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Analysis of Three-Dimensional Interface Corner Cracks

The growth of an interfacial crack, from the straight corner along the joint plane between the two long plates made of different materials is considered in this paper. Three different shapes of the crack front are analyzed: the concave, triangular and the quarter circular shape. Results are presented for variations of the normalized stress intensity factors of all the three crack loading Modes, as well as the normalized phase angles and the normalized energy release rate, all in terms of an angle, which measures the distance from the symmetry plane.

Keywords: *Interface crack, Three-dimensional crack, Corner crack, Phase angle, Stress intensity factors, Energy release rate.*

1. INTRODUCTION

In composite materials, which contain the bimaterial interface and are exposed to residual stresses or external loads, the fracture could occur either in one of the materials constituting the interface or along the interface itself [1]. The fracture mechanism depends on the sample geometry, loading, substrate toughness and interface toughness [2]. The comprehension of such mecha–nism is of utmost importance in many sector such as aerospace [3], automotive [4] and woodworking [5] to ensure safety in operational conditions.

In layered materials the mechanical integrity is determined by toughness of the interface between the various materials. The interface is a convenient place for fracture initiation, during the test, operation or storage of the layered component.

Mechanical properties of the multi-layered materials are frequently limited by the strength of the interface between the constituting materials. Very often the interface is a point where the initial crack appears. Delamination of the composite materials usually begins in corners. The singular stress field, which exists in ideally elastic bimaterial corner, corresponds to this interface destruction and it depends on material characteristics and the corner shape.

The initial progress in investigations of static fracture at the interface was provided in work by Rice [6] who defined the elastic fracture mechanics concepts for interfacial cracks. He defined the complex stress intensity factor for interfacial cracks and the so-called *mode mixity* or the *load phase angle*. The near tip stress field for an interface crack between the two dissimilar isotropic materials is a linear combination of the two types of fields, a coupled oscillatory field, defined by a complex stress intensity factor $K = K_1 + iK_2$ and a non-oscillatory field, scaled by a real stress intensity factor K_{III}.

Hutchinson and Suo [7] have considered the mixed mode crack propagation criteria in different types of solids, delamination of beams, interfacial crack problems and cracking and delamination of thin films.

Nikolic and Djokovic [8] were considering cracks in bimaterials and bicrystals with application of the Rice-Thomson model [9] to such materials.

Qu and Bassani [10] have considered the crack at the interface of the two anisotropic materials and they have obtained expressions for the stress fields at the crack tip, the corresponding stress intensity factors and the energy release rate. They also discussed some general problems related to the interface cracks – the oscillatory nature of the stress field, the crack faces contact, effect of the specimen size and dependence of the crack behavior on anisotropic materials properties.

Nikolic and Djokovic [11] were also considering application of the linear elastic fracture mechanics (LEFM) concept for interfacial cracks for the case of the cylindrical substrate delamination.

Barsoum and Chen [12] have analyzed the angular singularity of a bimaterial using the *Finite Element Iterative Method* (FEIM). They have shown that numerous bimaterial combinations do not exhibit the oscillatory character of the field in the vicinity of the angular singularity. In addition, they have shown that in such cases the singularity of the symmetric and anti-symmetric modes lies within range from 0.5 to 0.75, when the material mismatch becomes larger and that the complex power singularity strongly depends on the Poisson's ratio.

Nakamura [13] has conducted a 3D analysis of a bimaterial plate with a central crack. He has chosen the shear moduli and the Poisson's ratios of the two materials consituting the interface in such a way that, in plane strain conditions, the usual oscillatory stress field was obtained. He concluded that, since the experimental investigations have shown that the critical energy release rate strongly depends on the phase angles, one must take into account the phase angle variation along the crack front for accurate predictions of the interfacial crack behavior.

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Nakamura and Kamath [14] have conducted the three-dimensional analysis of the crack growth and destruction of a thin film on the rigid substrate. Their analysis pointed that the stress intensity factor along the leading edge of the interfacial crack, in the case when there is no the thin film decohesion, reaches the steady state value when the crack length reaches the value of about twice the thin film thickness. When there is a decohesion of the thin film, the stress intensity factor increases with the debonding opening angle increase. The equlibrium between the crack propagation and the thin film decohesion can bedetermined from the SIFs for the two cases and the fracture toughnesses of the thin film and the interface.

