Design of an H∞ PI controller with given relative stability and its application to the CSTR problem

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Abstract

H_∞ control theory has achieved a very high level, which is not followed by appropriate applications in industry. Some reasons are: synthesis of H_∞ controllers is complex and based on the numerical Nevanlinna-Pick procedure, high order of controllers (higher than the order of the process) and sensitivity to deviation of controller coefficients (fragile controller). On the other hand, PI (PID) controllers are still dominant in industry, which places the problem of design of fixed structure controllers at the forefront. The mentioned problem is very difficult. This paper proposes a simple interactive procedure for design of H PI controllers with the presence of constraints (given the relative stability) based on D-decomposition. The catalogue of responses to references, suppression of disturbances and minimum of H_∞ criteria of control is created. Selection of controllers, based on the catalogue, is left to the user. The application of H_∞ PI controllers to a CSTR (continuous stirred-tank reactor) is demonstrated.

Keywords: *H_∞ control, characteristics of H_∞ controller, low-order controllers, H_∞ PI controller, CSTR.*

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1. INTRODUCTION

Control theory is present in all fields of modern life. Applications are found in regulation of chemical processes, communication computer networks, robots, air-conditioning processes, mechatronic systems and many others. Classical control theory [1-2] solved the problem of robustness in design of controllers. The optimal control theory [3-4], in which optimality is guaranteed for precisely specified conditions, does not tolerate possible changes of those conditions. The initial enthusiasm for the use of optimal theory waned when it was confronted with practical applications since possible deviations from the optimal conditions considerably degraded performances of feedback systems. Neoclassicism in control theory was formed at the beginning of the eighties in the last century and it aimed at fusing optimality and robustness [5]. Intensive research resulted in a respectable theory of robust control systems. For example, a robust version of the Pontryagin's maximum principle was obtained [6]. Special attention was paid to a class of robust controllers known as H_∞ controllers [7-9]. These controllers are based on the theory of infinite-dimensional Hardy spaces [10].

H_∞ control theory is extremely complex and only experts in this field can deal with its possible applications. Main problems in this theory are as follows:

a) synthesis of H_∞ controllers is based on a complex procedure (Nevanlinna-Pick procedure [11-12]);
b) in the controller design, it is necessary to select weight functions in the frequency domain, which is a non-trivial task;
c) the H_∞ controller has a high order (higher than the order of the system), which is a consequence of the fact that the process transfer function is multiplied by the weight transfer function;
d) H_∞ controller coefficients have large dispersion of values (coefficients can differ by many orders of magnitude);
e) it was shown that H_∞ controllers (due to the property d)) stop being optimal or even stop stabilizing the system as a consequence of implementation of numerical values of controller coefficients, which can widely differ, causing errors; thus such controllers are termed fragile controllers [13].

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In the process industry, the application of PI (PID) controllers is still dominant. Different methods for synthesis of these controllers have been proposed. In [14], the possibility of synthesis of a robust PID controller by using the H_∞ loop shaping technique was considered. The *Vinnicombe* metric was used for describing the uncertainties of the model [15]. In [16], the approximate H_∞ loop shaping technique was used for synthesis of a robust PID controller. Synthesis of a robust PID controller based on the combination of the QFT (quantitative feedback theory) methodology [18-19] and the convex-concave procedure [20] has been described recently [17].

In a general case, design of fixed structure controllers (low-order controllers) is a very difficult problem. One of the possible approaches for solving this problem is the application of graphical methods (D-decomposition). Bases of this method were set in references [21-23]. The essence is to determine the area in the plane in which the system with closed feedback is stable. Parameters of the PI controller are determined by selecting a point in the plane. The method was applied to the design of PI controllers for hydraulic systems with long lines [24], as well as to the design of controllers for the TITO (two inputs – two outputs) system, which describes a distillation column (Wood-Berry model) [25]. The monograph [26] is dedicated to this type of problems.

This paper considers the design of H_∞ PI controllers by using a graphical method based on an interactive procedure. The initial idea presented previously [27] is expanded in the present work by introducing a constraint (relative stability) in the controller design. A procedure for creating a catalogue of responses to the Heaviside reference, the suppression of disturbances in the form of Heaviside function and the minimum of criteria of control for the selected network of points at which PI controllers are designed, is also proposed. The proposed methodology allows extremely simple interactive design of H_∞ controllers, where controller coefficients have a physical meaning. The problem of nominal performances is considered and it is shown that the proposed method provides the controllers that are considerably superior to standard H_∞ controllers.

Modern chemical industry is highly automated. A typical process in chemical industry is operation of a CSTR [28]. The mathematical model of this process is nonlinear, but its linearized variant is used in practice [29]. It is possible, by using methods of differential geometry to linearize a nonlinear model by feedback. In this paper we use the model proposed in [30], which is described by a linear system with a delay. The delay is approximated by the second-order Padé approximation. The designed H_∞ PI controller provides nominal performances of the system.

Great attention has been paid to the control of CSTR processes and different approaches have been developed such as the predictive control strategy [31] and the Hammerstein model [32], which yielded a robust adaptive controller.

The main contribution of the present paper is related to the design of H_∞ fixed structure controller as follows.

(i) Design of H_∞ PI controllers, using D-decomposition in the presence of constraints, which provides nominal system performance. The procedure has a general character and is applicable, without modification, to stable, unstable, non-minimum phase and integrator systems without and with time delay.

(ii) Creating a directory of responses to a reference and a disturbance and minimum of optimal criteria that allows, through an interactive mode, the choice of a suitable controller.

(iii) The proposed method is the basis for H_∞ PI controller design that ensures robust system performance (simultaneously providing nominal performance and robust system stability).

The paper is organized in the following way. Section 2 presents two results from the literature and they are analyzed in detail. Section 3 shows a short description of CSTR processes. It also introduces the Padé approximation for delay and considers possibilities of different approximations. Section 4 presents a new method of design of H_∞ PI controllers. The last section, Section 5, sums up the main results of the paper and perspectives for further research in this field.

