



## Multi-Criteria Decision Making Method for Models with the Dominant Criterion

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**Abstract.** In all practical problems associated with the selection and assessment, almost always criteria have different importance. The priorities of criteria are expressed by weighted coefficients. Every decision maker has to establish criteria priority scale. This can be done directly - degrees of criteria importance defined by judgment of experts, or indirectly - degrees of criteria importance calculated by alternatives themselves. Many multi-criteria decision methods allow some kind of compensatory between criteria. The low performance on an important criterion can be redeemed in overall aggregation by good performance on few other less important criteria. In this paper, we present the method which provides that such an important piece of information must be preserved: If an alternative does not satisfy a dominant criterion, then its overall aggregation value is zero.

### 1. Introduction

During the previous decades, many multi-criteria decision methods and techniques have been proposed and elaborated within the scientific disciplines, such as operations research, management science, computer science and statistics [1, 2, 5, 7, 14, 17, 18, 23]. Nowadays, many of these methods have an extensive software support. Multi-criteria decision analysis has been used in a wide variety of fields such as energy management, environmental planning, public services, healthcare, transportation, logistics, marketing, human resources management and finance [6, 9, 10, 12, 15, 16, 19–21, 24, 26, 27, 30, 31, 34, 35]. Multi-criteria decision analysis approaches have been widely used by public entities, firms and organisations [25]. For an overview of the available methods for solving multi-criteria decision problems we refer to Figueira et al. [8], Hwang and Yoon [11], Radojičić and Žižović [22], Triantaphyllou [28] and Zeleny [32].

Although multi-criteria decision problems could be very different in context, they share the following common features.

All criteria can be classified into two categories. Criteria that are to be maximized - the profit criteria category, and criteria that are to be minimized - the cost criteria category. In a natural way, any criterion of the second type can be transferred into a criterion of the first type.

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Almost always criteria have different importance. The priorities of criteria are expressed by weighted coefficients in normalized form (sum of all weights is 1). So, every decision maker has to establish criteria priority scale. This can be done directly - degrees of criteria importance defined by judgment of experts, or indirectly - degrees of criteria importance calculated by alternatives themselves.

One of the most important steps in many multi-criteria methods is normalization. Namely, different (quantitative) criteria can be expressed in different dimensions, i.e., in different measurement scales, and if we apply normalization process, all of them will be transformed into the same scale (usually [0,1]). Several normalization procedures are available in literature to eliminate computation problems caused by different measurement units (for example, see [3, 13]).

The crucial problem arises when we need to express performance value of alternatives in terms of qualitative criteria. This problem may become very difficult one (maybe even impossible). Every qualitative information need to be transformed into absolute quantitative value. The objectivity of this step always depends on decision-maker, i.e., on his subjective interpretation of numbers, and thus, it is always questionable. Can we notice low change in sensation? How many different qualitative objects can we distinguish? These questions are widely elaborated within psychological theories (see Milner, 1956) and some well known multi-criteria methods (such as AHP-method) are based on these researches (see Saaty [24] and Triantaphyllou [29]). All multi-criteria methods dealing with qualitative data information, eventually express them by using quantitative scale.

Further, using appropriate mathematical calculus, each alternative is assessed and the final rank of alternatives is obtained. Different multi-criteria methods are based on different utility functions used for calculation of overall values of alternatives. Selection of the method always depends on the type of problem that is to be solved, the knowledge and experience of the decision maker in the field of multi-criteria analysis, as well as technology issues under consideration.

## 2. Preliminaries

Our focus will be on the typical multi-criteria decision problem. Let there are  $m$  alternatives  $A_1, A_2, \dots, A_m$  to be assessed based on  $n$  criteria  $C_1, C_2, \dots, C_n$ . A decision matrix is a  $m \times n$ -matrix with each element  $a_{ij}$  being the  $j$ -th criteria performance value of the  $i$ -th alternative.