Lee and Rosakis [15] have studied the 3D field along the front of a bimaterial interfacial crack in order to determine the relevant fracture parameters for thin plates that were used as the laboratory samples. They concluded that: "*The 3D effects in interfacial cracks are not only caused by the presence of the cracks itself, but they are enhanced by the material mismatch along the interface*".

Gosz, Dolbow and Moran [16] have developed the new method for determining the stress intensity factors in the mixed mode conditions along the curved 3D interfacial crack front. That is the domain integral method, where the crack-tip contour integral is expressed as a volume integral over a finite domain surrounding the crack tip. For its derivation they imposed the auxiliary stress fields (the plane and anti-plane ones) along the curved crack front.

Chaudhuri [17] have analyzed the asymptotic stress field in the vicinity of the front of the five different penny-shaped discontinuities, including the penny-shaped crack, subjected to Modes I, II and III loadings. He concluded that the stress-singularities for the penny-shaped crack and anti-crack (i.e. the perfectly bonded thin rigid inclusion) are the same, the main difference being that for the anti-crack all the stress components depend on the Poisson's ratiounder the Modes I and II.

Ayhan and Nied [18] have demonstrated application of the enriched finite element approach as a very effective technique for obtaining the stress intensity factors for the general three-dimensional crack problems. They obtained a good agreement between different numerical solutions, except for the small zone near the free surface. The ascribed the difference to the fact that the previously published results have often neglected the change in the stress singularity at the free surface.

Ayhan, Kaya and Nied [19] have developed the efficient computer model, which gives the correct results of the asymptotic behavior at the crack tip, using the enriched element at the crack tip. In this formulation of the enriched element, the stress intensity factors K_I, K_{II} and K_{III} were considered as the additional degrees of freedom and they are obtained as the solution by the finite element method.

Nagai, Ikeda and Miyazaki [20] have analyzed the stress intensity factors for the 3D interfacial crack between dissimilar anisotropic materials in the presence of the thermal and mechanical stresses, by application of the M_1 integral method. The analysis was done for the cases of the double-edge cracks in jointed isotropic and anisotropic plates.

Koguchi and Yokoyama [21] have investigated the singular stress field at a vertex of an interface in the three-dimensional joints under the tensile loading by application of the boundary element method. They considered the three crack shapes – triangular, quarter circular and concave, for which they defined the dimensionless Mode II stress intensity factor, determined from the normalized stress distribution.

Saghafi et al. [22] have considered the effect of preload on impact response of composite laminates. They concluded that the delamination and the matrix cracks were the predominant modes of failure of the curved specimens. Also analyzing composite materials, Fragassa et al. [23] performed impact experiments on hybrid laminates reinforced with natural fibres, concluding that the crack inferred by the indenter is striclty related to the brittleness relationship between skin and core materials.

Saputra, Birk and Song [24] were dealing with calculations of the 3D fracture parameters of an interfacial crack and notches using the scaled boundary finite element method. They were computing the stress intensity factors and the T-stress. The semi-analytical solution is expressed as a matrix power function, which enabled obtaining the fracture parameters. They applied the proposed method to a through the thickness edge interface crack, a peny-shaped crack in a homogeneous block, a through the thickness crack in a homogeneous material and a double edge notch at a bimaterial interface.

Chiu and Lin [25] were considering the fracture mechanics parameters, including the strain energy release rate, the stress intensity factors and phase angles, along the curvilinear front of a three-dimensional bimaterial interface crack in electronic packages, by using the finite element method with the so-called *virtual crack closure technique* (VCCT). Validation for the procedure was done by comparing their numerical results to analytical solutions for the problems of interface crack subjected to either remote tension or mixed loading.

Veluri and Jensen [26] have analyzed the steady-state propagation of interface cracks in the thin surface layers or thin films close to three-dimensional corners. They were modeling the shape of the interface crack front and calculating the critical stress for the steady-state crack propagation. Estimates of the fracture mechanics para–meters, including the strain energy release rate, crack front profiles and the three-dimensional mode-mixity along the interface crack front, were obtained by the Finite Element Method application to the near field (crack tip) solutions based on the J-integral.

