and those who do not teach mathematics. A comparison has been made to determine whether there are statistically significant differences between the estimates of these two groups of teachers (Table 1).

Table 1

Statistically significant differences between the estimates of two groups of teachers

Estimates		1	2	3	4	5	*N*	*M*	χ^2
Estimate 1	NM	10 9.52%	15 14.29%	27 25.72%	35 33.33%	18 17.14%	105 100%	3.34	$\chi^2 = 13.146$ $df = 4$ $p = 0.011$
	MT	12 12.37%	21 21.65%	39 40.21%	16 16.49%	9 9.28%	97 100%	2.89	
Estimate 2	NM	9 8.57%	21 20.00%	33 31.43%	25 23.81%	17 16.19%	105 100%	3.19	$\chi^2 = 12.446$ $df = 4$ $p = 0.014$
	MT	13 13.40%	27 27.84%	40 41.23%	10 10.31%	7 7.22%	97 100%	2.70	
Estimate 3	NM	3 2.86%	12 11.43%	23 21.90%	37 35.24%	30 28.57%	105 100%	3.75	$\chi^2 = 17.139$ $df = 4$ $p = 0.002$
	MT	8 8.25%	24 24.74%	30 30.93%	21 21.65%	14 14.43%	97 100%	3.09	
Estimate 4	NM	2 1.90%	6 5.71%	34 32.39%	38 36.19%	25 23.81%	105 100%	3.74	$\chi^2 = 21.293$ $df = 4$ $p = 0.000$
	MT	9 9.28%	22 22.68%	30 30.93%	24 24.74%	12 12.37%	97 100%	3.08	
Estimate 5	NM	8 7.62%	12 11.43%	25 23.81%	40 38.09%	20 19.05%	105 100%	3.50	$\chi^2 = 25.228$ $df = 4$ $p = 0.000$
	MT	18 18.56%	22 22.68	34 35.05%	18 18.56%	5 5.15%	97 100%	2.69	
Estimate 6	NM	8 7.62%	10 9.52%	39 37.15%	22 20.95%	26 24.76%	105 100%	3.46	$\chi^2 = 15.447$ $df = 4$ $p = 0.004$
	MT	16 16.49%	23 23.71%	29 29.90%	18 18.56%	11 11.34%	97 100%	2.85	

Discussion

On the basis of the obtained results, it may be concluded that the evaluation of the importance of mathematical tasks on the PISA test by mathematics teachers was not rated with a high score. The overall average score was 2.89, indicating that mathematics teachers do not consider mathematical literacy testing through the tasks they were analysing as important for the students' future mathematical education. In each of the statements, over 60% of the respondents chose one of the first three grades, which we can interpret as a general disagreement with the concept of the mathematical tasks on PISA testing.

In the comments at the end of the questionnaire, one of the problems the respondents stated was the incompatibility of the tasks to the age. Special emphasis is placed on the tasks in the field of probability, which are studied in the higher grades of secondary schools both in Serbia and in most European schools. Some respondents noted that for 15 out of 78 tasks offered for the analysis, knowledge about the concept of probability was necessary. Thus, Serbian students were not able to solve 20% of the tasks, not because of the difficulty of tasks, but because of the fact that this field of mathematics was still unknown to them. In their comments, the teachers pointed out another group of tasks that they stated was not suitable for the students' age because almost the same tasks could be found in mathematics textbooks for the third and fourth grade of primary school. For all these reasons, teachers chose the lowest average grade (M=2.69) when assessing the extent to which mathematics curriculum matched the content of PISA tasks. It should be emphasized that teachers do not consider the percentage accounts nor the reading of data from the charts as particularly important mathematical competences for fifteen-year-olds although such tasks are in the focus of the PISA test.

On the basis of the obtained results, it may be noted that the evaluation of the importance of mathematical tasks in PISA tests by teachers who do not teach mathematics was rated with a higher score than when the assignments were evaluated by mathematics teachers. The overall average score was 3.50, indicating that teachers of other subjects assessed the tasks of the PISA test less strictly. Over 50% of the surveyed teachers believe that PISA tasks are applicable in everyday life, useful for students' further education, adjusted to the age of students and in the curriculum of mathematics education. The only slightly more unfavourable score (M = 3.19) refers to the ability to master mathematical content by learning from real-life situations.

The comments found in the questionnaires of non-mathematics teachers were largely affirmative. A large number of these respondents expressed satisfaction with their knowledge of mathematics because before analysing the PISA tasks they thought that they had long forgotten mathematics! The main reason why the present research also included non-mathematicians lies in the fact that they are, in the authors' opinion, excellent representatives of the public opinion and of the general attitude that is formed about PISA testing based on what can be heard in the media and from the main actors in education policy, who, unfortunately, could not be included in the research. As already mentioned, this position is, both in Serbia and in most other countries, a priori positive because the public, even the expert public, do not have enough data on which to make a well-founded conclusion.

Based on the obtained $\chi 2$ values with df = 4 at the level of 0.01, it may be concluded that there is a statistically significant difference in the following estimates:

– Mathematics tasks on PISA tests are useful for further mathematical education of students;
– Mathematics tasks on PISA tests are adjusted to fifteen-year-old students;
– The content of PISA tests in mathematics exists in the primary school mathematics curriculum;
– A percentage account and reading chart data are important mathematical competences that a fifteen-year-old student should possess.

A statistically significant difference is particularly noticeable in the assessment of the curriculum of mathematics education. Teachers who do not teach mathematics are not familiar with the content of the subject, but, naturally, they expect compatibility of the test with the contents of primary school mathematics. On the other hand, mathematics teachers have clearly identified tasks which do not match mathematics curriculum.

Some of the remarks that the teachers in the open-ended section of the questionnaire indicated will be considered. As already pointed out, a considerable number of comments made by teachers referred to the presence of probability tasks. These remarks were exclusively given by mathematics teachers because they were familiar with the content of this subject and knew that fifteen-year-olds were not yet familiar with this concept. It has to be noted that most probability problems are very simple, but students cannot solve them unless they have been, prior to testing, familiarised with the formula according to which the probability of an event is calculated as a quotient of the number of favourable and the number of total outcomes of an event. In order to understand the essence of the concept of probability and this formula, which is very simple to apply and useful for one fifth of the tasks of the PISA test, significant mathematical education is needed. How can teachers explain to a primary school pupil that, for example, the probability of getting a six when throwing a dice is one-sixth if in reality it happens that in ten throws a person does not get a six or gets it several times? It would just be a case of a "real-life situation" that could contradict teachers and confuse students if they are introduced with this concept "by force" and too soon. An example of such a task is the task of *Coloured Candies*.

Coloured Candies

Robert's mother lets him pick one candy from a bag. He can't see the candies. The number of candies of each colour in the bag is shown in the following graph (Figure 1).

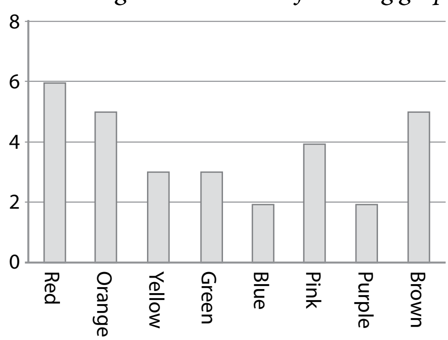

Figure 1. Coloured candies

What is the probability that Robert will pick a red candy?
A) 10% *B) 20%* *C) 25%* *D) 50%*

In addition to probability tasks, mathematics teachers also observed the existence of a large number of trivial tasks, which they also labelled as being inadequate. An example of such a task is the *Exports* task.

Exports

The graphic below shows information about exports from Zedland, a country that uses zeds as its currency (Figure 2).

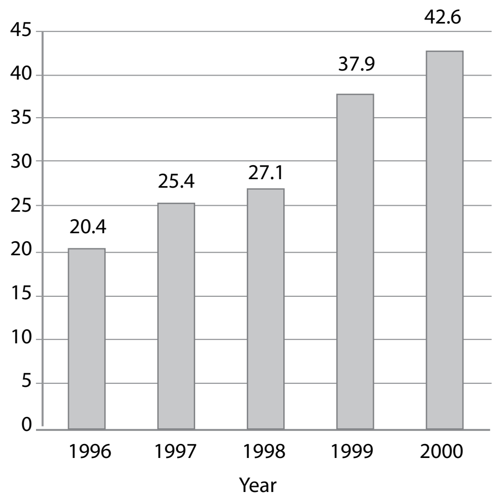

Figure 2. Exports

What was the total value (in millions of zeds) of exports from Zedland in 1998?

The authors of the PISA test ranked these assignments in the first two levels on a scale of achievement, where students are expected to "answer simple questions in a familiar context where all relevant information is given and questions are clearly formulated" (first level), and where " they can interpret and recognize situations in a context that does not require more than direct conclusions" (second level) (Baucal & Pavlović-Babić, 2010, p. 27). It should be noted that only in the description of the first level of achievement the respondents are expected to answer a question that is "clearly formulated". However, how is it possible to give an answer to a question that is not clearly formulated? Mathematics, in fact, teaches precision and rigor in the presentation and conclusion processes, and the students should transfer these experiences and knowledge, whenever possible, to "everyday life", and not transfer the non-obligatory and superficial mode of everyday speech in mathematics teaching.

As "problematic" tasks, respondents emphasized the tasks in which it was not clear what the students were supposed to do, to be more precise, the questions which were not "clearly formulated". Examples of such tasks are *Robberies* and *Test scores*.

Robberies

A TV reporter showed this graph and said: ''The graph shows that there is a huge increase in the number of robberies from 1998 to 1999" (Figure 3).

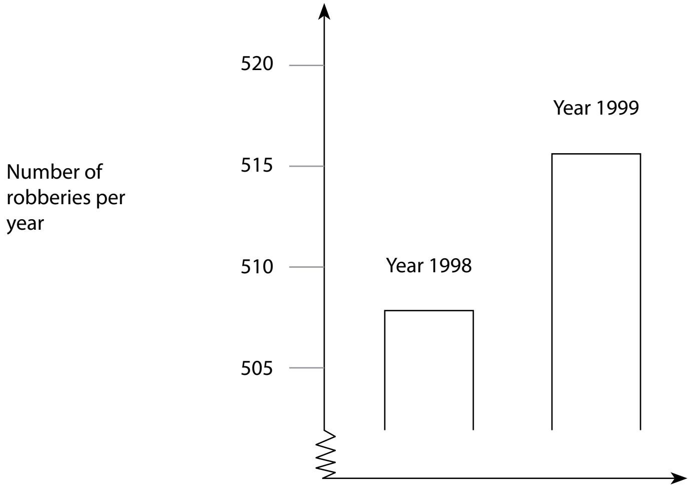

Figure 3. Robberies

Do you consider the reporter's statement to be a reasonable interpretation of the graph? Give an explanation of your answer.

The question "do you think" is not a mathematical question but a demand for students to express their personal attitudes. All three possible answers (not, yes, it cannot be said on the basis of the chart) could be supported by more or less common sense argumentation although here the desired response can also be guessed using common sense. Such an imprecise question places this task at the sixth, highest level of achievement. In the description of the sixth level of achievement, among other things, it is stated that students "can formulate and discuss with high accuracy the procedures they have applied, critically review findings, interpretations, arguments, as well as consider their suitability for solving complex problem situations" (Baucal, & Pavlović-Babić, 2010, p. 27). A debate and discussion on the arguments for and against an opinion can be a good exercise for developing students' different talents, but they cannot confirm their mathematical knowledge, which has already been discussed in the study *The Problem of Inadequate Use of the Mathematical Language in the PISA Test Task*, by Marković and Erić (2014). The respondents noticed a similar situation in the *Test scores* task.

Test scores

The diagram below shows the results on a Science test for two groups, labelled as Group A and Group B. The mean score for Group A is 62.0 and the mean score for Group B is 64.5. Students pass this test when their score is 50 or above.

