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Effects of Introducing Dynamic Constraints for Buckling to Truss Sizing Optimization Problems

In this paper the effects of adding buckling constraints to truss sizing optimization for minimizing mass are investigated. Introduction of buckling testing increases the complexity of the optimization process as Euler buckling criteria changes with each iteration of the optimization process due to the changes in element cross section dimensions. The resulting models which consider this criteria are practically applicable. For the purposes of showing the effects of dynamic constraints for buckling, optimal parametric standard test models of 10 bar, 17 bar, and 25 bar trusses from the literature are tested for buckling and compared to the models with the added constraint. Models which do not consider buckling criteria have a considerable number of elements which do not meet buckling criteria. The masses of these models are substantially smaller than their counterparts which consider buckling.

Keywords: Truss, sizing optimization, Euler buckling, dynamic constraints, genetic algorithm.

1. INTRODUCTION

Truss sizing structural optimization problems found in most of the literature consider the use of only stress and/or displacement constraints. Very few studies consider the addition of buckling constraints along with the stress and displacement. The addition of such a constraint considerably increases the complexity of the problem. The exclusion of a buckling constraint results in a practically unusable structure, which would not meet operational requirements.

Most studies published on the subject of sizing structural truss optimization use a variety of standard test examples, which consider only static constraints. Hasancebi and Azad [1] created and verified adaptive dimensional search (ADS), a new meta-heuristic method, which updates search dimensional parameters in every iteration. They investigated the capabilities and potentials of ADS in structural optimization, and tested their method on various standard test examples of trusses using stress and displacement constraints. Cheng et al. [2] tested their new hybrid harmony search algorithm on six test problems with static constraints achieving very competitive results. Degertekin et al. [3] applied teaching-learning based algorithm to optimize truss structure sizing and compared their results to other meta-heuristic method results. Sizing optimization done by Mortazavi and Toğan [4] showed the method of hyperspheres and showed promising results in using this method for truss optimization. Discrete sizing optimization of steel trusses was approached by Kazemzadeh et al. [5] using guided stochastic search (GSS) as a design-driven heuristic approach and tested it on 10, 117, 130, 392, and 354 member truss structures. Authors in [6] tested their developed hybridized genetic algorithm on various standard test examples. Farshi and Alinia-ziazi [7], applied the method of centers of force formation to solve truss sizing optimization problems with exceptional results. Testing for use on these problems by their respective authors was also conducted using hybrid harmony search [8], and teaching learning based algorithm [9]. Many other researchers have tried and tested various heuristic methods [10-15] aiming to improve convergence and minimize optimal weight by modifying, adapting and merging methods.

The most commonly used methods in the field are heuristic methods, however non-heuristic optimization methods have been used to solve structural optimization problems as well [16,17], though due to the complexity of sizing problems, these methods do not always give global solutions.

Effective optimization methods are constantly being investigated by researchers to solve intricate sizing optimization problems. Very few studies consider the addition of buckling constraints along with stress and displacement [18, 19]. These studies, however, work with a combination of topology, or sizing optimizations. The effects of adding a buckling constraint to just sizing optimization has not been explored in previous works.

The addition of buckling constraints, as proposed by this paper, allow for practical application of truss struc– tural optimization results. The increased complexity of adding such a constraint significantly increases calcu– lation times. This paper aims to show the difference in optimization results for just truss sizing by comparing

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optimal results from the literature which do not consider buckling with the same examples using the authors' algorithm both with and without considering buckling to show validity of the algorithm and the influence of the added constraint.

2. PROBLEM FORMULATION

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Sizing optimization considers cross section geometrical parameters as variables. The objective function aims to find the cross section area combination, which would minimize the construction's weight, cost, etc. Many researchers put considerable effort to solve this problem investigating various optimization methods. For typical truss sizing optimization found in the literature, the minimum weight design problem can be defined as:

$$\begin{cases} \min W(A) = \sum_{i=1}^{i=n} \rho_i A_i l_i \text{ with } A = (A_1, \dots, A_n) \\ \text{subjected to} \begin{cases} A_{\min} \le A_i \le A_{\max} & \text{for } i = 1, \dots, n \\ \sigma_{\min} \le \sigma_i \le \sigma_{\max} & \text{for } i = 1, \dots, n \\ u_{\min} \le u_j \le u_{\max} & \text{for } j = 1, \dots, k \end{cases} \end{cases}$$

where *n* is the number of truss elements, *k* is the number of nodes, l_i is the length of the *i*th element, A_i is the area of the *i*th element cross section, σ_i is the stress of the *i*th element, u_i is displacement of the *j*th node.