Kastratović et al. [27] developed a procedure to estimate mode I stress intensity factors in an elastic body subjected to remote uniaxial tensile loading. The method studied provided a fast and efficient assessment of stress intensity factors, even in the more complex 3D case that considers various damage sites.

There is a small number of papers in which are analyzed the stress intensity factors for the 3D interfacial crack, which starts from the bimaterial corner. One of the reasons for that is the fact that for the case of the interfacial crack the stress intensity factors for Mode I and Mode II are mutually coupled and do not have the same physical meaning as in the case of a homogeneous material.

In this paper are presented cases of the interfacial crack growth from the straight corner, along the joint plane between the two long plates made of different materials, for various shapes of the crack front. The three different shapes of the crack front are considered: the concave, triangular and the quarter circular shape.

2. PROBLEM FORMULATION

The stress field around the tip of a crack at the interface between the two materials is a linear combination of the two types of fields, the coupled oscillatory field, which is defined by the complex stress intensity factor and the non-oscillatory field, which is measured by the real stress intensity factor K_{III}. The stress field around the interfacial crack tip has the following form:

$$\sigma_{ij} = \frac{1}{\sqrt{2\pi r}} \begin{bmatrix} \mathrm{Re}(Kr^{i\varepsilon})\sigma^{I}_{\alpha\beta}(\theta,\varepsilon) \\ +\mathrm{Im}(Kr^{i\varepsilon})\sigma^{II}_{\alpha\beta}(\theta,\varepsilon) + K_{III}\sigma^{III}_{\alpha\beta}(\theta) \end{bmatrix} \quad (1)$$

where $\sigma^{I,II,III}_{\alpha\beta}(\theta,\varepsilon)$ are the angular functions, which correspond to the tensile forces (superscript I), the in-plane shear (superscript II) and the anti-plane shear (superscript III), [28]. Parameter ε is called the oscillatory index, it is a characteristics of the interface crack and is determined in [6] as:

$$\varepsilon = \frac{1}{2\pi}\ln\left(\frac{1-\beta}{1+\beta}\right) \quad (2)$$

where β is one of the two Dundurs parameters [29]:

$$\begin{aligned} \alpha &= \frac{\mu_2(\kappa_1+1)-\mu_1(\kappa_2+1)}{\mu_2(\kappa_1+1)+\mu_1(\kappa_2+1)} \\ \beta &= \frac{\mu_2(\kappa_1-1)-\mu_1(\kappa_2-1)}{\mu_2(\kappa_1+1)+\mu_1(\kappa_2+1)} \end{aligned} \quad (3)$$

where μ_i is the shear modulus, $\nu_i$is the Poisson's ratio, subscripts i = 1, 2 refer to material above and below the interface, respectively, while κ_i=3−4ν_i is valid for the plane strain state and κ_i=(3−ν_i)/(1+ν_i) is valid for the plane stress state.

In equation (1) K_{III} represents the Mode III stress intensity factor, which has the same form as for homogeneous solid. As opposite to homogeneous material, where Mode I and II factors are separated, K_I and K_{II}, for interfacial crack in-plane modes are coupled together, and given as [7]:

$$\begin{aligned} K_1 = \hat{K}_I &\equiv \mathrm{Re}(Kr^{i\varepsilon}) = K_I\cos(\varepsilon\ln r) - K_{II}\sin(\varepsilon\ln r) \\ K_2 = \hat{K}_{II} &\equiv \mathrm{Im}(Kr^{i\varepsilon}) = K_I\sin(\varepsilon\ln r) + K_{II}\cos(\varepsilon\ln r) \end{aligned} \quad (4)$$