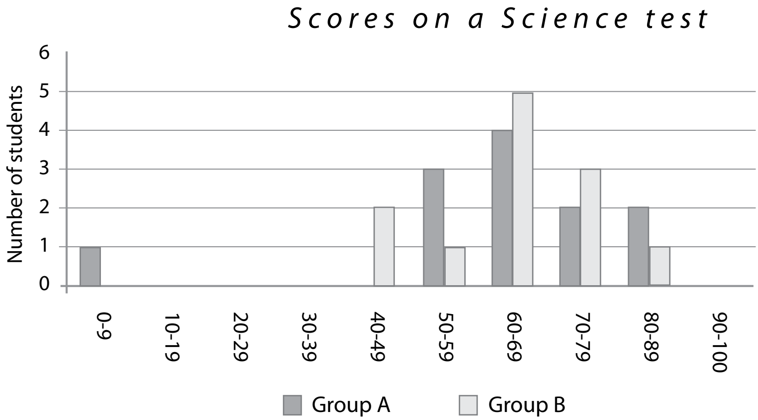

Figure 4. Test scores

Looking at the diagram (Figure 4), the teacher claims that Group B did better than Group A in this test. The students in Group A don't agree with their teacher. They are trying to convince the teacher that Group B may not necessarily have done better. Using the graph, provide one mathematical argument that the students in Group A could use.

This task may most obviously point to one of the key problems of PISA tests, which is inaccuracy. How does one evaluate which result is better when the term "*better result*" is not precisely formulated nor unambiguously determined? How one perceives *a better result* is not a mathematical problem, and no one's opinion or experience can be justified and defended by "mathematical arguments". Some think that the better result was achieved by the group with a higher average score, some prefer the group in which more students passed the test, or the group that had a higher average grade, not counting the student who did the worst, or the best in the test, etc. In this way we can name many reasons why we might consider a particular result as better than some other. All this leads to the conclusion that this task, despite the fact that it is accompanied by graphical representation of data, is not a mathematical task, and just like the previous one can cause further debate and discussion, and is definitely not the task on the basis of which one can judge whether a student can apply mathematics or not.

The task that most teachers described as "troublesome" for fifteen-year-olds was the *Earthquake* task.

Earthquake

A documentary was broadcast about earthquakes and how often earthquakes occur. It included a discussion about the predictability of earthquakes. A geologist stated: "In the next twenty years, the chance that an earthquake will occur in Zed City is two out of three". Which of the following best reflects the meaning of the geologist's statement?

A) $\frac{2}{3} \times 20 = 13.3$ *so between 13 and 14 years from now there will be an earthquake in Zed City.*

B) $\frac{2}{3}$ *is more than* $\frac{1}{2}$ *, so you can be sure there will be an earthquake in Zed City at some time during the next 20 years.*

C) The likelihood that there will be an earthquake in Zed City at some time during the next 20 years is higher than the likelihood of no earthquake.

D) You cannot say what will happen, because nobody can be sure when an earthquake will occur.

To solve this task, students need probability elements, which means that the task is not age adequate. Moreover, the formulation of the question "what reflects the meaning of the statement best" is mathematically neither precise nor unambiguous. Of course, the solution to this, as well as to most other tasks, can be intuitively and sensibly drawn from the offered answers. This task is classified at the fourth level of achievement, whose description states that students "can select and link data given in different ways, including symbolic ones, and link them directly with aspects of real-life situations" (Baucal & Pavlović-Babić, 2010, p. 27). Does this mean that students at this level of achievement are expected to link the number $\frac{1}{2}$ that represents the probability of an event with the statement that it is more likely that something will happen than that it will not happen? If students have not become familiar with the concept of probability in the education system, they simply cannot do it except by speculating or intuitively guessing, but if they have, then this statement is trivial and should not be in the high fourth place on the scale of achievement.

A similar problem is found in the task *Support for the president*.

Support for the President

In the country of Zedland, opinion polls were conducted to find out the level of support for the President in the forthcoming elections. Four newspaper publishers did separate nationwide polls. The results for the four newspaper polls are shown below:

Newspaper 1: 36.5% (poll conducted on January 6, with a sample of 500 randomly selected citizens with voting rights).

Newspaper 2: 41.0% (poll conducted on January 20, with a sample of 500 randomly selected citizens with voting rights).

Newspaper 3: 39.0% (poll conducted on January 20, with a sample of 1000 randomly selected citizens with voting rights).

Newspaper 4: 44.5% (poll conducted on January 20, with 1000 readers phoning the newspaper to vote).

In this task there is also a problem that is not mathematical. The task seeks a personal attitude, not an unambiguous solution to the task, therefore, there is again an imprecise formulation. Of course, the authors predicted only one correct answer - the third journal. This answer leads to common sense and the following arguments: the research was conducted just before the election (January 20), a large group of respondents (1000) were questioned, and only the citizens with the right to vote were questioned. However, it was not said how the survey was conducted for the third journal, whether the voters who were surveyed, although with the right to vote, were generally interested in voting.

Perhaps those who make phone calls to state their opinion are actually much more interested in the outcome of voting, so their voting is a better indicator of the final result. Of course, all this can be the subject of controversy and discussion, but not an exact mathematical task. It could also be questioned whether this task is adjusted to the age. Why would a fifteen-year-old need to know how surveys of the voters are conducted before the elections, and that it is necessary to question the accuracy of voters who vote by telephone?

Advocates and authors of the PISA test point out that the term mathematical literacy means "the ability of mathematical conclusion and the application of mathematical procedures and tools in describing, predicting and explaining phenomena rather than a set of school knowledge and skills." PISA measures learners' ability to respond to requirements relevant to everyday life, and to understand key concepts, and does not estimate the acquisition of knowledge relevant to specific areas" (Baucal & Pavlović Babić, 2010). Therefore, one of the first criteria that every task in the PISA test must meet is to be placed in a "real-life context". Critics of the Serbian school system emphasize that students memorise unnecessary data and learn theoretical facts which they then cannot apply in "everyday life", and that students' bad results in PISA tests are the best proof that they are right (Strategy for the development of education in Serbia until 2020, 2012). But, by chance or intentionally, it is forgotten that every formula taught in mathematics in both primary and secondary schools is derived and explained to students by referring to their previous mathematical knowledge and not "served" to be learnt as a ready-made formula. However, precisely such formulas whose meaning students do not understand can be seen in PISA tests.

In different tasks, students are offered formulas that more or less accurately describe some relationships in nature and real life: $d = 7.0 \cdot \sqrt{t - 12}$, (t - 12) (the relationship between the diameter of the circle shaped lichen and the number of years since the dissolution of ice) – the *Lichen* task, n/P=140 (the relationship between the number of steps per minute and step length in meters) – the *Walking* task, $D=dv/(60n)$ (formula for the infusion flow rate depending on the volume and number of hours) - the *Drip rate* task.

In each of these tasks, there are links between some structures for which students do not know how they have been obtained. The content of these tasks is placed within the theme that the authors of PISA testing call *Numbers and measures*. The tasks with this content require "understanding of numerical phenomena, quantitative relationships and patterns" (Baucal & Pavlović-Babić, 2010, p. 27). In each of the above-mentioned tasks, all the values are known except for the one that is required. By substituting known values in given equations in all three cases, trivial equations with one unknown variable are obtained. Students can solve such tasks mechanically using formulas, and have no need to think about them. Therefore, students solve them by using exactly the methods that PISA advocates and new education strategies criticize the most. However, the differences between these two cases of the application of the formula are essential. In school tasks, the required formulas are given to students with a complete explanation of their internal structure, meaning and reason, as well as the importance and place of that formula in their mathematical education. The real context of PISA tasks that allegedly exists is, however, just illusory because how many students in their everyday life think about the size and age of the lichen or the infusion flow rate? Can we imagine nurses who sit with a piece of paper, pencil and calculator, and for each patient in particular calculate the rate of infusion drop, or do they simply press a key on the computer keyboard and get the answer to that question? If these are real-life situations of a fifteen-year-old, the question is - what then is not their real-life situation? Have the advocates of this concept made the term *real-life situation* meaningless by means of its relativization?

One of these formulas (the *Walking* task) could be challenged because it is arbitrary and experiential, not a product of serious research. The question arises whether the n and P values representing the number of steps per minute and the length of the step in meters are directly or inversely proportional. When a man speeds up, do his steps become shorter or longer? Watching racewalking competitors, it seems that the steps are shorter, which makes the accuracy of the given formula questionable, as already discussed in the study by Marković and Pikula (2014). So, these tasks with dubious real-life contexts require nothing else from a student but to apply a simple formula whose nature is completely unclear to students.

The real-life context is also a justification for the tasks where there is a surplus of data, which has also been previously mentioned. Examples of such tasks are: *Staircase, Sailing ships, Power of the wind,* etc. In regular schooling, students do not have tasks with surplus data. Since in these tasks there is a text overload precisely because of the surplus data, students get confused thinking that they must have overlooked something. The fact that students fall into this trap does not necessarily reflect their mathematical ignorance, nor is it a consequence of learning without understanding, nor a key proof that they cannot deal with a real-life situation because at that moment, when they do the test, they are not on the stairs of a shopping mall, nor on the deck of a cargo ship, nor near a windmill. Their truly real-life situation at that moment is doing a test, and they perceive test tasks as correctly written mathematical tasks at school, and such tasks do not have unnecessary data. If they were really on board a ship, the ship's logbooks would have even more information about the ship than they were given in the task, but they would also know that all these data are not relevant to the problem that needs to be solved, for example, problem with the sail.

These tasks will not be listed because they are too long. However, it has to be pointed out that in the *Sailing ships* task the following situation is given. A ship whose name is "New Wave" spends 0.42 zeds (fictitious banknote) per litre of diesel. Due to high fuel prices, the owner decides to buy a kite-like sail that can reduce diesel consumption by about 20%. The data given in this assignment are: the price of diesel (0.42 zeds), the length of the ship (117 meters), the width of the ship (18 meters), its capacity (12,000 tons), the maximum speed (19 knots), the annual consumption of diesel without the kite (approximately 3,500,000 litres), and the cost of the kite sail equipment (2,500,000 zeds). The name of the ship is also indicated as well as the fact that the ship is a freighter. The question in the task is: After how many years will the cost of purchasing a kite-like sail pay off by saving diesel fuel?

When students manage to extract the necessary data out of this pile and solve this task, they get the answer that it is about 8.5 years, that is, between 8 and 9 years, which is also recognized as the correct answer. However, if the actual context is really insisted on, various problematic issues can be raised here. For example, what if the price of diesel for all those 8 or 9 years is variable, or will the weight of the kite-like sail reduce the carrying capacity of the ship and therefore its profit?

In the OECD brochures *PISA Released Items – Mathematics, 2006* and *PISA Released Mathematics Items, 2013* after each task an explanation is given as to how the tasks are scored. Although the method of evaluating tasks by the PISA team has not been the subject of a thorough research, some insights into this evaluation will be given. The first step in reviewing the tests is coding. Student responses are coded by pre-defined codes. This procedure is particularly sensitive when it comes to the answers to open-ended questions. Even though the coders were trained to minimize their subjectivity, they were, eventually, allowed to personally evaluate practically everything - whether the answers deserve "a maximum number of points", "a partial number of points" or "no points". The criteria are particularly vague for partially correct answers, i.e., for the "partial score". Thus, for example, in the *Lichen* task it is necessary to solve the elementary irrational equation $d = 7.0 \cdot \sqrt{t - 12}$ where $d = 35$. In the OECD PISA brochure *Released Items - Mathematics* (2006, p. 6) it is stated that partial points are assigned for "the correct replacement of values in a formula, but also for the wrong answer" or for answers t = 36 and t = 38 (although the only correct answer is t = 37), which "the student can come up with using the method of trial and error". No clear reasons are provided as to why partial points are assigned for an incorrect answer (36 or 38), which the student obtains through an unacceptable procedure (trial and error method). Partial points are assigned for the answer in which the student wrote, "$5 = \sqrt{t - 12}$", which leads to "$5 = \sqrt{t} - \sqrt{12}$". The student, making this gross material error and indicating that he/she does not know that the square root of a subtraction is not equal to the subtraction of square roots, is, according to the evaluators, partially mathematically literate although in fact he/she is absolutely mathematically uneducated.