2. DESIGN OF H_∞ STANDARD CONTROLLERS AND FIXED STRUCTURE CONTROLLERS

In this part of the paper we shall consider two published cases of H_∞ controllers in detail. Firstly, we shall briefly describe the result from [7, p. 80]. Now we shall introduce several terms connected with infinite-dimensional Hardy spaces [10].

The Hardy space H_∞ consists of all complex-valued functions $F(s)$ of a complex variable s, which are analytic and bounded in the open right half-plane, $\mathrm{Re}(s) > 0$. Bounded means that there is a real number b, such that

$$\left|F(s)\right| \le b, \quad \mathrm{Re}(s) > 0 \tag{1}$$

The least such bound b is the H_∞ - norm of F

$$\left\|F(s)\right\|_\infty = \sup\left\{\left|F(s)\right| \mathrm{Re}(s)>0\right\} \tag{2}$$

The important class of the function F is real-rational functions, *i.e.* rational functions with real coefficients. We denote the subset of an H$_\infty$ consisting of real-rational functions by RH_∞. If $F(s)$ is a real-rational function, then $F\in RH_\infty$ if and only if F is proper ($|F(\infty)| < \infty$) and stable (F has no poles in the closed right half-plane, Re(s) ≥ 0). By the maximum modulus theorem, it is possible to re-plane the open right half-plane in (1) by the imaginary axis.

$$\left\|F(s)\right\|_\infty = \sup\left\{\left|F(j\omega)\right| \omega\in R\right\} \tag{3}$$

Based on the relation (3), the norm of the system can be defined

$$\left\|G(s)\right\|_\infty = \sup_\omega\left\{\left|G(j\omega)\right|\right\} \tag{4}$$

where $G(s) \in RH_\infty$ is the transfer function of the control system.

The standard problem of control is presented in the Figure 1.

In Figure 1, $G(s)$ and $C(s)$ represent transfer functions of the system and controllers, respectively. In the block diagram, w is an exogenous input, which typically consists of a reference signal, disturbance and measurement noise; u is the control signal; z is the controlled output (typically, it is a tracking error (difference between the measured output y and the reference signal r)).

A specified form of the system presented in Figure 1 is shown in Figure 2 [7].

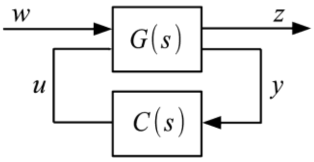

Figure 1. Standardized block diagram of the control system: G(s) *transfer function of the system, C(s) transfer function of the controller, w exogenous input, u control signal, z controlled output, and y measured output*

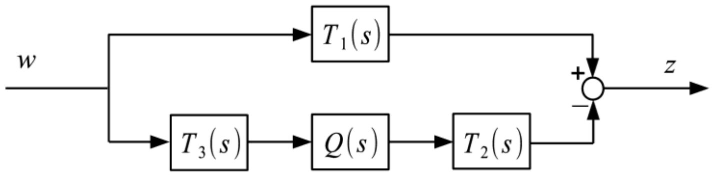

Figure 2. System in the model-matching form: w is an exogenous input, z is the controlled output, T_1(s), T_2*(s) and* T_3(s) *are given transfer functions and* Q(s) *is controller transfer function*

In Figure 2, the transfer function $T_1(s)$ represents a model, that should be matched by the cascade $T_2(s)$ - $Q(s)$ - $T_3(s)$ of three transfer functions $T_2(s)$, $T_3(s)$ and $Q(s)$. Here, $T_i(s)$ (i =1, 2, 3) are given and the controller $Q(s)$ is to be designed.

Now we shall introduce two functions, which play the key role in synthesis of H$_\infty$ controllers [33]. The first is the sensitivity function:

$$S(s) = \frac{1}{1 + C(s)G(s)} \tag{5}$$

which represents the transfer functions from the reference signal of the system to the regulation error and which is close to zero for low frequencies, and equals one for high frequencies. The second is the complementary sensitivity function (the transfer function from the reference signal to the output of the system):

$$T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)} \tag{6}$$

which is equal to one for low frequencies, and close to zero for high frequencies. These functions are related as:

$$S(s) + T(s) = 1 \tag{7}$$

By using the sensitivity function, following performances can be specified

$$\left|S(j\omega)\right| < \varepsilon \text{ for all } \omega \text{ in } [0,\ \omega_1] \tag{8}$$

where ε is a given number smaller than 1. Let us introduce the function $\varphi(j\omega)$ in the following way

$$\varphi(j\omega) = \begin{cases} 1, & \left|\omega\right| \le \omega_1 \\ 0, & \left|\omega\right| > \omega_1 \end{cases} \tag{9}$$

Now the relation (8) can be written in the form:

$$\left\|\varphi(j\omega)S(j\omega)\right\|_\infty < \varepsilon \tag{10}$$

Let us introduce the approximation for $\varphi(j\omega)$ in the form of the transfer function $W(s) \in RH_\infty$. The criterion given by the equation (10) now becomes

$$\left\| W(j\omega)S(j\omega) \right\|_{\infty} < \varepsilon \tag{11}$$

The criterion for design of H∞ controllers is obtained based on the relation (11):

$$J = \sup_{0 \le \omega \le \omega_1} \left| W(j\omega)S(j\omega) \right|, \quad J < \varepsilon \tag{12}$$

The control signal is obtained as a solution of the problem:

$$\min_C J = \min_C \sup_{0 \le \omega \le \omega_1} \left| W(j\omega)S(j\omega) \right| \tag{13}$$

In [7], the following non-minimum phase system is considered:

$$G(s) = \frac{(s\text{-}1)(s\text{-}2)}{(s+1)(s^2+s+1)} \tag{14}$$

where it is adopted that $\varepsilon = 0.1$, $\omega_1 = 0.01$ rad/s.

