



## New Weighted Sum Model

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**Abstract.** In this paper, a new weighting sum method for multi-criteria decision making is presented. The main advantage of this method is that it is easier for understanding and it can effectively be handled by a decision maker, so the obtained solution best suits his goal and his understanding of the problem.

### 1. Introduction

All criteria in a multi-criteria decision making problem can be classified into two categories. Criteria that are to be maximized (highest value is best value) and criteria that are to be minimized (lowest value is best value). An ideal solution to a multi-criteria decision making problem would maximize all criteria of the first type and minimize all criteria of the second type, but usually this solution is not possible to obtain. So, the question is what would be the best satisfying solution? This question may not always have a conclusive or unique answer. Whether a solution is satisfying depends on the level of the decision makers expectation. The goal of every analyst (the person using the method) is for his suggestion to be accepted as a valid solution by the decision maker. In order to reach that goal (and for the analyst to justify his work and achieve success), it is required to include the decision maker as much as possible, directly or indirectly, in the calculation process for solving the problem. Sometimes, this is not possible because of the limited time or the structure and complexity of the method from mathematical point of view. Therefore, it is necessary to find a solution reaching method where the decision maker can give his subjective judgments and to be included successfully and with ease.

There are many multi-criteria decision making methods available in the literature. Some of the most commonly used approaches are the Weighted sum model [7], the Weighted product model [2, 13], the analytic hierarchy process [17–20], the ELECTRE method [3, 16], the TOPSIS method [9], the PROMETHEE method [1], the VIKOR method [14], etc. There are also a number of papers, which are devoted to comparison of their characteristics and performances. Multi-criteria decision making methods differ in normalization processes performed to convert all criteria into a same unit [4, 11, 12], weighting techniques used for determination of the criteria importance [10, 22], method of aggregation of value functions assigned to each criterion [5], etc. This affects the core complexity of a method.

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In this article, we will present a new multi-criteria decision method where it is relatively easy to include the decision maker, his personal preference and his view on the problem. The method is consisted of two main parts, normalization and weighing processes. Finally, to illustrate the feasibility of our approach, we will apply our proposed method on a real application problem.

For an overview of the available methods for solving multi-criteria decision problems we refer to Figueira et al. [6], Hwang and Yoon [9], Radojčić and Žižović [15], Triantaphyllou [21] and Zeleny [23], and for an insight into practical applications we refer to [8].

## 2. Normalization of the Multi-Criteria Model

We will observe a multi-criteria model for ranking  $m$  alternatives  $(A_1, \dots, A_m)$  by  $n$  criteria  $(C_1, \dots, C_n)$  presented in Table 1. In this model, the degree in which alternative  $A_i$  ( $i = 1, \dots, m$ ) satisfies criterion  $C_j$ , ( $j = 1, \dots, n$ ) is denoted by  $a_{ij}$ . Without loss of generality, we can assume that the criteria are ordered based on importance, from the most important criterion  $C_1$  to the least important criterion  $C_n$ .

Table 1: Decision matrix

	$C_1$	$C_2$	$\dots$	$C_n$
$A_1$	$a_{11}$	$a_{12}$	$\dots$	$a_{1n}$
$A_2$	$a_{21}$	$a_{22}$		$a_{2n}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$A_m$	$a_{m1}$	$a_{m2}$	$\dots$	$a_{mn}$

For different criteria, the performance values of alternatives can be measured by different units. In order to have a valid comparisons, all elements of decision matrix need to be transferred into a same unit (the interval  $[0,1]$  is usually used as the basic unit interval). A lot of normalization methods have been developed. Some of most popular are vector normalization method, linear max-min normalization, linear sum based normalization, linear max normalization, Gaussian normalization, etc. The review of the literature on normalization methods can be found in [11].

Applying different normalization methods on a decision-making matrix can lead to different numerical results and finally affect alternatives order of preference. Therefore, a normalization method affects the quality of decision-making (see for example [12]). In this section, we will present one new normalization method and we will point out some of its advantages over most commonly known normalization methods.

Values of alternatives with respect to every criterion  $C_j$ , ( $j = 1, \dots, n$ ) are given in the  $j$ th column of Table 1. Clearly, these values can have different importance for the decision maker. Some values are extremely important for the decision maker, some are acceptable, some are barely acceptable, while some values are totally unacceptable. It is therefore, logical to define levels of acceptability for possible alternative values (this can be done even before observing the model itself in Table 1).