The solution for the crack at the interface is comp–letely determined if the individual stress intensity factors K_I, K_{II} and K_{III} are known, or equivalently, the energy release rate G and the phase angles ψ and φ. The energy release rate for the interfacial crack is, [7]:

$$G = \frac{1}{\cosh^2(\pi\varepsilon)}\cdot\frac{|K|^2}{E_*} + \frac{K^2_{III}}{2\mu_*} \quad (5)$$

where:

$$\frac{2}{E_*} = \frac{1}{\overline{E}_1} + \frac{1}{\overline{E}_2},\ \frac{2}{\mu_*} = \frac{1}{\mu_1} + \frac{1}{\mu_2}, \quad (6)$$

and where $\overline{E}_i = E_i/(1-\nu_i^2)$ is valid for the plane strain state and $\overline{E}_i = E_i$ for the plane stress state; μ_i is the shear modulus, E_i is the Young's elasticity modulus, ν_i is the Poisson's ratio, while the subscripts 1 and 2 refer to materials above and below the interface, respectively.

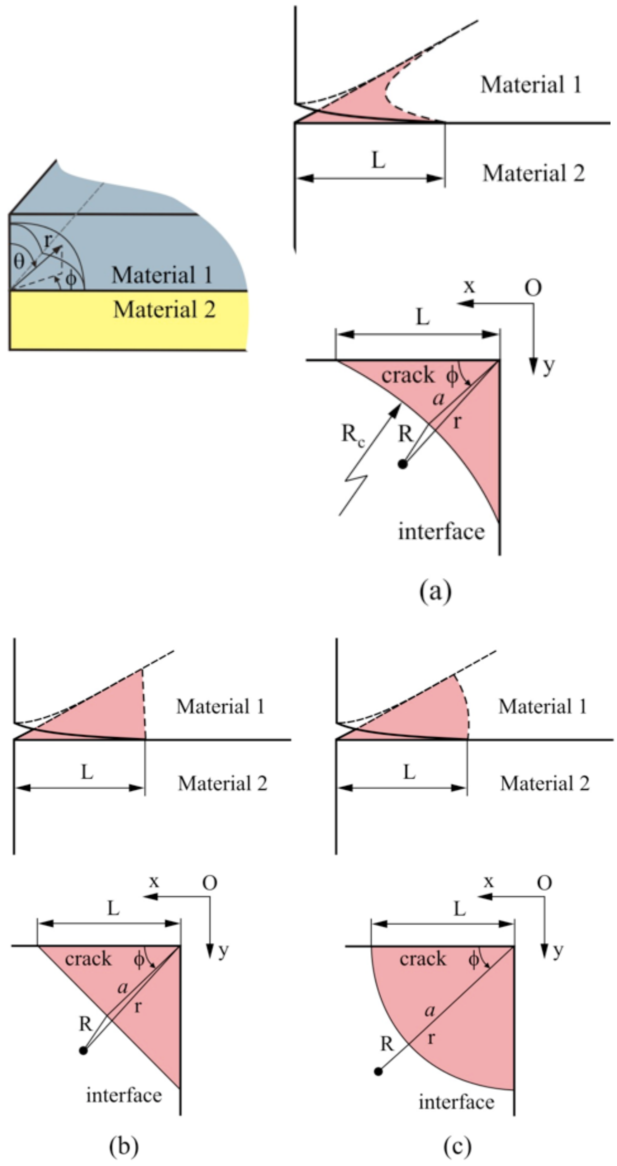

Figure 1. The three-dimensional bimaterial angle with a crack at the interface for different shapes of the crack front: (a) concave, (b) triangular and (c) a quarter circular.

For the case of the interfacial crack, $\varepsilon \neq 0$, influences of tension and shear around the crack tip are not separated. To measure the relative dependence of the shear on normal forces (or Mode II on Mode I) it is necessary to determine the characteristic length L. For the oscillatory field the mixed mode ψ and φ in the K space can be defined as [6]:

$$\mathrm{tg}\,\psi = \frac{\mathrm{Im}(KL^{i\varepsilon})}{\mathrm{Re}(KL^{i\varepsilon})},\quad \cos\varphi = \frac{K_{III}}{\sqrt{|K|^2 + K_{III}^2}} \tag{7}$$

The characteristic length L is chosen arbitrarily, but it could be invariant for a certain bimaterial combi–nation, i.e. L might not depend on the size and type of the sample. The reasonable selection for the value of L is a length of the elastic zone with respect to the sam–ple's size.