The weight function is defined as:

$$W(s) = \frac{(s+1)}{(10s+1)} \tag{15}$$

By applying the model-matching methodology (see Fig. 2), which is equivalent to the strategy presented by the relation (12) and which is based on the Nevanlinna-Pick numerical procedure, the transfer function of the controller $C(s)$ is obtained:

$$C(s) = 0.614 \frac{(s+0.3613)(s+1)(s^2+s+1)}{(s+0.004698)(s+0.528)(s^2+5.612s+9.599)} \tag{16}$$

Based on the equations (14) and (16), it can be seen that the order of the controllers is higher than the order of the process. One of the reasons is the application of the weight transfer function $W(s)$.

The minimum of the criterion J (relation (12)) is obtained in the following way:

```
% Frequency range
w = [0:0.0001:0.01];
% Definition of the variable s
s = zpk('s');
% Weight function
W = (s+1)/(10*s+1);
% Process
G = (s-1)*(s-2)/(s+1)/(s^2+s+1);
% Controller
C = 0.6114*(s+0.3613)*(s+1)*(s^2+s+1)/(s+0.004698)/(s+0.528)/(s^2+5.612*s+9.599);
% Sensitivity function
S = 1 / (1 + G*C);
% Amplitude and phase for the given frequency range
[amp, phase] = bode(W*S, w);
% Determination of the maximum amplitude and the corresponding
% frequency for the given range
J = max(amp)
w = w(amp==J)
```

The minimum value of the criterion (12) is 0.1191. The physical interpretation of this program is given by means of Bode diagrams and presented in Figure 3.

The control system, shown in Figure 4., described by the equations (14) and (16) is considered. The behaviour of the system is presented in the Figure 5.

In the continuation we shall consider the possibility of design of H∞ controllers by applying D-decomposition (H∞ PI controller). The transfer function of PI controllers is given as:

$$C_{PI}(s) = K_p + \frac{K_i}{s} \tag{17}$$

where K_p and K_i are proportional and integral gains, respectively. For the systems (14) and (17), the area of absolute stability is given in the Figure 6.

The point A_{PI} from Figure 6 is determined by the controller $C_{PI}(s)$. In reference [27], it was chosen that the point A_{PI} was selected by the coordinates (- 0.04747, 0.1328) in such a way that the structure of the PI controller is given by:

$$C_{\mathrm{PI}}(s)=-0.04747+\frac{0.1328}{s} \tag{18}$$

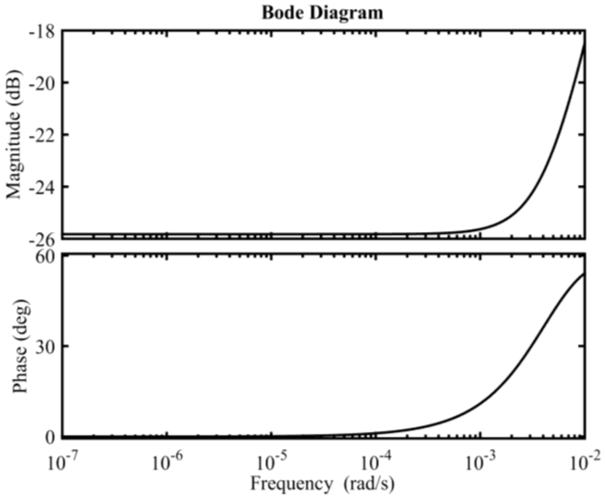

Figure 3. Calculation of min J *by means of Bode characteristics*

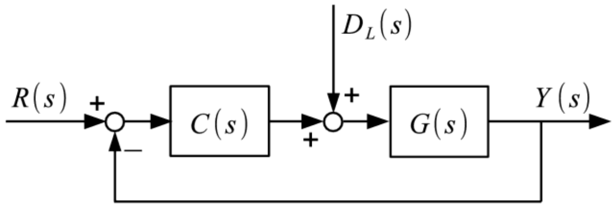

Figure 4. Feedback system: R(s) *- reference signal,* D_L(s) *– load disturbance,* Y(s) *– controlled output,* C(s) *– controller transfer function, and* G(s) *- plant transfer function*

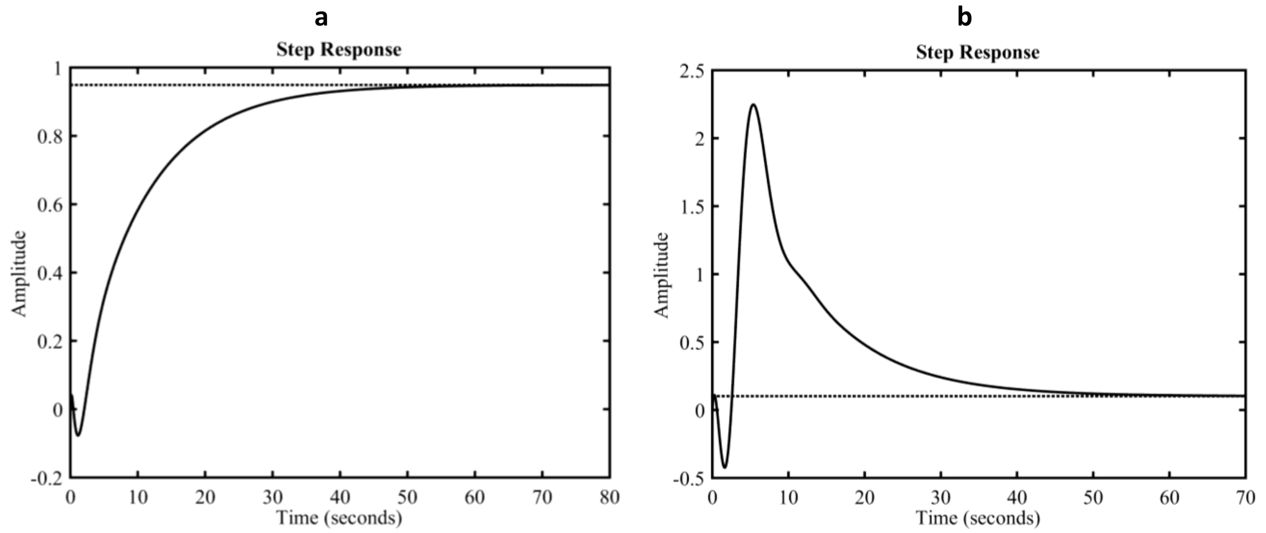

Figure 5. Characteristics of the control system defined by the relations (14) and (16): a) response to the Heaviside reference R(s) *at* (D_L(s)= 0)*;b) suppression of the load disturbance* D_L(s) *is in the form of the Heaviside fu at* (R(s)= 0)