The arbitrariness in accepting answers as correct is present in most of the tasks in which students are asked to provide some estimates. Thus, for example, in the *Shapes* task, the student is asked to describe the method of estimating the surface of an irregular body. As an example of the answer for which students should be assigned the maximum number of points, the following answer is given: "Draw a figure on a square network. Smaller squares mean greater precision." The instruction for evaluating this answer states: "Here the student's explanation is very brief, but we were lenient with the student and took into consideration his/her poor writing skills because we find the method offered by the student to be the right one" (PISA Released Items – Mathematics, 2006, p. 28). Unfortunately, there is no precise explanation for this type of assessment. In the previous two sentences, the student did not propose any method, but only started the explanation. His/her further flow of thought, which is not written, cannot be subject to evaluation. Furthermore, the procedure for solving a mathematical problem is either correct or incorrect, and can in no way be "considered" correct or incorrect.

The most obvious arbitrariness in evaluating responses is seen in the *Garage* task. In this task it is necessary to calculate the surface of the roof consisting of two matched rectangles, for which the length of one side is known and the other represents the hypotenuse of a single right triangle whose catheti are also known. The maximum number of points is given to the answer in which the student offers as a solution any value from 31 to 33 (note that calculators are allowed in PISA testing) although the roof surface can be precisely calculated. However, a much bigger problem are solutions that are assigned partial points: "The procedure does not show the use of the Pythagorean theorem, but acceptable values for the width of the roof (for example, any value of 2.5 to 3) were used, and the rest of the calculation was accurately performed" (PISA Released Items - Mathematics, 2013, p. 32).

The question that arises is what is actually estimated in this task. A partial number of points is given to the answer of a student who does not know how to apply Pythagorean theorem in the most obvious case, but knows roughly how wide one side of the garage roof should be! They are therefore giving points for experience and resourcefulness, not mathematical knowledge or any application of it.

These are only some examples where the lack of principled approach in evaluating responses as well as the complete arbitrariness in subjective evaluation of accurate or partially accurate responses can clearly be seen. Can such imprecision in assessment yield relevant data for any serious analysis? What is the value of complex statistical data analysis based on completely arbitrary and unreliable data?

In the end, the issue of students' motivation can also be discussed. Are they ready to invest maximum effort during the testing procedure doing some frequently boring, tiring tasks with text overload in order to achieve the best possible result that they cannot personally benefit from in any way? Lambić and Lipkovski (2011) rightly point to the importance of satisfaction and enjoyment as a motivational factor in solving the problem. It is known that PISA assesses student attitudes, beliefs and motivation for school learning, but a big problem is also the motivation of students for PISA testing, which they do not have to consider important because in their real-life situation this testing has no functional value. Wuttke (2007) also made some relevant observations regarding the problem of motivation and fatigue as significant factors in the performance of PISA testing.

Of course, one has to be aware of certain limitations of this research which was conducted by two independent researchers. First of all, the respondents had to spend a certain amount of time examining the 78 PISA test tasks. The question that arises is whether each respondent actually did it and how thorough they were. The second question is how many non-mathematics teachers were able to truly participate in the research and read and study all the tasks in detail. The third limitation relates to the fact that this research used a self-report survey and a numerical estimation scale that allowed personal attitudes to be expressed more than the authors may have wanted. The assessment of respondents who completed the questionnaire could be more or less subjective if, for example, they misunderstood a sentence, but also due to any opinion they had already formed about PISA testing.

Conclusion

The PISA project is an international evaluation macro-study, and this should always be kept in mind because this is why one should be very cautious in assessing its scope and possibilities. It does not really have, nor does it aspire to have, any practical value in terms of individual achievement of students or in terms of the impact on the actual teaching practice. We believe that any such attempt of literally transmitting or applying PISA philosophy in mathematics is wrong, and this view has been supported by the results of the research that has been conducted. Perhaps the biggest problem regarding the PISA project arises from the inconsistent and contradictory interpretation of the relations of applicable knowledge in mathematics, which according to the authors of PISA testing it measures, and its impact when it comes to teaching mathematics in school. The fact is that in the political discourse of many countries, including Serbia, the PISA project is perceived and interpreted in a simplified manner, at the same time as both a diagnosis and a cure. Of course, no one can live up to such expectations, including PISA. It is especially wrong to measure a complex system such as the education system by such simple means, like the average number of tasks solved in a test. It has already been illustrated how arbitrary the evaluation of the test tasks is, which indicates that the accuracy of a country's ranking is largely overestimated (Hopmann, 2008).

However, notwithstanding all these shortcomings, the PISA project has made a significant and stimulating impact with extremely positive consequences. Despite numerous disagreements pertaining to its scope and concrete results, no one will challenge the significance of PISA testing in launching the broadest discussion of the meta-evaluation type about the direction and meaning of the development of modern education.

References

Baucal, A., & Pavlović-Babić, D. (2010). *Nauči me da mislim, nauči me da učim*. Beograd: Institut za psihologiju Filozofskog fakulteta u Beogradu.

Bodin, A. (2005). *Ce qui est vraiment* évalué *par PISA en mathématiques. Ce qui ne l'est pas. Un point de vue français*. Retrieved from https://antoinebodin.files.wordpress.com/2016/07/ce-que-pisa-evalue_bull-apmep.pdf

Diels, H. (1983). *Predsokratovci I*. Zagreb: Naprijed.

Erić, M. (2011). *Pojam funkcionalnog znanja, Nastava i učenje*. Užice: Učiteljski fakultet.

Fernandez-Cano, A. (2016). A Methodological Critique of the PISA Evaluations. *RELIEVE*, *22*(1), art. M15.

Foucault, M. (2004). *Sécurité, territoire, population.* Paris: Gallimard.

Hopmann, S. T. (2008). No child, no school, no state left behind: schooling in the age of accountability. *J. Curriculum Studies*, *40*(4), 417–456. https://doi.org/10.1080/00220270801989818

Kosanović, G. (2018) *Novo PISA testiranje proveriće srpsko obrazovanje.* Retrieved from http://www.novosti.rs/vesti/naslovna/drustvo/aktuelno.290.html:718627-Novo-PISA-testiranje-proverice-srpsko-obrazovanje

Lambić, D., & Lipkovski, A. (2012). Measuring the Influence of Students' Attitudes on the Process of Acquiring Knowledge in Mathematics. *Croatian Journal of Education, 14* (1), 187-205.

Lukašjevič, J. (1997). *Stvaralački elementi u nauci, Filozofija nauke u prvoj polovini XX veka*. Beograd: Institut za filozofiju Filozofskog fakulteta u Beogradu.

Marković, O., & Erić, M. (2014). The problem of Inadequate Use of the Mathematical Language in the PISA Test Task. *Procedia – Social and Behavioral Sciences, 128*, 54-59. Retrieved from http://ac.els-cdn.com/S1877042814022083/1-s2.0-S1877042814022083-main.pdf?_tid=5a40d2a0-6b23-11e4-9387-00000aab0f6b&acdnat=1415876211_0e0ea7f1019c260ddf3841e5b856b956, https://doi.org/10.1016/j.sbspro.2014.03.117

Marković, O., & Pikula, M. (2014). Kritički osvrt na ideju primenljivog znanja u nastavi matematike. *Conference Proceedings* "Matematike i primene", Vol. IV (1). Beograd: Matematički fakultet.

PISA Released Items – Mathematics. (2006). Retrieved from https://www.oecd.org/pisa/38709418.pdf

PISA 2012 Released Mathematics Items. (2013). Retrieved from https://www.oecd.org/pisa/pisaproducts/pisa2012-2006-rel-items-maths-ENG.pdf

Popham, W. J. (2003). *Test better, teach better: the instructional role of assessment*. Alexandria, VA: Association for Supervision and Curriculum Development.

Sjoberg, S. (2007). PISA and "Real Life Challenges": Mission Impossible? In S. T. Hopmann, G. Brinek, & M. Retzl (Eds.), *PISA zufolge PISA - PISA According to PISA*. Vienna: LIT Verlag.

Skemp, R. (1993). *Theoretical foundations of problem solving*. University of Warwick. Retrieved from http://www.davidtall.com/skemp/sail/theops.html

Strategija razvoja obrazovanja u Srbiji do 2020. godine. (2012). "Službeni glasnik RS", 07/2012.

Šarčević, M. (2018) *Očekujemo izvestan napredak*. Retrieved from https://www.danas.rs/drustvo/sarcevic-ocekujemo-izvestan-napredak/

Wuttke, J. (2007). *Uncertainties and Bias in PISA*. In S.T. Hopmann, G. Brinek, & M. Retzl (Eds.), *PISA according to PISA: Does PISA keep what it promises* (pp. 241-264). Berlin and Vienna: LIT Verlag.

Olivera Marković
University of Kragujevac, Faculty of Education in Užice
Trg Svetog Save 36, 31000 Užice, Serbia
markovic@pfu.kg.ac.rs

Milenko Pikula
University of East Sarajevo, Faculty of Philosophy
Alekse Šantića 1, 71420 Pale, Bosnia and Herzegovina
mpikula@paleol.ney

Marina Zubac
University of Mostar, Faculty of Natural Sciences and Mathematics and Educational Sciences
Matice Hrvatske b.b. 88000 Mostar
Bosnia and Herzegovina
ante.zubac@tel.net.ba

Kritički prikaz matematičkih zadataka PISA testa

Sažetak

U ovom radu autori se bave kritičkom analizom nekih problema vezanih uz PISA testiranje iz matematike. Prije svega ukazano je na društvene, ekonomske i političke okolnosti pokretanja PISA projekta. Zatim su analizirani, po mišljenju autora, vrlo važni pojmovi za razumijevanje PISA filozofije, a to su primjenjivo znanje i rješavanje problema u stvarnoj životnoj situaciji. Usporedili smo dva različita načina definiranja ovih pojmova i pokazali koje su posljedice po učenje i nastavu matematike u oba slučaja. Izložili smo rezultate istraživanja koje smo proveli s nastavnicima matematike, ali i nastavnicima drugih predmeta. Istraživanje se odnosilo na utvrđivanje značaja i uloge matematičkih zadataka PISA testa za daljnje matematičko obrazovanje učenika. Rezultati istraživanja su pokazali da postoji statistički značajna razlika između procjena nastavnika koji predaju i nastavnika koji ne predaju matematiku. U radu smo analizirali i konkretne zadatke koje su učenici rješavali na PISA testiranju. Ukazali smo na izvjesne nedostatke i nepreciznosti matematičkih zadataka PISA testova, kao i na slabosti samog PISA testiranja.

Ključne riječi: *funkcionalno znanje; matematička pismenost; nastava matematike; stvarna životna situacija.*

Uvod

Prvo međunarodno PISA testiranje petnaestogodišnjih učenika organizirano je 2000. godine u organizaciji OECD-a. Od tada se, svake treće godine, provodi sada već u više od 70 zemalja s nakanom *procjenjivanja* čitalačke*, matematičke i prirodno-znanstvene pismenosti*. U svakom ciklusu testiranja dvije trećine pitanja odnosi se na jedno od navedenih područja, tako da se ta glavna područja naizmjenično smjenjuju. Godine 2018. PISA projekt postaje *punoljetan* i ulazi u *ozbiljne godine*, ali možemo li reći i da su sve dječje bolesti koje ga prate od nastanka preboljene? Jesu li otklonjene sve nedoumice i nejasnoće u vezi s motivom pokretanja i financiranja PISA testiranja od OECD-a, organizacije posvećene svjetskom ekonomskom razvoju? Jesu li nam danas pojmovi *funkcionalno znanje*, *znanje primjenjivo u svakodnevnom životu*, *matematička i znanstvena pismenost*... jasniji nego što su to bili kada su s velikom pažnjom ušli u obrazovnu raspravu svih sudionika svjetskog obrazovnog procesa? Zašto se u nekim zemljama, na primjer u Srbiji, PISA testiranje smatra prvorazrednim političkim događajem, a pomicanje za nekoliko mjesta na rang-listi viješću za naslovne stranice, a u drugim zemljama, kao na primjer u SAD-u, to prolazi relativno neopaženo, kako to tvrdi Hopmann (2008). Zašto nosioci obrazovne politike u mnogim zemljama smatraju imperativom da školske kurikule doslovno prilagode zahtjevima PISA testova i uz ovelik broj primjedbi i osporavanja od stručne i znanstvene javnosti (Hopmann, 2008)?