The propagation of the interface crack is analyzed in this paper, for the case when it starts from a right angle in the plane of a joint between the two plates made of different materials, for various shapes of the crack front, Figure 1.

As can be seen from Figure 1, the interface crack can have three different shapes: concave, triangular and a quarter circular. Based on geometries, presented in Figure 1, the distances from the crack tip for the con–cave, triangular and the quarter circular crack front shape, respectively are:

$$r = \sqrt{R^2 + a^2 - 2R\cdot a\cdot\cos\left(\varphi + \frac{R_c - a\cdot\cos\varphi}{R_c - a\cdot\sin\varphi}\right)} \tag{8}$$

$$r = \sqrt{R^2 + a^2 - 2R\cdot a\cdot\cos\left(\varphi + \frac{3\pi}{4}\right)} \tag{9}$$

$$r = R + a\,, \tag{10}$$

where a is the crack length, r, θ and □ are the polar co–ordinates, while R is the distance from the crack front and R_c is the radius of curvature of the concave crack front.

3. RESULTS AND DISCUSSION

In Figures 2, 3 and 4 are shown values of the nor–malized stress intensity factors along the interfacial crack front for the three shapes of the crack front for the Al_2O_3/steel interface. In calculations the following characteristics of the two materails were used: forAl_2O_3 - E_1 = 375 GPa, ν_1 = 0.27 and for steel -E_2 = 216 GPa, ν_2 = 0.28. Results are given in terms of angle θ, which is measured from the symmetry plane and normalized by the Mode I stress intensity factor for the penny-shaped crack $K_{IR} = (2/\pi)\sigma_0\sqrt{\pi a}$.

From Figures 2 and 3 one can see that results for $\hat{K}_I$ and $\hat{K}_{II}$ are symmetrical with respect to the θ = 45º plane, while from Figure 4, can be seen that for K_{III} the results are anti-symmetrical. The Mode I stress intensity factor increases as the crack front approaches the free surface for the case of the quarter circular crack front shape. For the cases of the concave and triangular crack front shapes, the Mode I SIF decreases with the crack front approaching to the free surface. The Mode II SIF remains almost constant along the whole crack front and its values are negative for all the three cases of the crack front shapes. The Mode III SIF has a value equal to zero in the symmetry plane and it increases as the front approaches the free surface.

In Figure 5 is presented variation of the normalized phase angle, $\hat{\psi} = \tan^{-1}(\hat{K}_{II}/\hat{K}_I)$ in terms of angle θ for the Al_2O_3/steel interface for all the three crack front shapes.

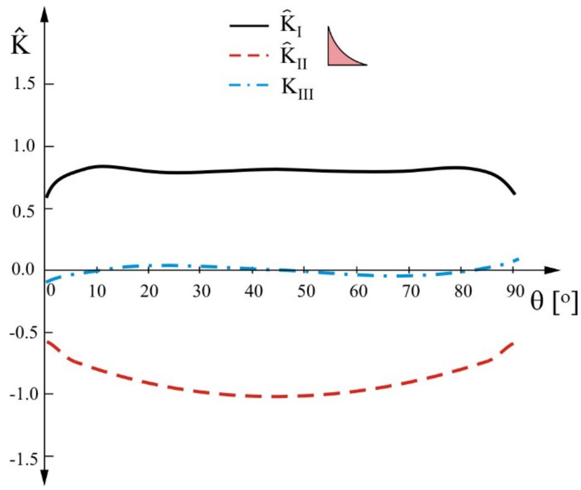

Figure 2. Variation of the normalized stress intensity factor in terms of angle θ for the concave shape of the crack front at the Al_2O_3/steel interface.