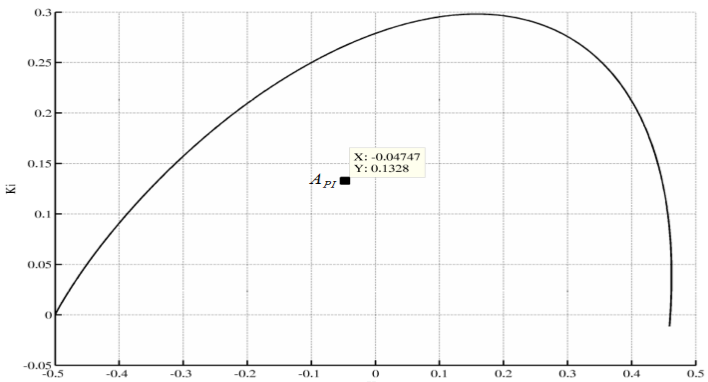

Figure 6. Area of absolute stability for the system described by the equations (14) and (17): K_p *and* K_i *are proportional and integral gain, respectively*

By using the relations (14) and (17), the sensitivity function is determined as:

$$S(s)=\frac{1}{1+C_{\mathrm{PI}}(s)G(s)} \tag{19}$$

For the function described by the equation (19), the minimum of the criterion based on the equation (12) was required. The procedure was realized by the following program:

```
% Frequency range
w = [0:0.0001:0.01];
% Definition of the variable s
s = zpk('s');
% Weight function
W = (s+1)/(10*s+1);
% Process
G = (s-1)*(s-2)/(s+1)/(s^2+s+1);
% Controller
C = -0.04747 + 0.1328/s;
% Sensitivity function
S = 1 / (1 + G*C);
% Amplitude and phase for the given frequency range
[amp, phase] = bode(W*S, w);
% Determination of the maximum amplitude and the corresponding frequency
% for the given range
J = max(amp)
w = w(amp==J)
```

The minimum value of the criterion (12) is 0.0373. It is seen that, in comparison with the standard H∞ controller, a value of the function *J* smaller by an order of magnitude is obtained with the H∞PI controller. The quantitative behaviour of the controller is shown in the Figure 7.

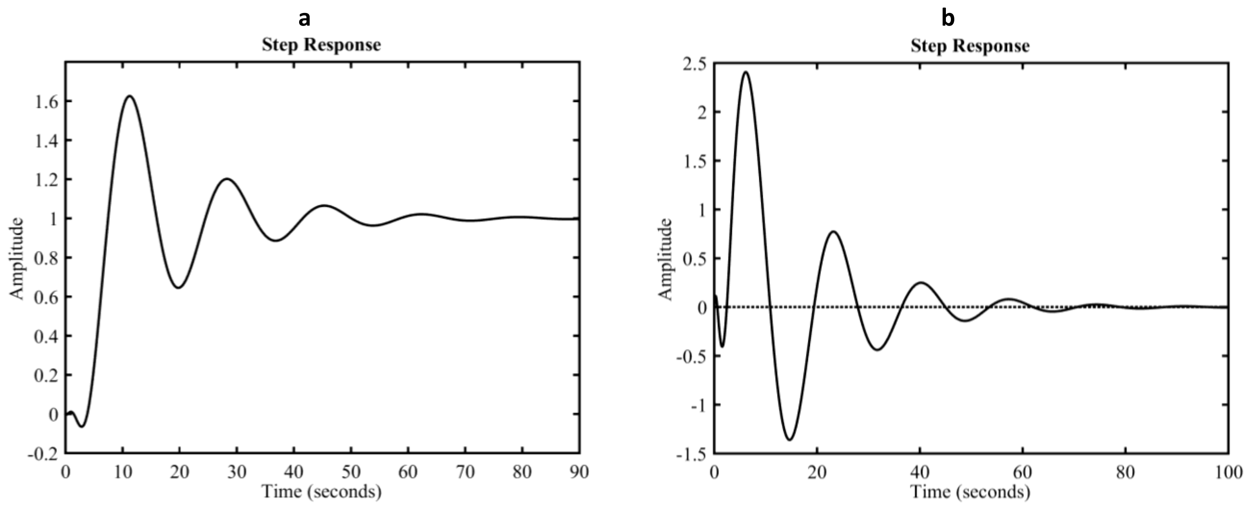

Figure 7. Characteristics of the control system defined by the relations (14) and (17): a) response to the Heaviside reference R*(s) at (*DL*(s)= 0); b) suppression of the load disturbance* DL*(s) is in the form of the Heaviside function at (*R*(s)= 0).*

Detailed consideration of two examples from the literature will serve as a basis for design of H∞PI controllers with the presence of constraints.

3. MATHEMATICAL MODEL OF A CSTR

Continuous stirred-tank reactors (CSTRs) have widespread application in industry and embody many features of other types of reactors [28].