Ako imamo na umu prirodu obrazovanja, njegovu važnost i značaj, sigurno ćemo se složiti s tvrdnjom koja jednako vrijedi i za Platonovu *Državu* kao i za bilo koju modernu državu, da se obrazovanje ne može promatrati izolirano od najvažnijih tijekova života jedne države, njezinih težnji i vrijednosti (Serder, Ideland, 2016). Prema tome, ono ne može biti izolirano ni od politike ako je politika izraz brige za vitalne potrebe jedne zajednice. Međutim, postavlja se pitanje na koji način shvaćamo prirodu obrazovanja i gdje vidimo njegovo mjesto u društvu, kao i kako određujemo njegovu ulogu u državi. Drugim riječima, u kojem pravcu treba ići obrazovna politika i u kojoj je mjeri ona odraz organskih i bitnih potreba društva iz kojega proizlazi, a koliko je odraz borbe moći, kako je to rekao Foucault (2004). U ovom slučaju vidimo da je politička organizacija, kao što je OECD, čiji je slogan: „Za bolju svjetsku ekonomiju", zainteresirana za jedan, po najvažnijim obilježjima akademski projekt. Međutim, OECD i njezine članice nisu započele PISA projekt zato što ih zanimaju temeljna istraživanja u području obrazovanja ili teorije učenja. Ta organizacija odlučila je uložiti u PISA testiranje zato što je obrazovanje ključno za ekonomiju, kako to tvrdi Sjoberg (2007). Tako uspostavljena, naizgled čvrsta veza između obrazovanja i ekonomije, osnovni je polazni aksiom. PISA polazi od osnovne premise da je ekonomsko bogatstvo u pozitivnoj korelaciji sa znanjem i kompetencijama učenika čiji su učinci, s druge strane, ovisni o određenim karakteristikama škola. Iz toga slijedi zaključak da se uvođenjem poželjnih vještina i znanja, bogatstvo zemlje povećava. Međutim, i polazna premisa i zaključci izvedeni na osnovi nje, u velikoj su mjeri pojednostavljeni, kako tvrdi Fernandez-Cano (2016) ili kako kaže Hopmann, PISA se „oslanja na „snažne pretpostavke" (Fertig, 2004) utemeljene na slabim podacima oslonjenim na svakodnevnu mudrost („Obrazovanje je važno, zar ne?", „Školske strukture stvaraju razliku, zar ne?"), ali ne nudi gotovo nikakvo empirijsko i povijesno istraživanje kojim bi ih potkrijepila. Međutim, to nije sprječavalo ni PISA istraživače, ni javnost da se ponašaju kao da su uzročni odnosi doista dani„ (Hopmann, 2008, str. 439). Upravo ta prividna sigurnost potkrijepljenja tablicama i rang-listama, dovela je do toga da se veza između učenika, škole i rezultata države, smatra samoočitom, a rezultati PISA testiranja nesumnjivom dijagnozom u koju se ne sumnja kao što se ne sumnja ni u mogućnost testa kao preciznog instrumenta s pomoću kojega dobivamo relevantne podatke.

Međutim, primjećujemo da su i prije pojave PISA testova neki znanstvenici osporavali standardne testove postignuća. Posebne kritike pretrpjeli su završni testovi znanja u američkom školskom sustavu. Potkraj XX. stoljeća ti su testovi dobili na važnosti i postali su mjerilo uspješnosti obrazovnog sustava, škola i nastavnika. Školama čiji su učenici na testovima bili lošiji, smanjivalo bi se financiranje, nastavnicima umanjivale plaće, a roditelji bi iz takvih škola ispisivali djecu, kaže Popham (2003). Budući da su rezultati testa izravno bili proporcionalni s rejtingom i materijalnom situacijom škole, a samim tim i nastavnika, nastavnici su svoj rad usmjerili osposobljavanju učenika za rješavanje završnih testova. Gradivo koje nije bilo obuhvaćeno testovima, tretiralo se kao drugorazredno i površno se obrađivalo, dodaje Popham (2003). Tako je u američkim školama u prvi plan stavljen završni test, koji je osporavan od mnogih znanstvenika, a ne obrada i istinsko znanje učenika. Izradom testova već od sredine XX. stoljeća bave se specijalizirane kompanije koje tom prilikom ostvaruju iznimno visok profit. Dodajmo još, prema tvrdnji Sjoberga (2007), da su ukupni troškovi testiranja PISA i TIMSS-a za sve zemlje u 2006. godini bili viši od 100 milijuna američkih dolara.

Dakle, u tom širem kontekstu treba promatrati i ovaj projekt jer njega ne provode nepristrani istraživači na području obrazovanja, već stručnjaci okupljeni oko neoliberalne ekonomske i političke paradigme čiji je cilj razvijanje sposobnosti i kompetencija kojima se promoviraju ekonomski i politički ciljevi OECD-a. Smatramo da je prirodna zainteresiranost za te ekonomske i političke ciljeve, kao što je i legitimno pravo svake vlade tražiti i dobiti relevantne informacije na osnovi kojih bi usmjeravala svoju politiku i uspostavljala prioritete u ekonomskom razvoju. Jedan od prioriteta svakako je težnja vlade u nekoj državi da osigura učinkovit rad obrazovnog sustava, što je potpuno razumljivo, pogotovo kada se ima na umu da je masovno obrazovanje iznimno skupo. To je osnovna činjenica koju autori PISA projekta ne skrivaju, ali se ona vrlo često zaboravlja u javnim i političkim raspravama o rezultatima i važnosti PISA testiranja. Međutim, velik broj autora koji se bave koncepcijom, dosezima i rezultatima PISA testiranja smatra da se ideja PISA projekta apsolutno ne može razumjeti neovisno o pitanjima koja se nalaze na dnevnom redu OECD-a, i izvan konteksta ekonomskog razvoja i konkurencije na globalnoj razini tržišne ekonomije, kako to kaže Sjoberg. (2007) Treba napomenuti i Hopmanovu studiju (2008) koja daje odličan pregled filozofskih i političkih teorija koje pokušavaju razumjeti PISA testiranje u kontekstu političkih i ekonomskih odnosa u svijetu.

Međutim, mi još uvijek nismo, uz sva istraživanja, dobili jasne i konačne odgovore, je li PISA projekt doista unaprijedio ekonomiju zemalja sudionica, u kojim segmentima i u kojoj mjeri. Može li se, na primjer, utvrditi jasna uzročno-posljedična veza između uspješnosti na PISA testiranju i ekonomske razvijenosti države? Međutim, dok čekamo te ekonomske pokazatelje, dobili smo mnoštvo povratnih informacija od stručne i znanstvene javnosti o nizu problema koje je izazvalo djelovanje PISA testiranja na školske sustave, posebno u državama kao što je Srbija koje su PISA projekt shvatile, po svoj prilici, kao najvažniji vrijednosni i politički okvir za razvoj obrazovnog sustava. „Krećemo u ciklus reformi i radi se upravo ono što PISA mjeri. Naš dosadašnji obrazovni sustav svodio se na prenošenje sadržaja učenicima, a novi koncept temelji se na planiranju i primjeni stečenog znanja", rekao je ministar obrazovanja Šarčević (2018). Savjetnica ministra Gordana Kosanović, govoreći o novom ciklusu PISA testiranja, rekla je „da je PISA testiranje indikator kvalitete sustava obrazovanja i da, uz testiranje čitalačke, matematičke, znanstvene i financijske pismenosti, učenici popunjavaju i upitnik, čijom analizom zemlje sudionice dobivaju brojne korisne podatke koje mogu upotrijebiti za donošenje obrazovnih politika – od toga što utječe na učenička postignuća, kakva je motivacija učenika, jesu li zadovoljni radom nastavnika, kakav je školski etos" (Kosanović, 2018).

Postoji jedna važna osobina koncepcije PISA projekta koja se vrlo često u raspravama potpuno zaboravlja – PISA testiranje, za razliku od TIMS-a, nije namijenjeno provjeravanju školskog znanja, već provjeravanju onoga što se zove funkcionalno, primjenjivo znanje koje nije čvrsto povezano s nacionalnim programima i školskim kurikulima. Naprotiv, naglašava se, kao što kaže Bodin (2005), da je riječ o znanjima koja u najvećoj mjeri nisu školska, već su rezultat osobnog iskustva stečenog u obitelji, druženja s vršnjacima ili boravka u široj kulturnoj sredini. Međutim, unatoč svim tim napomenama javnost i političari sve rezultate testiranja, i dobre i loše, pripisuju samo školi očekujući da upravo škola učini sve da se na sljedećem rangiranju njihova zemlja pojavi na što višem mjestu na rang-listi, kao što smo vidjeli u prethodnim izjavama. Visoko rangirane države, kao na primjer Finska, postaju uzor, a njihove škole predmet proučavanja drugih zemalja (Bodin, 2005). To, između ostalog, potvrđuje da je zanemarena osnovna namjera PISA testiranja ‚izmjeriti znanja koja su rezultat ekonomskih, kulturnih, povijesnih i političkih čimbenika koji oblikuju život mladih ljudi u jednoj zemlji', a neopravdano je naglašena i istaknuta uloga školskih znanja i institucija u kojima se stječu. Pojednostavljeno rečeno, tvrdimo da provjeravamo primjenjivost dijela znanja petnaestogodišnjaka koja oni stječu uglavnom putem izvanškolskih iskustava, a samo jednim manjim dijelom u školi. Nakon toga školu proglašavamo zaslužnom za rezultate, dobre ili loše, iako ona na osnovi prethodno rečenoga, to nikako ne može biti. Zatim postavljamo zahtjev za promjenom nastave u školi, tako da naši ispitanici na sljedećoj provjeri postignu što bolje rezultate rješavajući testove koji po riječima samih autora testa ne provjeravaju školska znanja. Moglo bi se reći da je u tom krugu nedosljednosti i kontradiktornosti izvor najvećeg broja problema koji se tiču samog razumijevanja PISA projekta i njegova mjesta u obrazovnim sustavima zemalja sudionica testiranja.

Izdvojit ćemo, za ovu priliku, dva problema koja čine misaoni koordinatni sustav PISA projekta: definiranje pojma *primjenjivo znanje* i *stvarna životna situacija*. Putem tih dvaju pojmova u mnogim se aspektima prelama najveći broj problema koje smo spomenuli. Pogledajmo u ovom kratkom povijesnom podsjećanju i popratnoj analizi koji se problemi javljaju u vezi s razumijevanjem tih pojmova.

Mnogi stari narodi, na primjer Babilonci, Kinezi, Egipćani, još dvije tisuće godina prije Krista znali su da je trokut sa stranicama 3, 4, 5 pravokutni trokut i koristili su se tim znanjem pri izgradnji vrlo složenih građevina kojima se i danas divimo. Međutim, tek je Pitagora to praktično pravilo i teorijski dokazao i na taj način omogućio potencijalno beskonačan broj sudova o slučajevima koje nikada nitko nije promatrao niti će to ikada činiti. Dodajmo još da je upravo Pitagorina škola zaslužna za uvođenje pojma *dokaz* u matematiku. Međutim, ta praktična korist koju teorija donosi sa stanovišta zdravorazumske logike potpuni je paradoks, jer zdrav razum praktičnu korist povezuje samo sa znanjem o pojedinačnim slučajevima. Jan Lukašjevič, govoreći o stvaralačkim elementima u znanosti, naprotiv, piše: "Danas su znanstvenici skloniji praktičnu vrijednost vidjeti u općenitosti. Opći sudovi, definiranjem uvjeta pod kojima se zbivaju fenomeni, čine mogućim predviđanje budućnosti, izazivanje korisnih fenomena i sprječavanje događanja štetnih." (Lukašjevič, 1997, str. 78). Možemo reći da je današnja matematika samo prirodni rezultat razvoja matematike od koje je Pitagora učinio znanost odvajajući je od trgovačkih poslova. Da se prisjetimo, sama riječ *aritmetika* odnosila se najprije na sposobnost upotrebe brojeva koja nema praktičnu upotrebnu vrijednost. Po Aristoksenovu svjedočenju ljudi su, do Pitagorina vremena, brojeve upotrebljavali isključivo u trgovačkim poslovima: „Znanost pak o brojevima čini se da je najviše počastio i unaprijedio Pitagora: odveo ju je iz trgovačke službe i sve je stvari uspoređivao s brojevima." (Diels, 1983, str. 398). Na taj su način upravo filozofija i matematika Stare Grčke učenjem o pojmu, kao vještini razlikovanja supstancijalnog od efemernog, i dokazu kao argumentiranom postupku, postavili temelje europskoj znanosti i kulturi. Što je ta razlika bitnog od nebitnog, kao što i Lukašjevič kaže, uspostavljena na općenitijoj razini, to je ona potencijalno praktično korisnija, jer se može primijeniti na veći broj slučajeva. Kada je u pitanju primjena takvog znanja, možemo sigurno tvrditi da ne postoji ni jedan razlog, ni logički ni životni, da onaj tko je istinski razumio neko pravilo na općoj razini nije to pravilo u stanju primijeniti na konkretnom životnom primjeru. U slučaju matematike, to znači da je preliminarni cilj nastave pomoći učenicima da izgrade koherentno i strukturirano znanje koje će tek tako postati *organom* (oruđem) za uspješno rješavanje problema u stvarnom životu. Pa čak i onih problema koji još ne postoje i koji će tek nastati, kako je to pokazano u radu *Pojam funkcionalnog znanja* M. Erića (2011).