Table 1: Decision matrix

	$C_1$	$C_2$	$\dots$	$C_n$
$A_1$	$a_{11}$	$a_{12}$	$\dots$	$a_{1n}$
$A_2$	$a_{21}$	$a_{22}$	$\dots$	$a_{2n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$
$A_m$	$a_{m1}$	$a_{m2}$	$\dots$	$a_{mn}$

Let  $w_1, w_2, \dots, w_n$  denote the weights assigned to criteria  $C_1, C_2, \dots, C_n$ . Each  $w_j$  reflects importance of  $C_j$  relative to the other criteria. The natural assumption is that the weights are normalized, i.e., they sum up to 1:

$$w_1 + w_2 + \dots + w_n = 1.$$

The criteria weights are set either in a direct or an indirect way. In the direct approach, weights are given by the user, while in an indirect method, preference information is collected through a pair-wise comparison of the previously selected alternatives. The outcome of the decision making process greatly depends on the objectivity of criteria weights. For an overview on various weighting methods we refer to [33].

In a multi-criteria value model, the overall value of alternative  $A_i$  is given by the utility function  $V(A_i)$  which is the result of the aggregation of the value functions  $V_j(A_i)$  assigned to each criterion  $C_j$ , i.e.

$$V(A_i) = M(V_1(A_i), V_2(A_i), \dots, V_n(A_i)).$$

The most commonly used aggregation function is the weighted sum  $V(A_i) = \sum_{j=1}^n w_j \cdot V_j(A_i)$  which is attractive due to its low complexity, but other aggregation functions can also be applied (see Hwang and Yoon [11]). For instance, very popular are non-additive approaches where aggregation function is not a linear combination of partial preferences (such as Chouquet integral [4]).

Notice that in case that the decision matrix is normalized according to some normalization methods, then the overall value of an alternative can be obtained by summing up its criteria performance values multiplied with the corresponding criteria weights, i. e.

$$V(A_i) = \sum_{j=1}^n w_j \cdot a_{ij}. \tag{1}$$

Once the aggregation function  $V$  is defined, the alternatives are automatically rank ordered by the partial order relation  $>_A$  on the set of alternatives  $A = \{A_1, A_2, \dots, A_m\}$  defined by

$$A_p >_A A_q \Leftrightarrow V(A_p) \geq V(A_q), \quad \text{for all } A_p, A_q \in A.$$

### 3. Dominant Criteria

Let  $C_j$  be the criterion which is the most dominant for the solution of multi-criteria problem, meaning that if its performance values of all alternatives are extremely low then problem should not be treated (it has no solution), or obtained solution of the problem has weak significance.

#### 3.1. Model 1

Let  $A_i$  be an alternative from starting set of alternatives  $A_1, A_2, \dots, A_m$ , and let  $w_1, w_2, \dots, w_n$  be the weighted coefficients associated to criteria  $C_1, C_2, \dots, C_n$  (they are given by judgement of decision maker or calculated by some of multi-criteria procedures, for example AHP method).

If the alternative  $A_i$  satisfies the most dominant criterion  $C_j$  with the degree  $a_{ij}$ , then we put

$$\begin{aligned} W_{jj}(A_i) &= 1 - a_{ij} + w_j a_{ij}, \\ W_{jk}(A_i) &= w_k a_{ij}, \quad \text{for all } k = 1, 2, \dots, n \text{ such that } k \neq j. \end{aligned}$$

In this way, each alternative  $A_i$  is associated with the set  $\{W_{j1}(A_i), W_{j2}(A_i), \dots, W_{jn}(A_i)\}$  of new weighted coefficients which correspond to dominant criterion  $C_j$ .

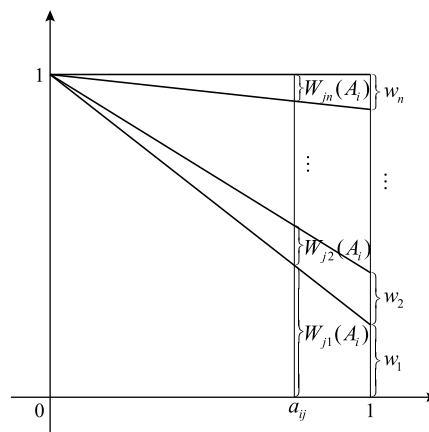


Figure 1: Graphical representation of weights by Model 1

**Theorem 3.1.** For each alternative  $A_i, i = 1, 2, \dots, m$ , the set of new weighted coefficients  $\{W_{j1}(A_i), W_{j2}(A_i), \dots, W_{jn}(A_i)\}$  is normalized and it sums up to 1, i.e.