Consider a simple liquid-phase, irreversible chemical reaction where chemical species A reacts to form species B. The reaction can be written as

$$A \rightarrow B \tag{20}$$

It is supposed that the reaction rate follows the first-order kinetics with respect to the component A

$$r = kc_A \tag{21}$$

where r is the reaction rate per unit volume, k is the reaction rate constant and c_A is the molar concentration of species A. For single phase reactions, dependence of the constant k on temperature, T, is given by the Arrhenius equation:

$$k = k_0 \exp\left\{-\frac{E}{RT}\right\} \tag{22}$$

where k_0 is the frequency factor, E is the activation energy and R is the gas constant. The parameters k_0 and E are determined by fitting the experimental data. The last two equations can be considered to be semi-empirical relations.

The schematic diagram of the CSTR is shown in Figure 8.

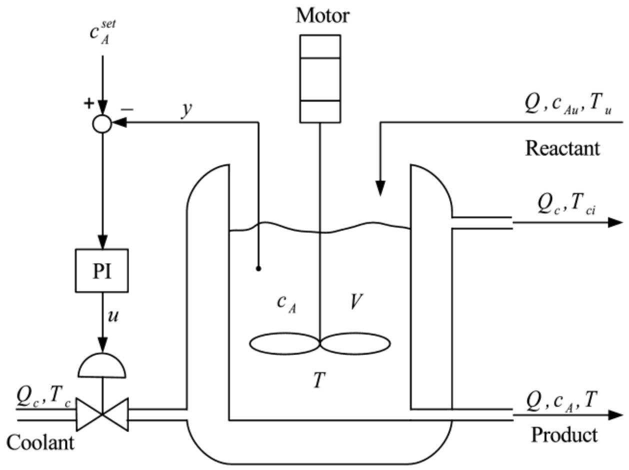

Figure 8. CSTR with cooling jacket:
c_{Au} – *molar concentration of the reactant in the feed CSTR,*
c_A – *molar concentration of the reactant in the CSTR and at the outlet,*
c_A^{set} - *set point for the molar concentration c_A in the CSTR,*
T_u - *temperature of the feed,*
T_c - *temperature of the coolant,*
T_{ci} - *output temperature of the coolant,*
T - *temperature in the CSTR,*
Q – *volumetric flow rate of the reactant and product,*
Q_c – *volumetric flow rate of the coolant,*
V – *volume of the reactor,*
y - *measurement of the concentration,*
u - *input signal,*
PI – *PI controller .*

The CSTR model development [34-35] is based on three assumptions:

(i) the CSTR is perfectly mixed,
(ii) densities of the feed and product streams are equal and constant,
(iii) liquid volume V in the CSTR is kept constant.

The mathematical model of CSTR has the form [29-30]:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1.308}{(13.515s+1)(6.241s+1)} e^{-4.896s} \tag{23}$$

The goal of the control systems is to control the CSTR composition (y) by manipulating the cool rate through the control signal (u). Different control strategies are presented in literature [31-32,34].

In this paper we shall approximate the element of delay by a second-order Padé approximation. The general form of Padé approximation, according to [37-38], is

$$e^{-T_d s} = \left(\frac{1 - \frac{sT_d}{2n} + \frac{1}{3}\left(\frac{sT_d}{2n}\right)^2}{1 + \frac{sT_d}{2n} + \frac{1}{3}\left(\frac{sT_d}{2n}\right)^2} \right)^n, T_d = -4.896 \tag{24}$$

In this paper, the approximation for n = 2 is used and, in that case, the transfer function or the process is:

$$G(s) = \frac{1.308}{(13.515s+1)(6.241s+1)} \cdot \left(\frac{1 - \frac{sT_d}{2n} + \frac{1}{3}\left(\frac{sT_d}{2n}\right)^2}{1 + \frac{sT_d}{2n} + \frac{1}{3}\left(\frac{sT_d}{2n}\right)^2} \right)^2, T_d = -4.896 \tag{25}$$

The application of Padé approximation for delay is widely used in chemical industry [38].

Note 1. Possible approximations for delay are:
A) Laguerre shift:

$$e^{-T_d s} = \lim_{n\to\infty}\left(\frac{1-\frac{sT_d}{2n}}{1+\frac{sT_d}{2n}}\right)^n \tag{26}$$

This type of approximation is applied in the robust control theory [37].
B) Kautz shift:

$$e^{-T_d s} \cong \left(\frac{1-\frac{sT_d}{2n}+\frac{1}{2}\left(\frac{sT_d}{2n}\right)^2}{1+\frac{sT_d}{2n}+2\left(\frac{sT_d}{2n}\right)^2}\right)^n \tag{27}$$

It was analytically shown that this type of approximation is more accurate than the Laguerre one [37].

4. DESIGN OF H$_\infty$ PI CONTROLLERS BY APPLYING D-DECOMPOSITION

We shall now present the procedure for design of H$_\infty$PI controllers, which will be used for regulating CSTR processes. Unlike the controllers in the previous chapter, here we shall introduce a constraint that relates to the relative stability of the system. The purpose of the constraint is to specify, in the s-plane, an area with complex-conjugate roots of the characteristic equation of the system. The constraint of this type has the form [39]:

$$0.7 \le \xi \le 1 \tag{28}$$

where ξ is the damping factor. The relation (28) defines the maximum allowed degree of oscillation of the transient mode, graphically shown in Figure 9.