Međutim, ono što se u PISA projektu pojavljuje kao problem jest što se problemima primjene matematike, učenju i nastavi matematike ne prilazi sa stajališta matematičara-praktičara iskusnog u nastavi, već sa stajališta tzv. *zdravog razuma*. Bodin (2005) svjedoči o snažnom otporu nastavnika prema pitanjima iz PISA jer smatraju da nisu kompatibilna s nastavnim programima. Pojednostavljeno rečeno, da pitanja nisu matematička. Takav otpor nastavnika matematike potvrđuje i naše istraživanje. Zapravo, problem se pojavljuje na dvije razine: jedan je problem na razini samog testa u kojem provjeravamo kako *zdrav razum* rješava krajnje pojednostavljene „matematičke probleme" u životnoj situaciji. Drugi problem koji se pojavljuje jest na razini koncepcije testa gdje nam taj isti *zdravi razum* daje svoje razumijevanje i svoj pogled na matematiku i nastavu matematike savjetujući nam što bi trebalo promijeniti u školi da bi on bio uspješniji u životnim situacijama. Taj problem već smo spomenuli. Mi smo ovdje *zdravim razumom* nazvali jednu *opću sposobnost* čije su prisustvo uočili i drugi autori. Wuttke (2007) piše da se primjerice 75% (Grčka) i 92% (Nizozemska) ukupne varijance kompetencija ispitanika može pripisati samo jednoj komponenti, što članovi PISA projekta odlučno odbacuju, čak i ismijavaju. Ideološki i strateški razlozi za taj sukob su očiti. Kada se utvrdi da PISA mjeri jedan opći faktor, opću sposobnost, teško je ne povezati taj faktor s istraživanjem opće inteligencije, što članovi PISA projekta doživljavaju kao prijetnju, jer naravno ni jedna vlada ne bi trošila milijune kako bi dobila obavijest o inteligenciji učenika. Moramo priznati, skloni smo podržati zapažanja Wuttkea, a smatramo da su naša istraživanja to i pokazala, jer razina matematičkog znanja potrebnog za rješavanje zadataka u PISA testu iznimno je niska.

Najčešće se u toj koncepciji sugerira, kako to Skemp (1993) odlično primjećuje, da je najbolji način kako ćemo učenike naučiti matematiku, taj da nastavu od samog početka oslonimo na stvarne životne situacije koje zahtijevaju znanja iz matematike za njihovo rješavanje. Međutim, čak i malo teorije bit će dovoljno da se pokaže da to nije najbolji put. Uzmimo za primjer, kaže Skemp (1993), općepoznatu zdravorazumsku činjenicu da željezo tone. I to je činjenica koju zdrav razum svakodnevno iskušava. Pa kako je onda moguće da postoje brodovi koji su izrađeni od željeza, a ipak ne tonu? Da bismo to razumjeli, neće nam biti dovoljni nikakvi pojedinačni slučajevi, nikakvo iskustveno zaključivanje. Potrebno je da znamo jednostavnu, ali moćnu teoriju – Arhimedov zakon. To je načelo koje nam omogućuje da predvidimo hoće li brod izrađen od željeza ili čak armiranog betona ploviti ili neće. S pomoću tog načela stići ćemo dalje nego što bismo to ikada mogli sa zdravim razumom, reći će Skemp (1993). Iskoristit ćemo taj Skempov primjer kako bismo napravili još jednu malu analizu. Zamislimo da će naš učenik jednog dana dobiti priliku popraviti neki brod kao inženjer brodogradnje. Kako će u tome uspjeti ako se on tijekom školovanja susreo samo s kvarovima *od jednog slučaja do drugog slučaja*, a baš ovaj kvar nije na popisu njemu poznatih slučajeva. Upravo ta vrsta konkretizacije, praktičnosti i učenja iz života sugerira se kao najbolji *methodos* (put) za nastavu matematike, kao što ćemo vidjeti u sljedećim analizama. U svim slučajevima PISA testova ta konkretna, stvarna životna situacija simulira se davanjem viška podataka u zadatku. Kako to Bodin (2005) s pravom primjećuje, u nekim slučajevima glavnu poteškoću s kojom se učenici suočavaju predstavlja samo razumijevanje teksta koji uopće nije matematički tekst. Naravno, i to može biti dio rješavanja problema, ali pravi matematički rad počinje kada se problem u potpunosti razumije. Ovdje možemo ukazati i na rad autora Anić i Pavlović Babić (2011) koji, nakon analize problema rješavanja zadataka u realnom kontekstu, daje prijedloge i za nastavnu praksu.

Naravno, postavlja se i pitanje zašto se u oblikovanju zadatka zaustavljamo samo na nekoliko suvišnih podataka kada je u stvarnoj životnoj situaciji, za koju pripremamo učenika, takvih podataka beskonačno mnogo: boja broda, kliktanje galebova iznad glave, šum valova, buka iz brodskog restorana i tako *ad infinitum*. Ako naš učenik, jednog dana inženjer, zahvaljujući svom obrazovanju ne bude sposoban sve to apstrahirati i usmjeriti se samo na *načelo* zahvaljujući kojem brod i plovi, onda će on moći danima, ako brod u međuvremenu ne potone, sjediti i analizirati životnu situaciju. Ta sposobnost, za koju ga školovanje i priprema, promatranje problema sa stajališta biti, upravo će njega i preporučiti za taj posao, a ne nekog zdravorazumskog prolaznika. Zašto bi se onda tijekom svog obrazovanja zadržavao na konkretnom nizu varijacija jedne situacije kada je vjerojatnost da će se baš s nekom od njih susresti – beznačajna, kako kaže Erić (2011).

Ako bismo željeli prethodno rečeno formalizirati i izdvojiti najvažnije korake u apstrahiranju koji se odnose na rješavanje problema, onda možemo reći da zapravo polazimo od toga da iz stvarnih situacija apstrahiramo konceptualni model, da nakon toga iz tog konceptualnog modela apstrahiramo pojmove koji su važni za naš problem, a da zatim iz tih pojmova izvlačimo matematički model. Kad na razini modela riješimo problem, onda obrnutim koracima primjenjujemo to rješenje na stvarnu životnu situaciju. Skemp (1993) također s pravom primjećuje da bismo u slučaju drugačijeg pristupa došli u bezizlaznu, situaciju slijepe ulice. Jer ako želimo bez teorije učiti preko konkretnih matematičkih zadataka, oni bi po prirodi stvari morali biti izolirani jedni od drugih,a tada bi učenik za svaki zadatak morao učiti novu metodu jer zdrav razum ne poznaje načela koja te zadatke svrstavaju u kategorije. U tom slučaju opterećenje memorije tolikim brojem „metoda", koje bi vrijedile samo za jedan slučaj, to bi vrlo brzo postalo nepodnošljivo. I ne samo to, rješavanje problema iz stvarnog svijeta neće osigurati dovoljno ponavljanja koja bi učvrstila novoformirane pojmove niti će vještinu njihove primjene učvrstiti kao dobro uspostavljenu rutinu.

Skemp (1993) predlaže način učenja koji bi najbolje izgradio apstraktne matematičke pojmove i strukturirano matematičko znanje:

1. Trebalo bi osigurati izvođenje pojmova uz najmanju količinu irelevantnih podataka, „niske buke", koje treba ignorirati pri izgradnji pojmova.
2. Trebao bi postojati velik broj primjera i pojmova koji su vremenski bliski.
3. Trebalo bi uvoditi samo jedan novi pojam koji se uči u danom vremenu.
4. Za uvođenje novih pojmova potrebno je postojanje odgovarajuće sheme, tako da se novi pojam može povezati s njom i na taj način omogućiti učenje s relacijskim razumijevanjem.

Sve navedene zahtjeve teško je ili nemoguće ostvariti s problemima „stvarnog svijeta„, kako kaže Skemp (1993).

Na takve ili slične primjedbe zagovornici PISA projekta najčešće odgovaraju da njihova namjera nije mjeriti školska matematička znanja petnaestogodišnjaka, jer se ona i ne mogu primijeniti u svakodnevnom životu. Njihov je cilj, kako kažu, mjeriti znanja koja je moguće primijeniti u stvarnoj životnoj situaciji. I doista, kad pogledamo zadatke koji ilustriraju tu koncepciju, vidimo da je riječ o matematički krajnje pojednostavljenim problemskim situacijama za čije se rješenje vrlo često koriste odgovori koji počinju sa čini mi se, *izgleda*, *vjerojatno je* ... Prirodno se nameće pitanje, kao što kaže Bodin (2005), treba li svako pitanje s brojevima smatrati matematičkim pitanjem? I na taj se način zatvara krug zbrke i konfuzije, što su implicitna, a što eksplicitna očekivanja od PISA projekta. Međutim, otvara se bitno pitanje zašto testiranje na tako jednostavnoj matematici uzimamo za glavni parametar na osnovi kojega odlučujemo u kojem pravcu mijenjamo naš rad u učionici kad su u pitanju matematički sadržaji? Zašto cijelo obrazovanje vrednujemo u skladu s rezultatima na PISA testiranju i unatoč tome što znamo da je školsko znanje zastupljeno samo djelomično, u tri područja čiju pismenost provjeravamo, a i u tim područjima provjeravamo samo jedan malen dio znanja primjenjiv u svakodnevnim životnim situacijama? Pokušat ćemo dobiti neke od mogućih odgovora na ta pitanja u analizi istraživanja koje smo proveli.

Metodologija istraživanja

Predmet ovog istraživanja su matematički zadaci PISA testa koji su se pojavljivali na PISA testiranjima, kao i na probnim PISA testiranjima u razdoblju od 2000. do 2012. godine. Istraživanje je provedeno kako bi se odgovorilo na pitanje kako nastavnici matematike, kao i ostali prosvjetni djelatnici koji su zaposleni u osnovnim i srednjim školama u Republici Srbiji, percipiraju ulogu PISA testiranja u matematičkom obrazovanju učenika. Zadaci istraživanja odnosili su se na:

- Utvrđivanje važnosti i uloge matematičkih zadataka PISA testa iz perspektive nastavnika matematike koji predaju taj nastavni predmet u osnovnim i srednjim školama;
- Utvrđivanje važnosti i uloge matematičkih zadataka PISA testa iz perspektive prosvjetnih radnika koji nisu matematičari, a zaposleni su u osnovnim i srednjim školama;
- Uspoređivanje procjene nastavnika koji predaju i nastavnika koji ne predaju matematiku o važnosti i ulozi matematičkih zadataka PISA testa.

Uzorak istraživanja

U istraživanju je sudjelovalo 97 nastavnika matematike i 105 nastavnika koji ne predaju matematiku. Od 105 nastavnika koji ne predaju matematiku 44 su učitelja, 16 je nastavnika srpskog jezika, 7 nastavnika kemije, 11 nastavnika fizike, 7 nastavnika engleskog jezika, 2 su nastavnika ruskog jezika, 8 je nastavnika informatike, 4 su nastavnika povijesti, 2 nastavnika geografije i 4 nastavnika biologije. Ispitivanje je provedeno u zimskom semestru 2017. godine u školama Zlatiborskog okruga, grada Čačka i grada Beograda. Nastavnici su sedam dana prije popunjavanja upitnika bili upoznati s linkovima gdje se nalaze PISA brošure: *PISA Released Items – Mathematics, 2006.* i *PISA Released Mathematics Items, 2013.*, kako na srpskom tako i na engleskom jeziku. Zamoljeni su da pogledaju i prouče 78 zadataka iz tih brošura.