For each criterion  $C_j$ , ( $j = 1, \dots, n$ ) of maximization type, the decision maker defines  $r$ , ( $r \in \mathbb{N}$ ) values  $Q_{j1} > Q_{j2} > \dots > Q_{jr}$ , such that  $Q_{j1}$  is assumed to be the decision makers ideal alternative value,  $Q_{j2}$  represents the lower limit of ideal values for decision maker,  $Q_{j,r-1}$  represents the upper limit for barely acceptable values for decision maker, and  $Q_{jr}$  is a lowest acceptable value for the decision maker.

Analogously, for each criterion of minimization type, the decision maker defines  $r$ , ( $r \in \mathbb{N}$ ) values  $Q_{j1} > Q_{j2} > \dots > Q_{jr}$  whose meanings are in reverse order with respect to previous list, i.e.  $Q_{j1}$  is assumed to be the decision maker nadir alternative value, while  $Q_{jr}$  is the ideal one.

For a criterion  $C_j$ , ( $j = 1, \dots, n$ ) of maximization type, these values  $Q_{jk}$ , ( $1 \leq k \leq r$ ) specify  $r - 1$  intervals

$$I_{j1} = [Q_{j2}, Q_{j1}], I_{j2} = (Q_{j3}, Q_{j2}], \dots, I_{j,r-1} = (Q_{jr}, Q_{j,r-1}],$$

and for a criterion of minimization type the corresponding intervals are

$$I_{j1} = [Q_{j2}, Q_{j1}), I_{j2} = [Q_{j3}, Q_{j2}), \dots, I_{j,r-1} = [Q_{jr}, Q_{j,r-1}].$$

In general, the number of these intervals can be different for different criteria and it depends on the decision maker preference on the criterion.

In the case that  $C_j$  is a criterion of maximization type, let  $p_{jk} : I_{jk} \rightarrow (0, 1]$ , ( $j = 1, \dots, n, k = 1, 2, \dots, r-1$ ) be a non-decreasing functions given by the decision maker and the analysts such that they satisfy the condition  $p_{jk}(x) > p_{j,k+1}(y)$ , for every  $x \in I_{jk}$  and  $y \in I_{j,k+1}$ , ( $k = 1, 2, \dots, r-2$ ). Further, let the functions  $f_j : \mathbb{R}_{\geq 0} \rightarrow [0, 1]$  be such that

$$f_j(a) = \begin{cases} 1, & a \geq Q_{j1}; \\ p_{jk}(a), & a \in I_{jk}, \quad k = 1, 2, \dots, r-1; \\ 0, & a \leq Q_{jr}. \end{cases} \tag{1}$$

In the case that  $C_j$  is a criterion of minimization type, then functions  $p_{jk}^* : I_{jk} \rightarrow (0, 1]$ , ( $j = 1, \dots, n, k = 1, 2, \dots, r-1$ ) are required to be non-increasing and to satisfy  $p_{jk}(x) < p_{j,k+1}(y)$ , for every  $x \in I_{jk}$  and  $y \in I_{j,k+1}$ , ( $k = 1, 2, \dots, r-2$ ). Further, let the functions  $f_j^* : \mathbb{R}_{\geq 0} \rightarrow [0, 1]$  be defined as

$$f_j^*(a) = \begin{cases} 0, & a \geq Q_{j1}; \\ p_{jk}^*(a), & a \in I_{jk}, \quad k = 1, 2, \dots, r-1; \\ 1, & a \leq Q_{jr}. \end{cases} \tag{2}$$

Now, we normalize the decision matrix given in Table 1, by

$$q_{ij} = f_j(a_{ij}), \text{ if } C_j \text{ is a criterion of maximization type;}$$

$$q_{ij} = f_j^*(a_{ij}), \text{ if } C_j \text{ is a criterion of minimization type.}$$

By this procedure, we obtain normalized decision matrix given by Table 2. All values in this matrix are elements of the real unit interval  $[0,1]$ . Moreover, all the criteria of minimization type are converted into the criteria of maximization type.