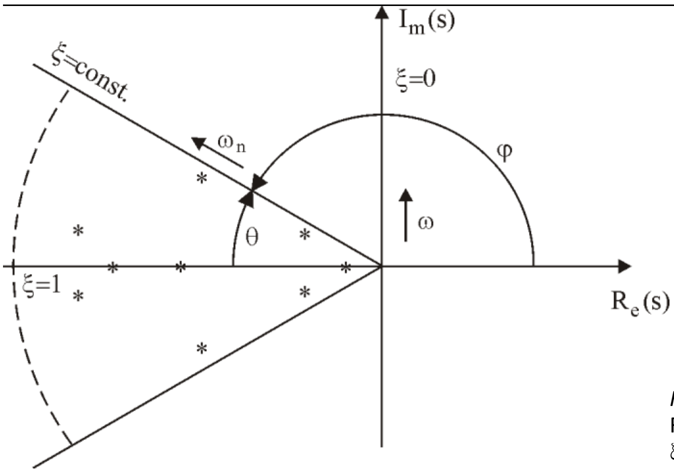

Figure 9. Area of the relative stability of the system: R_e real axis, I_m imaginary axis, ω_n natural frequency, ξ damping factor, θ and φ are angular coordinates

Having in mind the constraint given by the equation (28), the following criterion is minimized:

$$J = \sup_{0\le\omega\le\omega_1} \left\|S(j\omega)\right\| \tag{29}$$

Let us note that, instead of the constraint (28), a constraint that relates to the settling time may also be introduced. It is also possible to introduce both criteria simultaneously [24].

The procedure of design of H$_\infty$PI controllers is as follows:

1. select the area of relative stability of the system in the *s* plane;
2. choose the points on the curve ξ = const. and determine the parameters K_p and K_i of the controller for each point;
3. based on the transfer function of the $C(s)$ controller (obtained on the basis of the parameters K_p and K_i) and the transfer function of the $G(s)$ process, calculate the sensitivity of the system $S(s)$;

4. compute the minimum of the criterion J_{min} based on the equation (13), by applying MATLAB (Robust Control Toolbox);
5. create a catalogue which contains:
   a. graphical presentation of responses to the reference,
   b. graphical presentation of suppression of disturbances,
   c. minimum of the criterion J,
6. in accordance with engineering reasoning, choose the PI controller which provides nominal performances of the system.

The proposed procedure is an interactive graphical procedure, which is simple and enables engineers in industry to use the latest accomplishments in the automatic control theory in a comprehensible and easy way. In this case, it is H$_\infty$ optimization.

The advantages of the proposed procedure are as follows:

a) simple structure of the controller (PI controller), which allows physical interpretation of its parameters;

b) superiority of thus obtained controller in comparison with the original H$_\infty$ controller (which has a high order), which was shown for the case when there are no constraints (28);

c) elimination of the need to determine the weight function $W(s)$, which is a non-trivial problem [9].

It is necessary to note that the introduction of constraints to the criterion of control reduces performances of the system [40].

Note 2. This paper considers nominal performances of the system:

$$\|W_1(s)S(s)\|_\infty < 1$$

It is the first step in consideration of H$_\infty$ optimization. The next important problem is the robust stability of the system provided by the following criterion:

$$\|W_2(s)T(s)\|_\infty < 1$$

The final aim is robust performances of the system for which the criterion has the form:

$$\big\||W_1(s)S(s)| + |W_2(s)T(s)|\big\|_\infty < 1$$

The last criterion can be interpreted as simultaneous realization of nominal performances and robust stability of the system.

Simulation results were obtained for the example of the CSTR model. The parameter plane for the damping factor value $\xi = 0.7$ is shown in Figure 10.

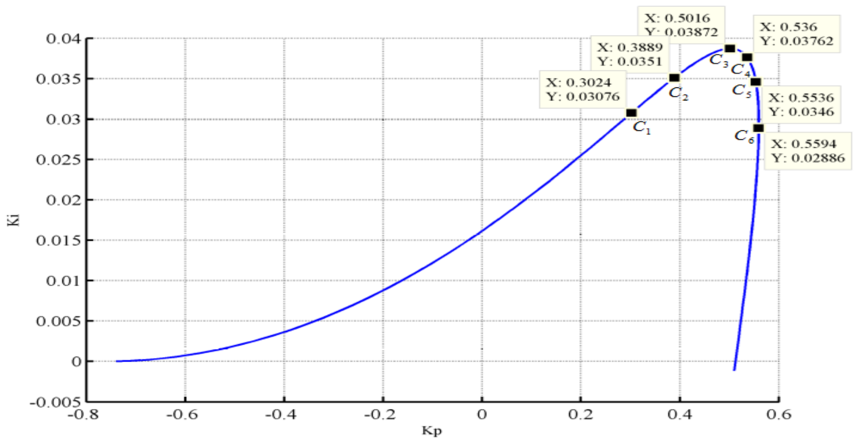

Figure 10. Parameter plane with the catalogue of PI controllers for ξ = 0.7: K_p and K_i are proportional and integral gain, respectively

Figure 10 shows the formation of the catalogue of 6 (C_1-C_6) PI controllers with the appropriate step. The minimum of the J criterion is calculated and the catalogue of responses per reference and suppression of the load disturbance is established for each of these controllers. The results are presented in Figures 11 and 12.

For the created catalogue of responses of the controllers from C_1 to C_6, the minimum of the J criterion was determined based on the appropriate program so that the result is:

J= min([J1,J2,J3,J4,J5,J6])= min([0.9981, 0.9978, 0.9976, 0.9976, 0.9978, 0.9982]) =(J3,J4)= 0.9976