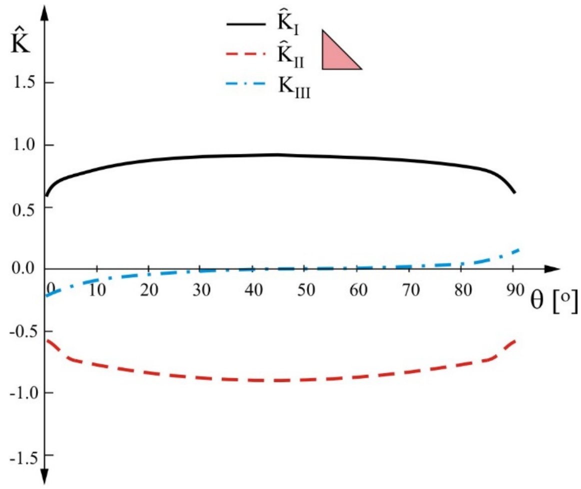

Figure 3. Variation of the normalized stress intensity factor in terms of angle θ for the triangular shape of the crack front at the Al_2O_3/steel interface.

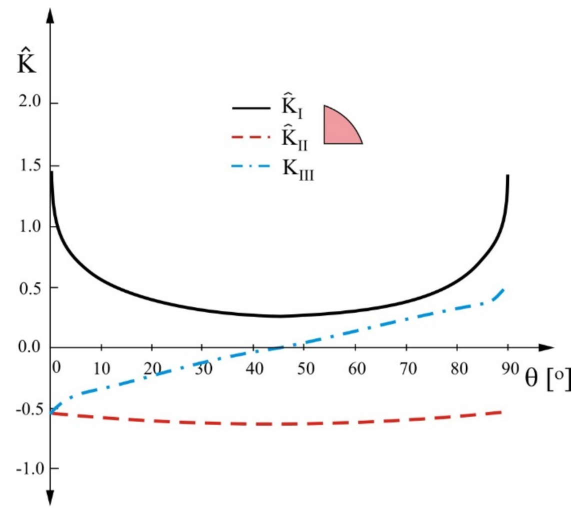

Figure 4. Variation of the normalized stress intensity factor in terms of angle θ for the quarter circular shape of the crack front at the Al_2O_3/steel interface.

From Figure 5 one can notice symmetry of results with respect to the θ = 45º plane. As in the case of the Mode I SIF variation with θ, it could be seen that the influence of the in-plane shear increases in the vicinity of the free surface.

In Figure 6 is presented the normalized phase angle, $\hat{\varphi} = \tan^{-1}(K_{III} / \hat{K}_I)$ in terms of angle θ for the Al_2O_3 /steel interface for all the three crack front shapes.

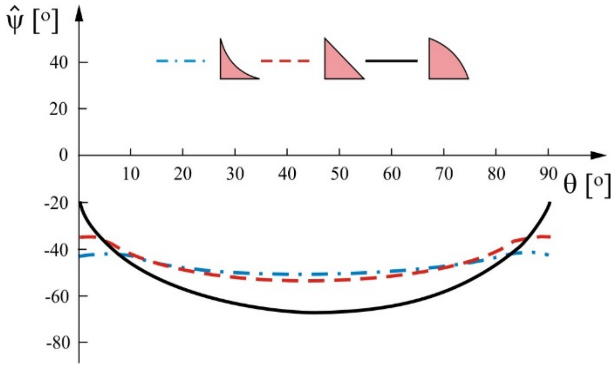

Figure 5. Variation of the normalized phase angle $\hat{\psi}$ in terms of angle θ for all the three crack front shapes at the Al_2O_3/steel interface.

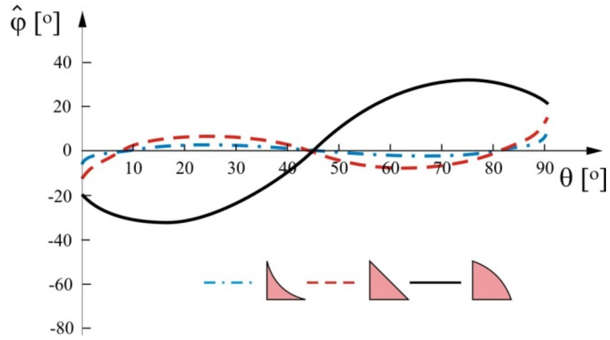

Figure 6. Variation of the normalized phase angle $\hat{\varphi}$ in terms of angle θ for all the three crack front shapes at the Al_2O_3/steel interface.