$$\sum_{k=1}^n W_{jk}(A_i) = 1.$$

*Proof.* Let alternative  $A_i$  satisfies the most dominant criterion  $C_j$  with the degree  $a_{ij}$ . Then we have

$$\begin{aligned} \sum_{k=1}^n W_{jk}(A_i) &= w_1 a_{ij} + \dots + w_{j-1} a_{ij} + (1 - a_{ij} + w_j a_{ij}) + w_{j+1} a_{ij} + \dots + w_n a_{ij} = \\ &= 1 - a_{ij} + a_{ij} \left( \sum_{k=1}^n w_k \right) = 1 - a_{ij} + a_{ij} = 1, \end{aligned}$$

and thus every coefficient  $W_{jk}(A_i)$  is normalized, i.e.  $W_{jk}(A_i) \in [0, 1]$ , and clearly they sum up to 1.  $\square$

**Theorem 3.2.** For  $i_1, i_2 \in \{1, 2, \dots, m\}$ , let the alternatives  $A_{i_1}$  and  $A_{i_2}$  be such that  $a_{i_1 j} < a_{i_2 j}$  and let  $C_j$  be the most dominant criterion. Then

$$W_{jj}(A_{i_1}) > W_{jj}(A_{i_2}) \quad \text{and} \quad W_{jk}(A_{i_1}) < W_{jk}(A_{i_2}), \quad \text{for } k \neq j, k = 1, 2, \dots, n.$$

*Proof.* If  $a_{i_1 j} < a_{i_2 j}$ , then holds

$$W_{jj}(A_{i_1}) = 1 - a_{i_1 j} + w_j a_{i_1 j} = 1 - a_{i_1 j} (1 - w_j) > 1 - a_{i_2 j} (1 - w_j) = 1 - a_{i_2 j} + w_j a_{i_2 j} = W_{jj}(A_{i_2}),$$

and for  $k \neq j, k = 1, 2, \dots, n$  it holds that  $W_{jk}(A_{i_1}) = w_k a_{i_1 j} < w_k a_{i_2 j} = W_{jk}(A_{i_2})$ .  $\square$

**Theorem 3.3.** Let  $C_j$  be the most dominant criterion with weight  $w_j < 1$ . Then, for each alternative  $A_i, i = 1, 2, \dots, m$ , such that  $a_{ij} < 1$  holds

$$W_{jj}(A_i) > w_j \quad \text{and} \quad W_{jk}(A_i) < w_k, \quad \text{for } k \neq j, k = 1, 2, \dots, n.$$

*Proof.* If  $a_{ij} < 1$ , then

$$W_{jj}(A_i) - w_j = 1 - a_{ij} + w_j a_{ij} - w_j = (1 - a_{ij})(1 - w_j).$$

Since  $a_{ij}, w_j \in [0, 1)$ , we have that  $W_{jj}(A_i) - w_j > 0$  and thus  $W_{jj}(A_i) > w_j$ . Also, if  $a_{ij} < 1$ , than for  $k \neq j$  we have  $W_{jk}(A_i) = w_k a_{ij} < w_k$ .  $\square$

By Theorems 3.2 and 3.3, we have that decreasing performance value of an alternative by the most dominant criterion linearly increase the weight of that dominant criterion and proportionally decrease the weights of other criteria.

**Theorem 3.4.** The following holds for an alternative  $A_i, i = 1, 2, \dots, m$  and the most dominant criterion  $C_j$ :

(i) If the alternative  $A_i$  completely satisfy the most dominant criterion  $C_j$ , then

$$W_{jk}(A_i) = w_k, \quad \text{for all } k = 1, 2, \dots, n.$$

(ii) If the alternative  $A_i$  does not satisfy the most dominant criterion  $C_j$ , then

$$W_{jj}(A_i) = 1 \quad \text{and} \quad W_{jk}(A_i) = 0, \quad \text{for all } k \neq j, k = 1, 2, \dots, n.$$

*Proof.* (i) If the alternative  $A_i$  completely satisfies the most dominant criterion  $C_j$ , i.e.  $a_{ij} = 1$ , then by definition of new weighted coefficients we have  $W_{jj}(A_i) = 1 - a_{ij} + w_j a_{ij} = w_j$  and for  $k \neq j$ ,  $W_{jk}(A_i) = w_k a_{ij} = w_k$ .