Instrument

Za potrebe našeg istraživanja oblikovali smo numeričku ljestvicu procjene. Pilot-inačicu ove ljestvice ispitali smo na uzorku studenata I. i II. godine Pedagoškog fakulteta u Užicama. Na osnovi njihovih komentara i odgovora kompletirali smo završnu inačicu ljestvice. Ljestvica sadrži popis od 6 procjena koje su ponuđene nastavnicima na vrednovanje:

- Znanja koja su potrebna za rješavanje PISA testa iz matematike primjenjiva su u svakodnevnom životu;

– Učenje preko stvarnih životnih situacija dobar je način ovladavanja matematičkim sadržajima u višim razredima osnovne škole;
– Zadaci PISA testova iz matematike korisni su za daljnje matematičko obrazovanje učenika;
– Zadaci PISA testova iz matematike prilagođeni su uzrastu petnaestogodišnjaka;
– Sadržaj PISA testova iz matematike dio je kurikula nastave matematike u osnovnoj školi;
– Postotni račun i čitanje podataka s grafikona važne su matematičke kompetencije koje petnaestogodišnji učenik treba posjedovati.

Slaganje s određenom procjenom ispitanici su ocjenjivali ocjenama od 1 do 5 (1 je najnepovoljnija ocjena procjene, a 5 najpovoljnija ocjena procjene). Na kraju je nastavnicima ostavljena mogućnost da, ako žele, daju komentare otvorenog tipa o danim zadacima. U istraživanju je primijenjena deskriptivna metoda, a da bi se utvrdilo postoji li značajna razlika percepcije matematičkih zadataka PISA testa od nastavnika matematike i nastavnika drugih školskih predmeta, izrađen je χ^2 test.

Rezultati istraživanja

Prvi zadatak istraživanja bio je ispitati kako nastavnici matematike procjenjuju zadatke PISA testa. Od 97 ispitanih nastavnika njih je 82 već prije imalo priliku upoznati se sa zadacima koji su im bili dani na uvid. Podaci do kojih smo došli istraživanjem prikazani su u tablici 1. U tablici vidimo pojedinačne vrijednosti za svaku procjenu i prosječne rezultate (M) za svaku od navedenih procjena.

Drugi zadatak našeg istraživanja odnosio se na utvrđivanje kako nastavnici koji ne predaju matematiku procjenjuju zadatke PISA testiranja. Od 105 ispitanih nastavnika njih se 26 već prije imalo prilike upoznati sa zadacima koji su im bili dani na uvid. Podaci do kojih smo došli istraživanjem prikazani su u tablici 1.

Treći zadatak istraživanja odnosio se na uspoređivanje procjene nastavnika koji predaju i nastavnika koji ne predaju matematiku o značajnosti i ulozi matematičkih zadataka PISA testa. Uspoređivanje ćemo izvršiti utvrđivanjem postojanja statistički značajne razlike između procjena tih dviju grupa nastavnika (Tablica 1).

Tablica 1

Rasprava

Na osnovi dobivenih rezultata zaključujemo da vrednovanje značajnosti matematičkih zadataka PISA testa od nastavnika matematike nije ocijenjeno visokom ocjenom. Ukupna srednja ocjena 2,89, ukazuje na to da nastavnici matematike testiranje matematičke pismenosti u zadacima koje su proučavali ne ocjenjuju kao značajno za budućnost matematičkog obrazovanja učenika. U svakoj od procjena više od 60% ispitanika dalo je jednu od prve tri ocjene, što možemo tumačiti općim neslaganjem s koncepcijom matematičkih zadataka PISA testiranja.

U komentarima na kraju upitnika ispitanici kao jedan od problema navode neprilagođenost zadataka dobi. Tu se posebno naglašavaju zadaci iz područja vjerojatnosti koje se u srpskim, ali i u većini europskih škola, poučava u višim razredima srednje škole. Pojedini su ispitanici primijetili da je za 15, od ukupno za analizu ponuđenih 78 zadataka, bilo potrebno poznavanje pojma vjerojatnosti. Dakle, 20% zadataka naši učenici nisu mogli rješavati, i to ne zbog težine zadataka, već zbog činjenice da im to područje matematike još uvijek nije poznato. Nastavnici su u svojim komentarima istaknuli još jednu skupinu zadataka za koju su naveli da nije prikladna dobi učenika jer se gotovo isti takvi zadaci nalaze u udžbenicima matematike za III. i IV. razred osnovne škole. Zbog svih tih razloga nastavnici su najnižom od svih drugih srednjih ocjena (2,69) procijenili podudaranost kurikula nastave matematike i sadržaja PISA zadataka. Treba istaknuti kako nastavnici postotni račun, kao i čitanje podataka s grafikona, ne smatraju posebno važnim matematičkim kompetencijama za jednog petnaestogodišnjaka iako su takvi zadaci u fokusu PISA testa.

Na osnovi dobivenih rezultata primjećujemo da je vrednovanje značajnosti matematičkih zadataka u PISA testovima od nastavnika koji ne predaju matematiku ocijenjeno većom ocjenom nego kad su zadatke procjenjivali nastavnici matematike. Ukupna srednja ocjena 3,50 ukazuje na to da nastavnici drugih nastavnih predmeta s manjom strogošću ocjenjuju zadatke PISA testa. Više od 50% ispitanih nastavnika smatra da su PISA zadaci primjenjivi u svakodnevnom životu, korisni za daljnje školovanje učenika, prilagođeni dobi učenika i da se nalaze u kurikulu nastave matematike. Jedino malo nepovoljnija procjena (M = 3,19) odnosi se na mogućnost ovladavanja matematičkog sadržaja učenjem iz stvarnih životnih situacija.

Komentari koji su se našli u upitnicima nastavnika koji nisu matematičari bili su u najvećoj mjeri afirmativni za PISA testiranje. Velik broj tih ispitanika izrazio je zadovoljstvo zbog svog znanja matematike jer su prije proučavanja PISA zadataka mislili da su matematiku davno zaboravili! Glavni razlog zbog kojeg smo proveli istraživanje na nematematičarima u tome je što su oni, po našem mišljenju, odlični predstavnici javnog mnijenja i zastupnika općeg stava koji se formirao o PISA testiranju na osnovi onoga što se o tome može čuti u medijima i od glavnih aktera obrazovne politike koji nam, nažalost, nisu bili dostupni da bismo ih uključili u istraživanje. Kao što smo već rekli, taj stav je, i u nas i u većini drugih zemalja, *a priori* pozitivan. Kažemo *a priori* jer javnost, čak i ona stručna, ne raspolaže s dovoljno podataka na osnovi kojih bi mogla donijeti utemeljen zaključak.

Na osnovi dobivenih svih vrijednosti χ^2 uz df=4 na razini 0,01 zaključujemo da postoji statistički značajna razlika u sljedećim procjenama:

– Zadaci PISA testova iz matematike korisni su za daljnje matematičko školovanje učenika;
– Zadaci PISA testova iz matematike prilagođeni su dobi petnaestogodišnjaka;
– Sadržaj PISA testova iz matematike nalazi se u kurikulu nastave matematike u osnovnoj školi;

– Postotni račun i čitanje podataka s grafikona važne su matematičke kompetencije koje petnaestogodišnji učenik treba imati.

Posebno je statistički značajna razlika uočljiva u procjeni koja se tiče kurikula nastave matematike. Nastavnici koji ne predaju matematiku nisu upoznati sa sadržajem tog predmeta, ali prirodno očekuju usklađenost testa sa sadržajima osnovnoškolske matematike. S druge strane, nastavnici matematike jasno su uočili zadatke u kojima je usklađenost s programom matematike izostala.

Promotrit ćemo neke od primjedbi koje su nastavnici navodili u otvorenom dijelu upitnika. Kao što smo već prije istaknuli, znatan broj komentara nastavnici su uputili na prisutnost zadataka vjerojatnosti. Te primjedbe navodili su isključivo nastavnici matematike jer su oni upoznati sa sadržajem tog nastavnog predmeta i znaju da se petnaestogodišnjaci s tim pojmom još nisu susreli. Primijetili smo da je većina zadataka iz vjerojatnosti vrlo jednostavna, ali učenici ih ne mogu riješiti osim ako su se prije testiranja upoznali s formulom za vjerojatnost nekog događaja koja se računa kao količnik broja povoljnih i broja mogućih ishoda tog događaja. Za razumijevanje biti pojma vjerojatnosti i te formule, koja je vrlo jednostavna za primjenu i korisna za petinu zadataka PISA testa, potrebno je značajnije matematičko obrazovanje. Kako bismo objasnili učeniku osnovne škole da je, na primjer, vjerojatnost za dobivanje šestice pri bacanju igraće kockice jedna šestina ako u stvarnosti imamo situaciju gdje iz deset bacanja ni jednom ne dobijemo šesticu ili da je dobijemo nekoliko puta? Upravo bi nas primjer iz „stvarne životne situacije„ mogao demantirati, a učenike zbuniti ako ih s tim pojmom upoznamo „na silu" i prerano. Primjer takvog zadatka je zadatak *Obojeni bomboni*.

Obojene bombone

Majka daje Milanu vrećicu bombona iz koje odabire jedan bombon. On ih ne može vidjeti. Na sljedećem grafikonu vidi se koliko je u vrećici bilo bombona svake boje.

Prikaz 1

Kolika je vjerojatnost da će Milan odabrati crveni bombon?
A) 10% *B) 20%* *C) 25%* *D) 50%*

Osim zadataka *vjerojatnosti,* nastavnici matematike upozorili su i na postojanje velikog broja trivijalnih zadataka, koje su također označavali neprimjerenim za pojedine uzraste. Primjer takvog zadatka je zadatak *Izvoz.*

Izvoz

Sljedeći Prikaz 2 sadrži podatke o izvozu iz Zedlanda, zemlje u kojoj se koristi valuta zed.

Prikaz 2

Koliko je iznosila ukupna vrijednost (u milijunima zeda) izvoza iz Zedlanda u 1998?

Autori PISA testa takve zadatke svrstavaju u prve dvije razine na skali postignuća gdje se od učenika očekuje da „mogu odgovoriti na jednostavna pitanja u poznatom kontekstu gdje su sve relevantne informacije zadane, a pitanja jasno formulirana" (prva razina), kao i da „mogu interpretirati i prepoznati situacije u kontekstu koje ne zahtijevaju više od izravnog zaključivanja" (druga razina) (Baucal, Pavlović-Babić, 2010, str. 27). Primijetili smo da se samo u opisu prve razine postignuća od ispitanika očekuju odgovori na pitanje koje je „jasno formulirano". Međutim, kako je uopće moguće dati odgovor na pitanje koje nije jasno formulirano? Upravo nas matematika uči preciznosti i strogosti u izlaganju i zaključivanju i učenici bi ta iskustva i znanja trebali prenijeti, kad je to moguće, u „svakodnevni život" a ne da neobaveznost i površnost svakodnevnog govora prenesu u nastavu matematike.

Kao „problematične" zadatke ispitanici ističu zadatke u kojima nije jasno što se od učenika traži, točnije rečeno, u kojima pitanja nisu „jasno formulirana". Primjeri takvih zadataka su zadaci *Pljačke* i *Rezultati testa*.

Pljačke

Jedan TV reporter prikazao je sljedeći grafikon i rekao: „Ovaj grafikon pokazuje da je u razdoblju od 1998. do 1999. došlo do golemog porasta broja pljački."

Prikaz 3

Smatrate li da je reporterova izjava realno tumačenje tog grafikona? Navedite objašnjenje kojim ćete potkrijepiti svoj odgovor.