From Figure 6 one can notice that results for normalized phase angle $\hat{\varphi}$ are anti-symmetrical results with respect to the θ = 45º plane, as in the case of the Mode III SIF variation with θ.

Taking into account that the energy release rate determines the crack propagation, in Figure 7 is presented normalized energy release rate $\hat{G}$, in terms of angle θ for the Al_2O_3/steel interface for all the three crack front shapes. The energy release rate values are normalized by the value $G_{IR} = K_{IR}^2 / E_*$.

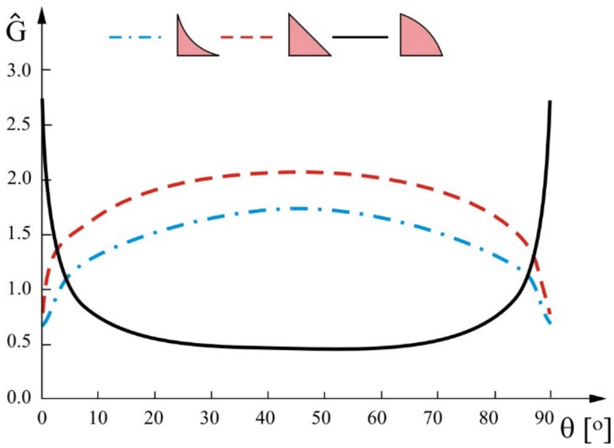

Figure 7. Variation of the normalized energy release rate $\hat{G}$ in terms of angle θ for all the three crack front shapes at the Al_2O_3/steel

From Figure 7 can be noticed that the crack of the quarter circular shape rapidly advances close to the free surface and slower in the middle. The opposite tendency can be seen for the cracks of the concave and triangular shape. From these one can draw a conclusion on the shape of the crack propagation. For the quarter circular crack front will tend to flatten from quarter circular to triangular shape. For the case of the concave front, thecrack would also "end up" as triangular. From these considerations, one can conclude that the triangular shape of the front is of the most stable.

4. CONCLUSIONS

The propagation from a right angle of the corner interface crack along the joint plane between the two plates made of different materials was here analyzed. The three different shapes of the crack front were considered: concave, triangular and quarter circular shape.

Results are presented for variations of the normalized stress intensity factors of all the modes, the normalized phase angle and normalized energy release rate, in terms of angle θ, which measures the distance from the symmetry plane.

Based on results for all the three shapes of the crack front one can notice the difference in the stress field that exists for all the three shapes of the crack front close to the free surface. Reasons for such a behavior can be explained by the change of the singular field around the crack tip at the free surface, since it is not of the $1/\sqrt{r}$ form.

Another conclusion, drawn from the energy release rate variations, is that the triangular shape of the crack front is the most stable one.

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NOMENCLATURE

a	Crack length
r	Polar coordinate
E	Young modulus
G	Energy release rate
K_I	Stress intensity factor for Mode I crack propagation
K_{II}	Stress intensity factor for Mode II crack propagation
K_{III}	Stress intensity factor for Mode III crack propagation
K_1,K_2	In-plane stress intensity factors for the interfacial crack
L	Characteristic length
R	Distance from the crack front
R_c	Crack curvature radius

Greek symbols

α, β	Dundurs parameters
ε	Oscillatory index
θ, □	Polar coordinates
μ	Shear modulus

ν	Poisson ratio
σ	Normal stress
ψ, φ	Phase angles
ε	Oscillatory index
σ	Stress

Superscripts

I	Tensile force
II	In-plane shear force
III	Anti-plane shear force

АНАЛИЗА ТРОДИМЕНЗИОНАЛНИХ ИНТЕРФЕЈСНИХ ПРСЛИНА ИЗ УГЛА

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У овом раду је анализиран раст интерфејсне прслине из угла две спојене дугачке плоче које су направљене од различитих материјала. Анализирани су различити случајеви облика фронта прслине и то: конкавни, троугаони и конвексни (облик четвртине круга).
Приказани су резултати промене нормализованог фактора интензитета напона за сва три Мода оптерећења прслине, као и нормализованих фазних углова и нормализоване брзине ослобађања енергије, а све у функцији угла који мери растојање од равни симетрије.