(ii) If alternative  $A_i$  does not satisfy the most dominant criterion  $C_j$ , then clearly  $a_{ij} = 0$ . By definition of new weighted coefficients we have  $W_{jj}(A_i) = 1 - a_{ij} + w_j a_{ij} = 1$  and  $W_{jk}(A_i) = w_k a_{ij} = 0$ , for  $k \neq j$ .  $\square$

The aggregation function, defined by

$$\tilde{V}(A_i) = \sum_{k=1}^n W_{jk}(A_i) \cdot a_{ik}, \quad (2)$$

has the following property.

**Theorem 3.5.** *If an alternative  $A_i$ , for some  $i \in \{1, 2, \dots, m\}$ , does not satisfy the most dominant criterion  $C_j$ , then the overall value of  $A_i$  is equal to zero, i.e.*

$$\tilde{V}(A_i) = 0.$$

*Proof.* By Theorem 3.4, we have  $W_{jj}(A_i) = 1$  and  $W_{jk}(A_i) = 0$ , for  $k \neq j$ , and thus

$$\tilde{V}(A_i) = \sum_{k=1}^n W_{jk}(A_i) \cdot a_{ik} = \sum_{k=1, k \neq j}^n W_{jk}(A_i) \cdot a_{ik} + W_{jj}(A_i) \cdot a_{ij} = 0,$$

which completes proof of the theorem.  $\square$

As a consequence of the previous observation, we have that non-dominant criteria have no influence on alternative scoring in the case that this alternative does not satisfy the most dominant criterion.

### 3.2. Model 2

Here we will modify the procedure presented in Model 1 to obtain the following property: If performance value of an alternative by the most dominant criterion is lower than a given value  $a$ , then all other criteria are disregarded and overall value of that alternative is equal to zero.

Let  $C_j$  be the most dominant criterion, and let  $a$  be a real number such that  $0 < a < 1$ . If an alternative  $A_i$  satisfies the most dominant criterion  $C_j$  with the degree  $a_{ij} < a$ , then we put

$$Q_{jj}(A_i) = 1 \quad \text{and} \quad Q_{jk}(A_i) = 0 \quad \text{for all } k = 1, 2, \dots, n \text{ such that } k \neq j,$$

otherwise, if  $a_{ij} \geq a$ , we put

$$Q_{jj}(A_i) = 1 + (w_j - 1)((a_{ij} - a)/(1 - a)),$$

$$Q_{jk}(A_i) = w_k((a_{ij} - a)/(1 - a)), \quad \text{for all } k = 1, 2, \dots, n \text{ such that } k \neq j.$$

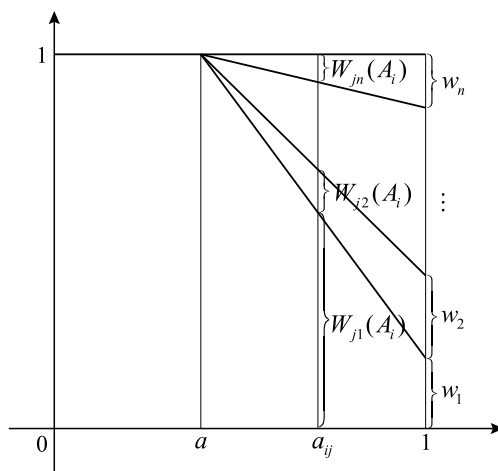


Figure 2: Graphical representation of weights by Model 2

**Theorem 3.6.** *The set of new weighted coefficients  $\{Q_{j1}(A_i), Q_{j2}(A_i), \dots, Q_{jn}(A_i)\}$  is normalized for each alternative  $A_i, i = 1, 2, \dots, m$ , and it sums up to 1, i.e.*

$$\sum_{k=1}^n Q_{jk}(A_i) = 1.$$

*Proof.* Similar to the proof of Theorem 1.  $\square$

Let the aggregation function be defined by

$$\widetilde{V}(A_i) = \sum_{k=1}^n Q_{jk}(A_i) \cdot a_{ik}. \tag{3}$$

Then, for  $\widetilde{V}$ , we can obtain the analogies of Theorems 2-5 if we replace  $W$  by  $Q$  and  $\widetilde{V}$  by  $\widetilde{V}$ .