2.1 Euler Buckling Constraint

Many optimal solutions to truss sizing optimization problems have small cross sections of elements subjected to large compression forces. It is hypothesised that these are weak points in the structure, therefore consequent effects of buckling should be tested during the optimization process to avoid unusable results. Since the Euler critical buckling load equation (3) considers cross sectional characteristics, and sizing optimization creates a new set of cross sections in each iteration for all elements, buckling needs to be checked for each iteration. The proposed Euler buckling constraint defined by Euler's critical load is given in the following expressions:

$$\left|F_{Ai}^{comp}\right| \le F_{Ki} \text{ for } i = 1, \dots, n \tag{2}$$

$$F_{Ki} = \frac{\pi^2 \cdot E_i \cdot I_i}{l_i^2} \tag{3}$$

where F_{Ai}^{comp} is the axial compression force, F_{Ki} is Euler's critical load, E_i is the modulus of elasticity, and I_i is the minimum area moment of inertia of the cross section of the of the *i*th element. The condition from equation (2) will be added to the existing constraints from equation (1).

As the buckling constraint changes with each itteration, this constraint is considered a dynamic constraints, and its addition drastically increases the complexity of the optimization problem. The addition of dynamic constraints complicates the optimization process, and requires the use of adequate methods.

2.2 Optimization

Optimization is the process of finding solutions from a group of alternative possible solutions. These solutions necessitate better characteristics of the construction, while at the same time decreasing invested effort and expended costs. The complex problem of truss sizing optimization is best conducted using heuristic optimi–zation. Heuristic methods are preferred when it comes to engineering problems due to their favourable characteristics, such as their ability to work with a large number of variables, overcoming local extremes, speed and efficiency of work, low threshold of needed facts about the problem in order to find a solution, etc.

For the purposes of this research Genetic algorithm (GA) is used. GA is a heuristic method for optimizing whose operation is based on mimicking natural processes [20]. The algorithm contains three basic operators: selection, crossover, and mutation (figure 1).

The process of transferring genetic information through generations is called selection. Crossover represents the process/operations between two parents, where an exchange of genetic information and new generations are made. A random change in the genetic structure of some individuals for overcoming early convergence is created by the mutation operator.



Figure 1. Genetic algorithm

Algorithm operation is based on survival of the fittest individuals through evolution that exchange genetic material. Selection ranks individuals in the population using values from the fitness function, which defines the ability/quality of the individual.

3. TEST EXAMPLES AND ANALYSIS

The most commonly used sizing optimization problems for 2D and 3D trusses are 10, 15, 17, 18, 25, 52, 72 bar, etc. For the purposes of this research, the 10, 17, and 25 bar truss standard test models were considered. In addition to testing optimal results of these models from the literature, models were optimized by the authors of this paper using genetic algorithm without the buckling constraint, and tested for buckling. The same genetic algorithm model was also made with the Euler buckling dynamic constraint to show the influence of the added constraint on the benchmark test models from [6]. As the benchmark models are given in English units, appropriate unit conversions were conducted in order to use SI units.

For the 10 bar truss the initial model bar and node layout is given in figure 2. This cantilever truss has 10 independent variables. The material of the truss elements is Aluminum 6063-T5 whose characteristics are: Young modulus 68947MPa, and density of 2.7g/cm³. Point loads are P_1 =444.82kN, P_2 =0kN in the first load case (LC1), and are P_1 =667.233kN, and P_2 =222.411kN in the second load case (LC2), as shown in figure 2. The model is limited to a maximal displacement of ±0.0508m of all nodes in all directions, axial stress of ±172.3689MPa for all bars, and minimum radius of all members is limited to 4.5225mm. Optimization results using GA for this model are shown in figure 5.