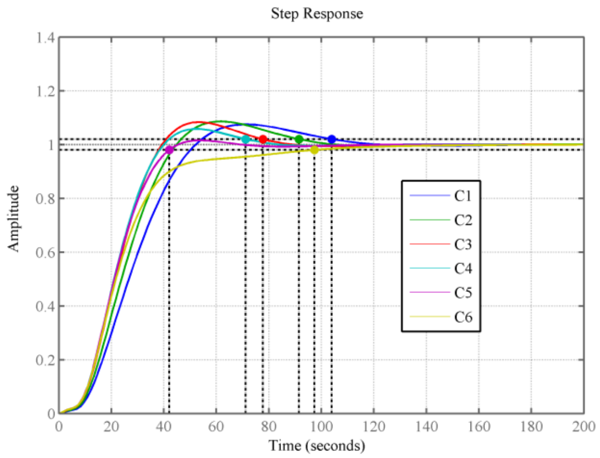

Figure 11. Catalogue of responses per reference for the controllers C_1-C_6

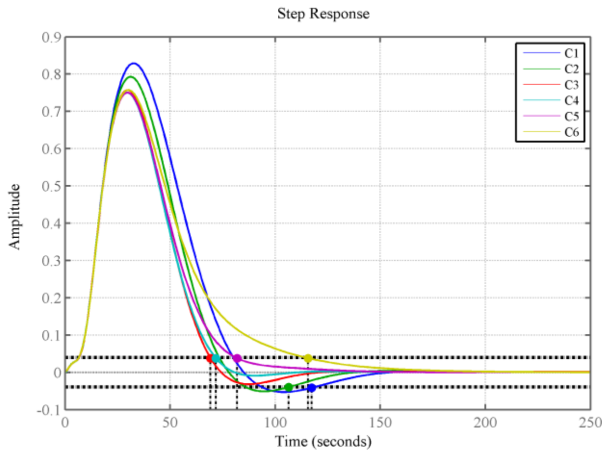

Figure 12. Catalogue of responses per suppression of the load disturbance for the controllers C_1-C_6

Based on this result, it can be seen that two controllers fulfil the condition of the criterion minimum for the catalogue of controllers thus created. Hence, it is obvious that this catalogue with the corresponding smaller step can cover a large number of controllers in the vicinity of the point with the maximum integral gain. It is necessary to determine a part of the curve where all controllers fulfil the condition of the minimum of the J criterion, surely up to a certain order.

Figure 13 presents the parameter plane for the damping coefficient value ξ = 0.7 where all controllers fulfil the minimum of the J criterion with the value of J=0.9976.

From Figure 13, it can be seen that a new, narrower catalogue of PI controllers with the range of proportional gain from K_{pmin} *to* K_{pmax} can be determined, and that all these controllers fulfil the minimum of the J criterion.

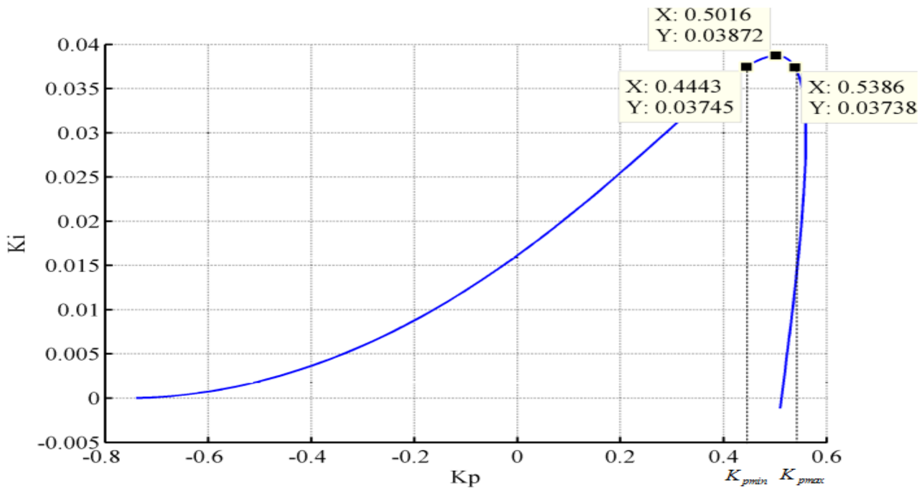

Figure 13. Part of the curve for ξ = 0.7 where all controllers fulfil the minimum of the J criterion: K_p and Ki are proportional and integral gain, respectively

Figures 14 and 15 present results per reference and suppression of the load disturbance for the catalogue of the controllers C_1-C_3.

Based on the results presented in Figures 14 and 15, it can be seen that the controller C_3 (K_{pmax}, K_i) provides a somewhat better result from the aspect of settling time and overshoot if compared with the other controllers, which fulfil the minimum of the *J* criterion.

This procedure has a general character and is applicable without modification to stable, unstable, non-minimum phase and integrator processes with and without delay. It is interesting to observe that changing the time delay in the considered system affects the response of a closed loop. Three time delays were considered for the given process and the results of the controller parameters for all three time delays are given in Table 1.

Table 1. Controller parameters for three different time delays

T_d = 4.896 s	$T_{d1} = T_d$	$T_{d2} = 1.5T_d$	$T_{d3} = 2T_d$
Kp	0.5386	0.3924	0.3408
Ki	0.03738	0.02855	0.02429

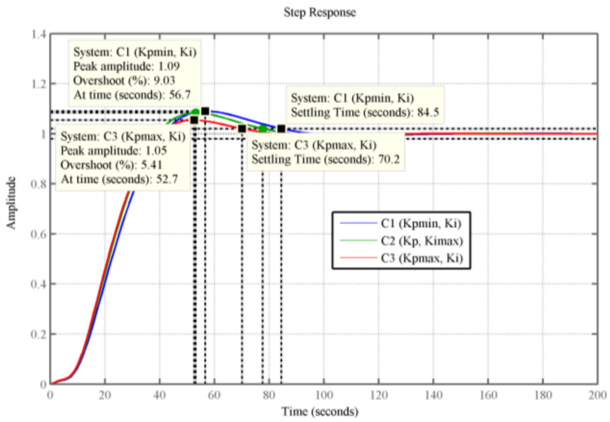

Figure 14. Catalogue of responses per reference for the controllers C_1-C_3

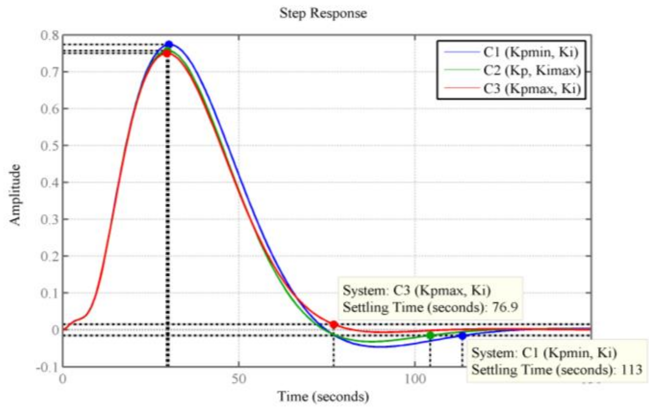

Figure 15. Catalogue of responses per suppression of the load disturbance for the controllers C_1-C_3

Results of the impact of increased time delays on the closed-loop response are shown in Figure 16.