**Theorem 3.7.** *Let  $0 < a < 1$  be the minimal suitable value of the most dominant criterion  $C_j$ . If an alternative  $A_i, i = 1, 2, \dots, m$ , satisfies the criterion  $C_j$  with the degree  $a_{ij} < a$ , then the overall value of  $A_i$  is equal to zero, i.e.*

$$\widetilde{V}(A_i) = 0.$$

*Proof.* By analogue of Theorem 3.4, we have  $Q_{jj}(A_i) = 1$  and  $Q_{jk}(A_i) = 0$ , for all  $k \neq j, k = 1, 2, \dots, n$ , and thus  $\widetilde{V}(A_i) = 0$ .  $\square$

By the constructions, Model 1 and Model 2 have the following common property.

**Theorem 3.8.** *The rank of alternatives from the set  $\{A_1, A_2, \dots, A_m\}$  remains the same in the case that the starting set of alternatives is expanded by a set of new alternatives  $\{B_1, B_2, \dots, B_s\}$ .*

*Proof.* Let  $[a_{ij}]_{i=1, j=1}^{m, n}$  and  $[b_{kj}]_{k=1, j=1}^{s, n}$  be two normalized decision matrices and let  $[c_{ij}]_{i=1, j=1}^{m+s, n}$  be a decision matrix such that, for all  $j = 1, 2, \dots, n$ , holds

$$\begin{cases} c_{ij} = a_{ij}, & \text{for all } i = 1, 2, \dots, m, \\ c_{ij} = b_{kj}, & \text{for all } i = m + k, k = 1, 2, \dots, s. \end{cases}$$

Let  $A = \{A_i = (a_{ij}), i = 1, 2, \dots, m\}$  and  $C = \{C_i = (c_{ij}), i = 1, 2, \dots, m + s\}$  be two set of alternatives and let  $>_A$  denote the partial order on the set  $A$  induced by the function  $\widetilde{V}_A$  (respectively  $\widetilde{\widetilde{V}}_A$ ), and let  $>_C$  denote the partial order relation on the expanded set  $C$  induced by the function  $\widetilde{V}_C$  (respectively  $\widetilde{\widetilde{V}}_C$ ). If  $A_p$  and  $A_q$  are two alternatives from the starting set of alternatives  $A$ , then  $\widetilde{V}_A(A_p) = \widetilde{V}_C(A_p)$ , so  $A_p >_A A_q$  if and only if  $A_p >_C A_q$ .  $\square$

It is well known that most of the multi-criteria decision methods suffer from a structuring problem in the sense that it is possible to obtain a reverse rank of alternatives by the introduction of new alternative options. By Theorem 3.8, the multi-criteria decision method presented by models 1 and 2 preserve that rank so that there are no possibilities of favoring or manipulating alternative ranking by taking new alternatives into account.

**Corollary 3.9.** *No alternative can be favored by adding new alternatives into multi-criteria model.*

*Proof.* By Theorem 3, adding new alternatives to the multi-criteria model does not rearrange the rank of previously introduced alternative choices, so no alternative can be favored.  $\square$

**Example 3.10.** It can be noticed that many multi-criteria decision methods which are based on an additive aggregation function allow some kind of compensation between criteria. The low performance of an important criterion can be redeemed in the overall aggregation by the good performance of a few other less important criteria which will be shown in this example.

Let two alternatives  $A_1$  and  $A_2$  evaluated by the bases on five criteria  $C_1, C_2, \dots, C_5$  induce decision matrix presented by Table 2. Clearly, for the alternatives  $A_1$  and  $A_2$ , incomparability holds for all pair-wise comparisons.

Table 2: Decision matrix of Example 3.10

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$
$A_1$	0.40	0.60	0.50	0.60	0.50
$A_2$	0.10	0.80	0.90	0.80	0.80

If  $C_1$  is the most dominant criterion with the weight  $w_1 = 0.4$  and  $C_2, C_3, C_4$  and  $C_5$  are less important criteria associated with weights  $w_2 = w_3 = w_4 = w_5 = 0,15$ , then using weighted sum as overall values of  $A_1$  and  $A_2$  we obtain  $V(A_1) = 0.49$  and  $V(A_2) = 0.55$ . Therefore, alternative  $A_2$  is preferred over alternative  $A_1$ , but alternative  $A_2$  barely meets the most dominant criterion  $C_1$ , so the question arises whether obtained order of alternatives is correct.