Figure 2. Initial 10 bar truss model

For the 17 bar truss the initial model bar and node layout is given in figure 3. For this example the material characteristics are: Young modulus 206842.719MPa, and density of 7.4g/cm³. A single point load of 444.82kN is applied in node 9, as shown in figure 2. Each bar cross section is an independent variable limited to a minimal radius of all members limited to 4.5225mm for full circular profiles. This example does not have a stress constraint, the only constraint is a displacement limitation for all nodes of ± 0.0508 m of all nodes in both x and y directions. Optimization results using GA for this model are shown in figure 6.



Figure 3. Initial 17 bar truss model

The 25 bar truss initial model bar and node layout is given in figure 4. The material of the truss elements is Aluminum 6063-T5, the same as for the 10 bar truss. This example has two load cases, which are given in table 1. This space truss has members cross sections grouped as follows: 1 (A₁), 2 (A₂ – A₅), 3 (A₆ – A₉), 4

 $(A_{10} - A_{11})$, 5 $(A_{12} - A_{13})$, 6 $(A_{14} - A_{17})$, 7 $(A_{18} - A_{21})$, 8 $(A_{22} - A_{25})$. The model is limited to a maximal displacement of ±0.00889m of all nodes in all directions, member stress limitations for bar groups are given in table 2, and minimum radius of all members is limited to 1.433mm. Optimization results using GA for this model are shown in figure 7.

Table 1. Lead conditions for 25 bar truss example.

Node	LC 1 components P_{x}, P_{y}, P_{z} [kN]	LC 2 components P_{x}, P_{y}, P_{z} [kN]
1	0, 20, -5	1, 10, -5
2	0, -20, -5	0, 10, -5
3	0, 0, 0	0.5, 0, 0
6	0, 0, 0	0.5, 0, 0

Table 2. Member stress limitation for the 25 bar truss.

Mambar groups	Compressive stress	Tensile stress		
Weinber groups	limitation [kN]	limit [kN]		
$l(A_l)$	241.951	40		
$2(A_2, A_5)$	79.9102	40		
$3(A_{6}, A_{9})$	119.314	40		
$4(A_{10}, A_{11})$	241.951	40		
$5(A_{12}, A_{13})$	241.951	40		
6 (A ₁₄₋ A ₁₇)	46.6017	40		
$7(A_{18}A_{21})$	47.9806	40		
8 (A22- A25)	76.4077	40		



Figure 4. Initial 25 bar truss model

The parametric models and optimization in this research are all done in Rhinoceros 5.0 software using Grasshopper, Galapagos optimization, and Karamba plugins. Files were created in this program for all three models. Galapagos optimization uses GA as its optimization method. Cross section parameters from the optimal models taken from the literature [6-9] are input into the same files, and the buckling conditions are checked for all bars. Various methods' results from the literature are compared to the GA used in this paper, for use both with and without the added constraint, to verify the created GA. All solutions which do not meet constraint criteria are penalized by assigning a large value.

4. RESULTS

Optimization was conducted according to the parameters set in the previous section using GA. For the same models optimization was repeated with the dynamic constraints for critical buckling load added to the same algorithm. Comparison of results from the literature, which use hybrid simulated annealing genetic algorithm (H-SAGA) [6], force method [7], hybrid harmony search (HSS) [8], and teaching-learning-based optimization (TLBO) [9], are given in tables 4 and 5 for 10 bar truss

load cases 1 and 2 respectively. Table 3 gives optimization results for the 17 bar, and table 6 for the 25 bar trusses. For bars that do not meet buckling conditions the values of their cross sections are given in bold. In the case of the 25 bar truss, in table 6, as the bars are grouped, the specific bars from each group, which do not meet buckling constraints are listed in bold.