Pitanje „smatrate li" nije matematičko pitanje, nego zahtjev za iznošenje osobnih stavova učenika. Sva tri moguća odgovora (ne; da; ne može se reći na osnovi grafikona) mogla bi biti potkrijepljena jačom ili slabijom zdravorazumskom argumentacijom, iako se ovdje i poželjan odgovor također zdravorazumski naslućuje. Takvo neprecizno pitanje svrstalo je taj zadatak u šestu, najvišu razinu postignuća. U opisu šeste razine postignuća, između ostalog, stoji da učenici „mogu formulirati i s visokom preciznošću raspravljati o postupcima koje su primijenili, da kritički razmatraju nalaze, interpretacije, argumente, uključujući i razmatranje njihove prikladnosti za rješavanje kompleksnih problemskih situacija". (Baucal, Pavlović-Babić, 2010, str. 27). Rasprava o argumentima za i protiv nekog stava može biti dobra vježba za razvijanje različitih učeničkih talenata, ali ne može biti i potvrda njihova matematičkog znanja, o čemu je bilo riječi u radu *The problem of Inadequate Use of the Mathematical Language in the PISA Test Task*, autora Marković, i Erić (2014). Sličnu situaciju ispitanici su uočili u zadatku *Rezultati testa*.

Rezultati testa

Na donjem dijagramu prikazani su rezultati testa iz fizike koji su radile dvije grupe, označene kao Grupa A i Grupa B. Srednji rezultat za grupu A je 62,0, a za grupu B 64,5. Učenici prolaze test ako osvoje 50 ili više bodova.

Prikaz 4

Gledajući grafikon, nastavnik tvrdi da je na tom testu grupa B postigla bolji rezultat od *grupe A. Učenici u grupi A ne slažu se s nastavnikom. Oni ga pokušavaju uvjeriti da grupa B možda nije postigla bolji rezultat. Koristeći se grafikonom, navedite jedan matematički argument koji bi mogli upotrijebiti učenici grupe A.*

Taj zadatak možda najviše upućuje na jedan od ključnih problem PISA testova, a to je nepreciznost. Kako procijeniti koji je rezultat bolji kad sam pojam „*bolji rezultat*" nije precizno formuliran i jednoznačno određen. Kako tko doživljava *bolji rezultat* nije matematički problem i ne može se nečije mišljenje ili doživljaj opravdavati i braniti „matematičkim argumentima". Za nekoga je bolji rezultat postigla grupa koja ima veći prosječan broj bodova, za nekoga ona kod koje je više učenika položilo test, ili ona grupa koja ima veću prosječnu ocjenu ne računajući onog učenika koji je uradio najlošije, odnosno najbolje test itd. Možemo tako navoditi još mnogo razloga zašto bismo neki rezultat mogli smatrati boljim od drugog. Sve nas to dovodi do zaključka da taj zadatak, i uz to što je popraćen grafičkim prikazivanjem podataka, nije matematički zadatak i kao prethodni može biti povod za neku raspravu, ali ne zadatak na osnovi kojeg ćemo ocjenjivati zna li učenik primijeniti matematiku ili ne zna.

Zadatak koji je veći broj ispitanih nastavnika naveo kao „problematičan" za petnaestogodišnjaka je zadatak *Potres*.

Potres

Prikazana je dokumentarna emisija o potresima i koliko često se oni događaju. Raspravljalo se o tome mogu li se potresi predvidjeti. Jedan je geolog izjavio: „U sljedećih dvadeset godina, šanse da se u Zedgradu dogodi potres su dva od tri". Koja od sljedećih izjava najbolje odražava smisao izjave tog geologa?

A) $\frac{2}{3} \times 20 = 13{,}3$ *prema tome, u razdoblju od 13 do 14 godina od ovog trenutka u Zedgradu će se dogoditi potres.*

B) $\frac{2}{3}$ *je više nego* $\frac{1}{2}$*, prema tome, možemo biti sigurni da će se u Zedgradu dogoditi potres u sljedećih 20 godina.*

C) Vjerojatnost da će se potres dogoditi u sljedećih 20 godina veća je nego vjerojatnost da neće biti potresa.

D) Ne može se reći što će se dogoditi zato što nitko nije siguran kada će doći do potresa.

Za rješavanje tog zadatka potrebni su nam elementi vjerojatnosti, što znači da zadatak nije primjeren za predviđenu dob učenika. Također, ni formulacija pitanja što „najbolje odražava smisao izjave" nije matematički precizna i jednosmislena. Naravno, rješenje tog, kao i većine drugih zadataka, intuitivno se i zdravorazumski može naslutiti iz ponuđenih odgovora. Navedeni zadatak svrstan je u četvrtu razinu postignuća u čijem opisu stoji da učenici „mogu selektirati i povezivati podatke zadane na različite načine, uključujući simboličke, i povezujući ih izravno s aspektima situacija iz stvarnog života". (Baucal, Pavlović-Babić, 2010, str. 27). Znači li to da se od učenika, na toj razini postignuća, očekuje povezivanje broja 2/3, koji predstavlja vjerojatnost nekog događaja, s tvrdnjom da je vjerojatnije da će se taj događaj ostvariti nego da neće? Ako učenici nisu u školskom sustavu upoznati s pojmom vjerojatnosti, oni to jednostavno ne mogu učiniti osim da nagađaju ili intuitivno naslućuju, ali ako jesu, tvrdnja je trivijalna i na ljestvici razina postignuća ne bi trebala zauzimati visoko četvrto mjesto.

Sličan problem nalazimo i u zadatku *Podrška predsjedniku*.

Podrška predsjedniku

U državi Zedland obavljena su ispitivanja javnog mnijenja o podršci predsjedniku na budućim izborima. Četiri časopisa obavila su odvojena nacionalna ispitivanja. Rezultati ispitivanja četiri novinska istraživanja su sljedeći:

Prvi časopis: 36.5% (ispitivanje obavljeno 6. siječnja na uzorku od 500 slučajno odabranih građana s pravom glasa).

Drugi časopis: 41% (ispitivanje obavljeno 20. siječnja na uzorku od 500 slučajno odabranih građana s pravom glasa).

Treći časopis: 39% (ispitivanje obavljeno 20. siječnja na uzorku od 1 000 slučajno odabranih građana s pravom glasa).

Peti časopis: 44.5% (ispitivanje obavljeno 20. siječnja tako što je 1 000 čitatelja telefoniralo redakciji i glasalo).

Ako se izbori održavaju 25. siječnja, rezultati kojeg časopisa najbolje oslikavaju podršku predsjedniku? Navedite dva razloga koji podržavaju vaš stav.

I u ovom zadatku imamo problem koji nije matematički. Traži se osobni stav, a ne jednoznačno rješenje zadatka, dakle ponovno imamo nepreciznu formulaciju. Naravno, autori su ipak predvidjeli samo jedan točan odgovor – treći časopis. Do tog odgovora vode nas zdrav razum i sljedeći argumenti: istraživanje je provedeno neposredno prije izbora (20. siječnja), u pitanju je velika grupa ispitanika (1000), i ispitivani su samo građani s pravom glasa. Međutim, nije rečeno kako je treći časopis provodio ispitivanje, jesu li ispitivani glasači, iako s pravom glasa, uopće bili zainteresirani za glasanje. Možda su oni koji su telefonski iskazivali svoje mišljenje zapravo bili puno zaineresiraniji za ishod glasanja pa je njihovo glasanje bolji pokazatelj konačnog rezultata. Naravno, sve navedeno može biti predmet polemike i rasprave, ali nikako egzaktan matematički zadatak. Također, možemo postaviti pitanje je li taj zadatak prilagođen dobi ispitanika? Zašto bi petnaestogodišnjak morao znati kako funkcionira anketiranje građana pred izbore i kako je potrebno sumnjati u točnost glasača koji glasaju telefonskim putem?

Zagovornici i autori PISA testa ističu da se pod pojmom matematičke pismenosti podrazumijeva sposobnost matematičkog zaključivanja i primjene matematičkih postupaka i alata u opisivanju, predviđanju i objašnjavanju pojava, a ne skup školskih znanja i vještina. „PISA mjeri osposobljenost učenika da odgovore na zahtjeve relevantne za svakodnevni život i razumijevanje ključnih koncepata, a ne procjenjuje usvojenost znanja koja su specifična za određena područja" (Baucal, Pavlović Babić 2010). Zbog toga je jedan od prvih kriterija koje mora ispuniti svaki zadatak koji se nađe na PISA testu taj da zadatak treba biti smješten u „stvarni životni kontekst". Kritičari našeg školskog sustava ističu da učenici u našim školama memoriraju nepotrebne podatke i uče teorijske činjenice koje potom ne znaju primijeniti u „svakodnevnom životu", a kako su loši rezultati učenika na PISA testovima najbolji dokaz da su u pravu (Strategija razvoja obrazovanja u Srbiji do 2020. godine, 2012). Međutim, slučajno ili namjerno, zaboravlja se činjenica da je svaka formula koja se uči iz matematike i u osnovnim i u srednjim školama učenicima objašnjena i izvedena pozivajući se na njihova prethodna matematička znanja, a ne „servirana„ kao gotova formula za usvajanje. Upravo takve formule čiji smisao učenici ne razumiju mogu se vidjeti na PISA testovima.

U različitim zadacima učenicima su ponuđene formule koje precizno ili manje precizno opisuju neke odnose u prirodi i realnom životu: $d = 7{,}0 \cdot \sqrt{t - 12}, (t \geq 12)$ (veza između promjera lišaja oblika kruga i broja godina proteklih od otapanja leda) – zadatak *Lišajevi*, $n/P=140$ (veza između broja koraka u minuti i duljine koraka u metrima) – zadatak Šetnja, $D=dv/(60n)$ (formula za brzinu kretanja infuzije u ovisnosti o volumenu i broju sati) – zadatak *Brzina kapanja infuzije*.

U svakom od tih zadataka nalaze se veze između nekih veličina za koje učenici ne znaju kako su dobivene. Sadržaji tih zadataka smješteni su u tematsko područje koje autori PISA testiranja nazivaju *Brojevi i mjere*. Putem zadataka s tim sadržajima „traži se razumijevanje numeričkih fenomena, kvantitavnih odnosa i obrazaca". (Baucal, Pavlović-Babić, 2010, str. 27). U svakom od navedenih zadataka poznate su sve veličine osim jedne koja se traži. Zamjenom poznatih vrijednosti u danim jednakostima u sva tri slučaja dobivaju se trivijalne jednadžbe s jednom nepoznanicom. Učenici takve zadatke mogu rješavati mehanički, šablonski primjenjujući dane formule i nemaju potrebe o njima razmišljati. Dakle, učenici ih rješavaju upravo onim metodama koje zagovornici PISA testiranja i novih strategija obrazovanja najviše kritiziraju. Međutim, razlike između ta dva slučaja primjene formula su osnovne. U školskim zadacima potrebne formule dane su učenicima s potpunim objašnjenjem njihove unutarnje strukture, smisla i razloga, kao i značaju i mjestu te formule u njihovom matematičkom obrazovanju. Realni kontekst koji kod PISA zadataka navodno postoji ovdje je, ipak, samo prividan. Jer koliko učenika u svom svakodnevnom životu razmišlja o odnosu veličine i starosti lišaja ili o potrebnoj brzini kapanja infuzije? Možemo li zamisliti i bolničarke koje s papirom, olovkom i kalkulatorom sjede i za svakog pacijenta posebno računaju brzinu kapanja infuzije ili one pritiskom na jednu tipku računala dobivaju odgovor na to pitanje? Ako su ovo stvarne životne situacije petnaestogodišnjaka, postavlja se pitanje – što onda nije njegova stvarna životna situacija. Jesu li takvim relativiziranjem pojma životna situacija upravo zagovornici te koncepcije potpuno obesmislili sam pojam?

Jedna od ovih formula (zadatak Šetnja) mogla bi se osporiti jer je proizvoljna i iskustvene prirode, a ne rezultat ozbiljnog istraživanja. Postavlja se pitanje jesu li veličine n i P, koje predstavljaju broj koraka u minuti i duljinu koraka u metrima, izravno ili obrnuto proporcionalne. Kad čovjek ubrzava hod, postaju li njegovi koraci kraći ili dulji? Gledajući natjecatelje u brzom hodanju, čini nam se ipak kraći, što točnost dane formule dovodi u pitanje, o čemu je bilo riječi u radu Marković, Pikula (2014). Dakle, ti zadaci sa sumnjivim realnim kontekstima, ne traže ništa drugo od učenika nego najjednostavniju primjenu formule čija je priroda učenicima potpuno nejasna.