From Figure 16 it can be seen that when the time delay increases, the system slows down, which is logical, but the design procedure itself provides a good result from the aspect of changing the time delay. Comparison of the proposed design procedure with Ziegler-Nichols and Tyreus-Luyben method is shown in Figure 17, based on the controller parameters given in Table 2.

As shown in Figure 17, the proposed method provides a much better result than the two methods most commonly used as a starting point for controller tuning in industrial processes.

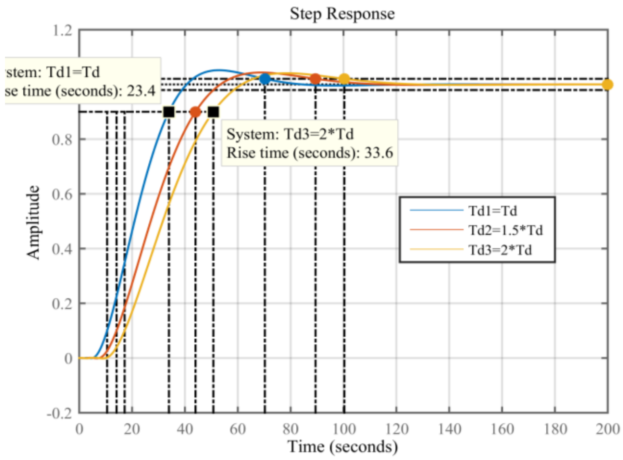

Figure 16. The system response for three different time delays

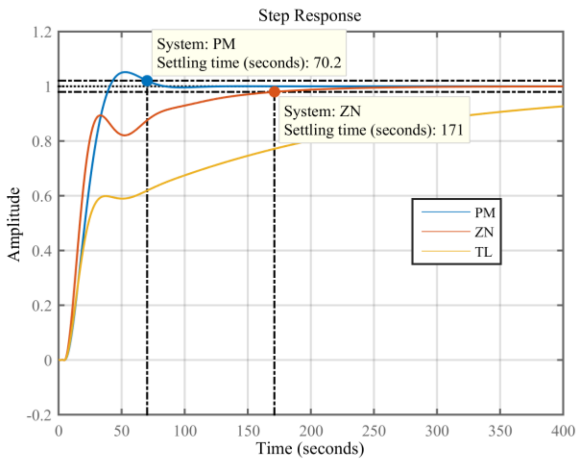

Figure 17. Comparison of system responses for the proposed method (PM), Ziegler-Nichols (ZN) and Tyreus-Luyben (TL) methods

Table 2. Comparative controller parameters

Method	K_p	K_i
ZN (Ziegler-Nichols)	0.985	0.025
TL (Tyreus-Luyben)	0.6656	0.0067
PM (Proposed method)	0.5386	0.03738

5. CONCLUSIONS

This paper proposes a simple interactive procedure, based on D-decomposition, for the design of H$_\infty$PI controllers. The design of controllers includes a constraint in the form of given relative stability, which is the indicator of oscillation of the transient process. The catalogue of responses to the reference, suppression of the load disturbance and minimum of the H$_\infty$ criterion of control is created through simulation. The controller, which is estimated to be the most adequate for the given process, is selected based on the catalogue. The obtained controller is of a much lower order than the original H$_\infty$ controller which is obtained by the application of very complex numerical procedures. The proposed procedure provides a better result as compared to some other procedures that are often used for controller design in industry. Also, the parameters of H∞PI controllers have a physical meaning.

This paper considers the problem of nominal performances of the system. Further extension of the results will be connected with robust stability and robust performances of the system.

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SAŽETAK

PROJEKTOVANJE H$_\infty$ PI REGULATORA SA ZADATOM RELATIVNOM STABILNOŠĆU I NJEGOVA PRIMENA NA PRIM PROBLEM

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(Naučni rad)

Teorija H$_\infty$ upravljanja je dostigla vrlo visok nivo, koga ne prati odgovarajuća primena u industriji. Neki od razloga su: složena sinteza H$_\infty$ regulatora zasnovana na numeričkoj Nevenlina-Pik (engl. Nevanlinna–Pick) proceduri, visok red regulatora (veći od reda procesa) i osetljivost na odstupanje koeficijenata regulatora (engl. *fragile controller*). S druge strane, u industriji su i dalje dominantni PI (PID) regulatori, što u prvi plan postavlja problem projektovanja regulatora fiksne strukture. Navedeni problem je vrlo komplikovan. U ovom radu je predložena jednostavna interaktivna procedura za projektovanje H$_\infty$ PI regulatora uz prisustvo ograničenja (zadata relativna stabilnost) zasnovana na D-dekompoziciji. Formira se katalog odziva na referencu, potiskivanje poremećaja i minimuma H$_\infty$ kriterijuma upravljanja. Izbor regulatora, na osnovu kataloga, prepušta se korisniku. Demonstrirana je primena H$_\infty$PI regulatora na protočni reaktor sa idealnim mešanjem (PRIM). Dobijeni regulator projektovan ovom metodom je mnogo nižeg reda od originalnog H$_\infty$ regulatora koji se dobija primenom vrlo složenih numeričkih procedura.

Ključne reči: H$_\infty$ upravljanje, osobine H$_\infty$ regulatora, regulatori niskog reda, H$_\infty$ PI regulator, protočni reaktor sa idealnim mešanjem PRIM