Radius			GA with
of bar	H-SAGA[6]	GA	buckling
[mm]			constraint
1	56.996	53.7856	50.3976
2	4.646	21.0949	14.8867
3	49.693	53.0201	54.496
4	4.532	4.532	22.3092
5	40.818	45.1648	42.2716
6	33.705	26.9179	4.9622
7	49.316	45.359	43.5632
8	4.532	4.532	12.2252
9	40.421	36.2076	31.5096
10	4.532	27.1101	33.5869
11	28.914	35.3051	42.0052
12	4.532	17.6662	33.6485
13	34.166	34.3093	31.0825
14	28.693	31.0538	36.3276
15	33.866	28.2859	44.0668
16	4.532	21.496	46.5286
17	33.853	18.8083	34.9367
Weight [kg]	1169.705	1183.071	1507.665

Table 3. O	ptimization	results	of the	17	bar	truss

Table 4. Optimization results of the 10 bar for LC1.

Radius of bar [mm]	H-SAGA [6]	Force method [7]	HHS [8]	TLBO [9]	GA	GA with buckling constraint
1	78.99075	79.16929	83.06698	79.04962	77.561	48.7244
2	4.522325	4.531675	18.23964	4.531675	4.5317	40.9627
3	68.88255	69.03037	68.57669	69.08925	71.614	116.3289
4	55.7414	55.91287	54.00114	56.1776	60.994	70.7638
5	4.522325	4.531675	18.23964	4.531675	9.2417	4.8576
6	10.66253	10.6422	18.23964	10.86752	4.677	44.0836
7	39.06304	39.15876	40.45647	39.08921	36.948	73.4453
8	65.646	65.72367	68.57669	65.61781	69.1574	92.3201
9	66.2688	66.49282	67.21561	66.49838	66.7315	29.7619
10	1.430085	4.531675	18.23964	4.531675	4.5317	105.932
Weight [kg]	2294.568	2295.645	2490.556	2295.611	2327.910	4759.458

Table 5. Optimization results of the 10 bar for LC2.

Radius of bar [mm]	H-SAGA [6]	Force method [7]	HHS [8]	TLBO [9]	GA	GA with buckling constraint
1	69.05795	69.50916	79.14231	69.50473	69.781	45.217
2	4.522325	4.531675	4.531675	4.531675	4.5317	50.072
3	72.3275	72.0723	70.20441	70.84648	71.064	114.176
4	54.7631	52.41094	53.6195	54.52907	57.5154	81.026
5	4.522325	4.531675	4.531675	4.531675	4.5317	34.5492
6	20.06964	20.11268	10.13313	20.24092	20.1103	68.914
7	50.0844	50.47094	39.24546	50.32813	52.322	61.104
8	50.85815	51.3191	66.4474	51.04727	56.5379	102.37
9	64.50015	64.61476	66.4474	64.65225	64.193	48.84
10	4.522325	4.531675	4.531675	4.531675	4.5317	93.033
Weight [kg]	2120.738	2121.814	2298.5	2122.044	2165.659	5104.395

Table 6.	Optimization	results	of the	25 ba	r truss.
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	H-SAGA [6]		Force method [7] HHS		S[8] TLBO)[9]		βA	GA with buckling constraint	
Bar group	R [mm]	Buckling bars	R [mm]	Buckling bars	R [mm]	Buckling bars	R [mm]	Buckling bars	R [mm]	Buckling bars	R [mm]
1	1.43	n/a	1.43304	n/a	1.433042	1	1.433042	n/a	1.433	n/a	11.741
2-5	20.1555	2, 3,5	20.3774	2, 3,5	7.849092	2,3,5	20.62385	2,3,5	16.014	2, 3,5	34.1507
6-9	24.7595	6-8	24.6133	6-8	26.42398	6-8	24.64248	6-8	28.185	6-8	32.8214
10-11	1.43	10-11	1.43304	10-11	4.531675	10-11	1.433042	10-11	1.433	10-11	3.8506
12-13	1.43	12-13	1.69560	12-13	20.76674	12	1.433042	12-13	1.433	12-13	6.5588
14-17	11.798	14,16,17	11.8865	14, 16, 17	14.33042	14,16,17	11.89597	14,16,17	12.985	14,16,17	23.082
18-21	18.4995	18-20	18.4468	18-20	10.13313	18-20	18.24471	18-20	21.652	18-20	37.5147
22-25	23.3405	23, 24	23.3854	22-24	26.42398	23-25	23.44589	23	20.925	23-25	34.4423
Weight [kg] 247.1532		247.3756	.375671 219.9240812		247.2303196		261.2695		690.6489		

Figures 5, 6 and, 7 show the visual difference in optimal cross section thicknesses for the 10, 17, and 25 bar trusses respectively.