Realni životni kontekst opravdanje je i za zadatke u kojima se javlja višak podataka, o čemu smo već govorili. Primjeri takvih zadataka su zadaci: S*tubište*, *Snaga vjetra*, *Brodovi na jedro* itd. U redovnom školovanju učenici ne susreću zadatke s viškom podataka. Budući da su ti zadaci, upravo zbog viška podataka, preopterećeni dugim tekstom, učenici ostaju zbunjeni pred zadacima tog tipa misleći da nešto očigledno predviđaju. Upadanje učenika u tu zamku ne mora biti odraz njihova matematičkog neznanja, niti je to posljedica učenja bez razumijevanja, niti je ključni dokaz da se učenici ne znaju snaći u stvarnoj životnoj situaciji, jer oni u tom trenutku kada rješavaju test nisu ni na stubištu nekog trgovačkog centra, niti na palubi teretnog broda, niti pokraj neke vjetrenjače. Njihova istinski realna životna situacija u tom trenutku je izrada testa i o zadacima u testu razmišljaju kao o korektno napisanim matematičkim zadacima u školi, a takvi zadaci nemaju nepotrebnih podataka. Da se doista nalaze na brodu, u brodskim knjigama imali bi još više podataka o brodu nego što je dano u zadatku, ali bi isto tako znali da svi ti podaci nisu bitni za problem, na primjer, s jedrom, koji moraju riješiti.

Ove zadatke nećemo navoditi jer su predugi. Recimo samo da je u zadatku *Brodovi na jedro* dana sljedeća situacija. Brod čije je ime „Novi val" troši 0,42 zeda (fiktivna novčanica) po litri dizela. Zbog visoke cijene goriva vlasnik odlučuje da će kupiti zmajoliko jedro koje može smanjiti potrošnju dizela za približno 20%. Dani podaci u ovom zadatku su: cijena dizela (0,42 zeda), dužina broda (117 metara), širina broda (18 metara), nosivost (12 000 tona), maksimalna brzina (19 čvorova), godišnja potrošnja dizela bez zmaja (približno 3 500 000 litra), cijena opremanja zmajolikim jedrom (2 500 000 zeda). Naveden je i naziv broda i da je brod tipa teretnjak. Pitanje u zadatku glasi: Poslije koliko će se godina, otprilike, uštedom dizelskog goriva isplatiti cijena kupovine zmajolikog jedra?

Kad učenici iz šume podataka izdvoje potrebne podatke i taj zadatak riješe, dobivaju odgovor da je u pitanju otprilike 8,5 godina, odnosno između 8 i 9 godina, što se također priznaje kao točan odgovor. Međutim, ako baš inzistiramo na realnom kontekstu, onda puno toga možemo problematizirati. Na primjer, što ako cijena dizela svih tih 8 ili 9 godina bude promjenjiva, ili hoće li težina zmajolikog jedra smanjiti nosivost broda, a samim tim i njegovu zaradu?

U OECD brošurama *PISA Released Items – Mathematics, 2006.* i *PISA Released Mathematics Items, 2013.* poslije svakog je zadatka dano objašnjenje kako se taj zadatak boduje. Iako način vrednovanja zadataka od PISA tima nije bio predmet našeg detaljnog istraživanja, dat ćemo neke uvide u to vrednovanje. Prvi je korak pri pregledavanju testova kodiranje. Odgovori učenika kodiraju se tako što im se dodjeljuju unaprijed definirani kodovi. Taj postupak posebno je osjetljiv kod odgovora na pitanja otvorenog tipa. Iako su ocjenjivači obučavani tako da se njihova subjektivnost svede na najmanju mjeru, njima je na kraju ostavljeno da osobno procjenjuju praktično sve – jesu li dani odgovori za „maksimalan broj bodova", „djelomičan broj bodova" ili „bez bodova". Posebno je nejasan kriterij za polovično točne odgovore, tj. za „djelomičan broj bodova". Tako je, primjerice, u zadatku *Lišajevi* potrebno riješiti elementarnu iracionalnu jednadžbu $d = 7{,}0 \cdot \sqrt{t - 12}$

gdje je $d = 35$. U OECD brošuri PISA Released Items – Mathematics, navodi se da se djelomičan broj bodova dobiva za „ispravnu zamjenu vrijednosti u formuli, ali pogrešan odgovor" ili za odgovore $t = 36$, odnosno $t = 38$ (iako je jedini točan odgovor $t = 37$), do kojih „učenik može doći metodom pokušaja i pogrešaka". Ovdje nisu jasni razlozi zašto se djelomičnim brojem bodova nagrađuje netočan odgovor (36 ili 38) do kojeg je učenik došao neprihvatljivim postupkom (metoda pokušaja i pogrešaka). Djelomičnim brojem bodova ocijenjen je odgovor u kojem je učenik napisao „$5 = \sqrt{t - 12}$ iz čega slijedi $5 = \sqrt{t} - \sqrt{12}$". Učenik je, čineći grubu materijalnu pogrešku i pokazujući da ne zna da kvadratni korijen razlike nije jednak razlici kvadratnih korijena, prema ocjenjivačima, djelomično matematički pismen, iako je zapravo apsolutno matematički neobrazovan.

Proizvoljnost u priznavanju odgovora prisutna je u većini zadataka u kojima se od učenika traže neke procjene. Tako se, primjerice, u zadatku *Figure* od učenika traži da opiše metodu procjene površine jedne nepravilne figure. Kao primjer odgovora za koje učenika treba nagraditi maksimalnim broj bodova dan je sljedeći odgovor: „Nacrtati figuru na kvadratnoj mreži. Manji kvadrati znače veću preciznost." U naputku za ocjenu tog odgovora stoji: „Ovdje je učenikovo objašnjenje jako kratko, ali smo bili popustljivi prema njegovoj slaboj vještini pisanja jer metoda koju je učenik ponudio smatramo ispravnom". (PISA Released Items – Mathematics). Nažalost, nije dano preciznije objašnjenje za takav vid ocjenjivanja. Učenik u prethodne dvije rečenice nije ponudio nijednu metodu, već samo započeo s objašnjenjem. Njegov daljnji tijek misli, koji nije napisan, ne može biti predmet ocjenjivanja. Dalje, postupak za rješavanje nekog matematičkog zadatka ili je točan ili netočan, a nikako ne može biti „smatran" točnim ili netočnim.

Najočiglednija proizvoljnost u procjenjivanju odgovora vidi se u zadatku *Garaža*. U tom zadatku potrebno je izračunati površinu krova koji se sastoji od dva sukladna pravokutnika čija je jedna stranica poznata, a druga predstavlja hipotenuzu jednog pravokutnog trokuta čije su katete također poznate.

Maksimalnim brojem bodova ocjenjuje se odgovor u kojem učenik kao rješenje ponudi bilo koju vrijednost od 31 do 33 (kalkulatori su na PISA testiranju dopušteni) iako se površina krova može precizno izračunati. Međutim mnogo je problematičnije što se vrednuje djelomičnim brojem bodova: „Postupak ne prikazuje upotrebu Pitagorina poučka, ali su upotrijebljene prihvatljive vrijednosti za širinu krova (primjerice, bilo koja vrijednost od 2,5 do 3), a ostatak računa je točno obavljen". (*PISA Released Items – Mathematics*, 2013, str. 32).

Nameće se pitanje što se u ovom zadatku zapravo ocjenjuje. Djelomičnim brojem bodova ocijenjen je odgovor učenika koji ne zna primijeniti Pitagorin poučak u navedenom primjeru, ali zna koliko bi otprilike morala biti široka jedna strana krova garaže! Boduju se, dakle, iskustvo i snalažljivost, a ne matematičko znanje ni bilo kakva njegova primjena.

To su samo neki primjeri u kojima se jasno vidi nedostatak principijelnosti u vrednovanju odgovara, kao i potpuna proizvoljnost u osobnom procjenjivanju točnih, odnosno djelomično točnih odgovora. Može li nas takva nepreciznost u ocjenjivanju dovesti do relevantnih podataka za bilo kakvu ozbiljnu analizu? Kakva je vrijednost složenih statističkih obrada zasnovanih na potpuno proizvoljnim i nepouzdanim podacima?

Na kraju, možemo postaviti pitanje što je s motivacijom učenika? Jesu li spremni uložiti maksimalan trud na testiranju radeći često dosadne, zamorne, preopterećene suvišnim tekstom, zadatke ne bi li postigli što bolji rezultat koji njima osobno ništa ne donosi? Lambić i Lipkovski s pravom ukazuju na važnost zadovoljstva i uživanja kao čimbenika motivacije prilikom rješavanja problema (Lambić, Lipkovski, 2011). Znamo da PISA procjenjuje stavove, uvjerenja i motivaciju učenika za školsko učenje, ali je također velik problem motivacija učenika za PISA testiranje koje oni ne moraju smatrati bitnim jer u njihovoj realnoj životnoj situaciji ovo testiranje nema nikakvu funkcionalnu vrijednost. O problemu motivacije, ali i o zamoru kao važnom čimbeniku uspešnosti prilikom izrade testa, važna je zapažanja dao i Wuttke (2007).

Naravno, moramo biti svjesni i određenih ograničenja koje ima ovo istraživanje koja su provela dva neovisna istraživača.

Prvo, ispitanici su prije popunjavanja upitnika morali izdvojiti određeno vrijeme za proučiti 78 zadataka PISA testiranja. Postavlja se pitanje je li to učinio svaki ispitanik i koliko temeljito.

Drugo je pitanje koliko su nastavnici koji ne predaju matematiku bili u stanju istinski sudjelovati u istraživanju i detaljno pročitati i proučiti sve zadatke.

Treće ograničenje odnosi se na činjenicu da smo se koristili samopopunjavajućim anketnim upitnikom i numeričkom ljestvicomu procjene koja omogućuje da osobni stavovi ponekad dođu do izražaja više nego što smo to htjeli. Procjena ispitanika koji su popunjavali upitnik može biti više ili manje subjektivna ako su, primjerice, pogrešno razumjeli neku rečenicu, ali i shodno prethodnom stavu koji su imali uopće o PISA testiranju.

Zaključak

PISA projekt međunarodna je evaluacijska makrostudija, što treba stalno imati na umu, i upravo zato treba biti iznimno oprezan pri ocjeni njezinih dosega i mogućnosti. Ona doista nema, niti na to pretendira, praktične vrijednosti u pogledu pojedinačnog postignuća učenika ili u pogledu utjecaja na konkretnu nastavnu praksu. Smatramo da je svaki takav pokušaj doslovnog prenošenja ili primjene PISA filozofije na nastavu matematike pogrešan i taj smo stav potkrijepili rezultatima istraživanja koje smo proveli. Možda najveći problem u vezi s PISA projektom i nastaje zbog nedosljednog i kontradiktornog tumačenja odnosa primjenjivog znanja iz matematike, koje po riječima autora PISA testiranja ono mjeri, i njegove povratne sprege prema nastavi matematike u školi. Činjenica je da se u političkom diskursu mnogih zemalja, uključujući Srbiju, PISA projekt doživljava i tumači na jedan pojednostavljen način, u isto vrijeme kao dijagnoza i kao lijek. Naravno, takva očekivanja nitko ne može ispuniti pa ni PISA. Posebno je pogrešno kada jedan tako složen sustav, kao što je obrazovni sustav, mjerimo nečim jednostavnim kao što je prosječan broj riješenih zadataka na testiranju. Već smo rekli koliko je sama evaluacija zadataka na testiranju proizvoljna, odakle proizlazi da je i točnost rangiranja zemalja u velikoj mjeri precijenjena (Hopmann, 2008).

Međutim, uz sve nedostatke PISA projekt ostvario je važan i stimulativan utjecaj s iznimno pozitivnim posljedicama. Usprkos brojnim neslaganjima s obzirom na dosege i konkretne rezultate, nitko neće osporiti važnost PISA testiranja za pokretanja najšire rasprave metaevaluacijskog karaktera o pravcima i smislu razvoja suvremenog obrazovanja.