On the other hand, new weighted coefficients (according to Model 1) for the alternative  $A_1$  are  $W_1(A_1) = 0.76$  and  $W_2(A_1) = W_3(A_1) = W_4(A_1) = W_5(A_1) = 0.06$ , and therefore  $\widetilde{V}(A_1) = 0.448$ . Similarly, new weighted coefficients (according to Model 1) for the alternative  $A_2$  are  $W_1(A_2) = 0.94$  and  $W_2(A_2) = W_3(A_2) = W_4(A_2) = W_5(A_2) = 0.015$ , and therefore  $\widetilde{V}(A_2) = 0.145$ . Thus, the alternative  $A_1$  is preferred over the alternative  $A_2$ . If the minimal suitable value for the dominant criterion is  $a = 0.3$ , than according to Model 2 we have that  $\widetilde{\widetilde{V}}(A_1) = 0.41$  and  $\widetilde{\widetilde{V}}(A_2) = 0.1$ , so alternative  $A_1$  is preferred over the alternative  $A_2$ .

**Example 3.11.** Here we will observe one hypothetical example to demonstrate how a small change in alternative performance by dominant criterion result in changes of weighted coefficients and overall values. We will consider ten alternatives  $A_1, A_2, \dots, A_{10}$  evaluated by five criteria  $C_1, C_2, \dots, C_5$ . The degrees in which alternatives satisfy criteria are presented in Table 3.

Table 3: Decision matrix of the Example 3.11

	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	C <sub>5</sub>
A <sub>1</sub>	1.0	0.8	0.6	0.7	0.5
A <sub>2</sub>	0.9	0.8	0.6	0.7	0.5
A <sub>3</sub>	0.8	0.8	0.6	0.7	0.5
A <sub>4</sub>	0.7	0.8	0.6	0.7	0.5
A <sub>5</sub>	0.6	0.8	0.6	0.7	0.5
A <sub>6</sub>	0.5	0.8	0.6	0.7	0.5
A <sub>7</sub>	0.4	0.8	0.6	0.7	0.5
A <sub>8</sub>	0.3	0.8	0.6	0.7	0.5
A <sub>9</sub>	0.2	0.8	0.6	0.7	0.5
A <sub>10</sub>	0.1	0.8	0.6	0.7	0.5

Let C<sub>1</sub> be the most dominant criterion and let  $w_1 = 0.40$ ,  $w_2 = w_3 = 0.20$ ,  $w_4 = 0.15$  and  $w_5 = 0.05$ . Then overall values calculated by aggregation functions (1) and (2) are given in Table 4. It can be noticed that  $\min\{V(A_i), \tilde{V}(A_i)\} = V(A_i)$ , for  $i = 1, 2, 3, 4$ , and  $\min\{V(A_i), \tilde{V}(A_i)\} = \tilde{V}(A_i)$ , for  $i = 5, 6, \dots, 10$ .

In the case that the minimal suitable value for the dominant criterion C<sub>1</sub> is  $a = 0.3$ , then by Model 2, we obtain the overall values of aggregation function (3) which are presented in Table 4.

Table 4: Overall values of alternatives in Example 3.11

	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub>	A <sub>5</sub>	A <sub>6</sub>	A <sub>7</sub>	A <sub>8</sub>	A <sub>9</sub>	A <sub>10</sub>
V(·)	0,81	0,77	0,73	0,69	0,65	0,61	0,57	0,53	0,49	0,45
$\tilde{V}$ (·)	0,81	0,783	0,744	0,693	0,63	0,555	0,468	0,369	0,2364	0,135
$\tilde{\tilde{V}}$ (·)	0,81	0,789	0,75	0,694	0,61	0,531	0,42	0,3	0,2	0,1

#### 4. Conclusion

In this paper we observed multi-criteria models in which one criterion dominates the others. We presented two multi-criteria methods for identifying the most preferred alternative choice (or for ranking the alternatives), which gives fruitful and more accurate information on the observed alternatives and corresponding preference of the most dominant criterion.

Note that the weighted coefficients and the aggregation function discussed in this paper will be studied from a more general point of view in our further research.

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