Figure 5. Optimization results for 10 bar truss using a) GA, and b) GA with buckling constraint for LC1 and LC2 respectively



Figure 6. Optimization results for 17 bar truss using a) GA, and b) GA with buckling constraint





Figure 7. Optimization results for 25 bar truss using a) GA, and b) GA with buckling constraint

5. CONCLUSION

After testing optimal results from the literature and the optimized model it is evident that a buckling condition is necessary in structural optimization of trusses. All tested solutions without the constraint have more than one bar which do not meet the buckling criteria. The use of just a single bar in a truss which does not meet buckling criteria would result in a compromised structure, which is unusable in practice. Therefore it can be concluded that truss sizing optimization results which do not use buckling constraints are not practically applicable.

The weights of all optimal solutions without the constraint vary by 195.988kg (~8%) for 10 bar for LC1 and 177.762kg (~8%) for LC2, 13.365kg (~1%) for 17 bar, and 41.345kg (~17%) for 25 bar trusses. The average weight of all the examples from the literature differ from the GA solution given in this paper by 16.18kg (~1%) in load case 1 and 0.12kg (~0%) in load case 2 for the 10 bar, 13.365kg (1%) for the 17 bar, and 20.85kg (9%) for the 25 bar truss. Variances between the methods used in the literature and GA in this paper are very small. Due to this small difference, the comparison between the GA optimal results with and without buckling constraints can be considered valid. The weight of the optimal model which considers buckling using GA differs from its GA counterpart by 2431.548kg (104%) in load case 1 and 2938.736kg (136%) in load case 2 for the 10 bar, 324.594kg (27%) for the 17 bar, and 429.379kg (164%) for the 25 bar truss. While the 25 bar truss model has predefined static constraints for compressive forces, they still allow for buckling in optimal models from the literature.

Optimization results which use, the herein proposed, the buckling dynamic constraints give significantly larger weights of models, but compared to examples that do not have this constraint, they meet buckling conditions. All cross sections of bars subjected to buckling in the optimal models without the added constraint were considerably increased in the solutions which had buckling constraints. The larger optimal weights due to the consideration of buckling can be decreased by adding simultaneous topological and sizing optimization. This must be conducted in a single stage optimization approach to ensure the best solution combination is achi-eved. Creating such a process would further increase the complexity of the problem, but would eliminate unused elements, and modify the shape, all with the objective of decreasing overall weight. This approach will also be the authors' focus of further research in this field.

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УТИЦАЈ УВОЂЕЊА ДИНАМИЧКИХ ОГРАНИЧЕЊА НА ИЗВИЈАЊЕ КОД ПРОБЛЕМА ОПТИМИЗАЦИЈЕ ПОПРЕЧНИХ ПРЕСЕКА РЕШЕТКАСТИХ НОСАЧА

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У овом раду истражени су утицаји додавања ограничења на извијање проблему оптимизације попречних пресека за минимизацију масе. Увођење провере на извијање повећава комплексност оптимизационог процеса пошто Ојлеров критеријум извијања се мења из итерације у итерацију оптимизације услед промена димензија попречних пресека елемената. Резултујући модели, који узима у обзир овај критеријум, су практично применљиви. За потребе приказивања утицаја динамичког ограничења на извијање, оптимални параметарски стандардни тест модели решеткастих носача са 10, 17 и 25 штапова из литературе су проверени на извијање и упоређени са моделима код којих је то ограничење узето у обзир. Модели који не узимају у обзир извијање имају елементе који не задовољавају критеријум извијања. Масе ових модела су знатно мање од истих који узимају у обзир извијање.