Table 2: Normalized decision matrix

	$C_1$	$C_2$	$\dots$	$C_n$
$A_1$	$q_{11}$	$q_{12}$	$\dots$	$q_{1n}$
$A_2$	$q_{21}$	$q_{22}$	$\dots$	$q_{2n}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$A_m$	$q_{m1}$	$q_{m2}$	$\dots$	$q_{mn}$

### 3. Weighted Coefficients

In this section, we will present a new procedure for calculation of weighting coefficients in a multi-criteria decision making model. This procedure is based on the pairwise comparisons between the most important criterion  $C_1$  and the remaining  $n - 1$  criteria  $C_2, \dots, C_n$ .

Let  $p_{1k} \in (0, 100]$ , ( $k = 2, \dots, n$ ) be the value of importance of the criterion  $C_1$  with respect to the criterion  $C_k$ . Then  $p_{k1} = 100 - p_{1k}$  represent the value of importance of criterion  $C_k$  with respect to criterion  $C_1$ . These values are given in Table 3.

Table 3: The most important criterion w.r.t. other criteria

$p_{12}$	$p_{13}$	$p_{14}$	$\dots$	$p_{1n}$
$p_{21}$	$p_{31}$	$p_{41}$	$\dots$	$p_{n1}$

Clearly, the following holds

$$50 \leq p_{12} \leq p_{13} \leq \dots \leq p_{1n} < 100 \quad \text{and} \quad 50 \geq p_{21} \geq p_{31} \geq \dots \geq p_{n1} > 0.$$

As we assumed above, criteria  $C_1, \dots, C_n$  are listed in order of importance, and thus  $C_i$  is of greater or equal importance as  $C_{i+1}$ , for each  $i = 1, \dots, n - 1$ . Now, using Table 3, it is possible to calculate the value of importance of the criterion  $C_2$  with respect to criteria  $C_3, \dots, C_n$ , further, the value of importance of criterion  $C_3$  with respect to criteria  $C_4, \dots, C_n$ , etc., and finally the value of importance of criterion  $C_{n-1}$  with respect to criterion  $C_n$ . This procedure is given by following recursive formula:

$$p_{j+1,k} = \frac{100(p_{jk} : p_{kj}) \cdot (p_{j,j+1} : p_{j+1,j})}{1 + (p_{jk} : p_{kj}) \cdot (p_{j,j+1} : p_{j+1,j})}, \quad p_{k,j+1} = 100 - p_{j+1,k}, \quad \text{for } j = 2, \dots, n - 1, k = j + 1, \dots, n. \quad (3)$$

In this way we can form a triangular Table 4 (which also includes Table 3).

Table 4: Pairwise comparison of criteria

$p_{12}$	$p_{13}$	$p_{14}$	$\dots$	$p_{1,n-1}$	$p_{1n}$
$p_{21}$	$p_{31}$	$p_{41}$	$\dots$	$p_{n-1,1}$	$p_{n1}$
	$p_{23}$	$p_{24}$	$\dots$	$p_{2,n-1}$	$p_{2n}$
	$p_{32}$	$p_{42}$	$\dots$	$p_{n-1,2}$	$p_{n2}$
		$p_{34}$	$\dots$	$p_{3,n-1}$	$p_{3n}$
		$p_{43}$	$\dots$	$p_{n-1,3}$	$p_{n3}$
			$\dots$		
				$p_{n-2,n-1}$	$p_{n-2,n}$
				$p_{n-1,n-2}$	$p_{n,n-2}$
					$p_{n-1,n}$
					$p_{n,n-1}$

**Theorem 3.1.** *The following holds for every  $j = 1, 2, \dots, n - 1$ :*

$$50 \leq p_{j,j+1} \leq p_{j,j+2} \leq \dots \leq p_{jn} < 100 \quad \text{and} \quad 50 \geq p_{j+1,j} \geq p_{j+2,j} \geq \dots \geq p_{n,j} > 0. \quad (4)$$

*Proof.* Clearly, the assertion holds for  $j = 1$  and suppose that the assertion holds for some  $j - 1 \in \{1, 2, \dots, n - 1\}$ . Then by (4) we obtain

$$p_{j,j+1} = \frac{100\delta}{1 + \delta}, \quad \text{for } \delta = \frac{p_{j-1,j+1}}{p_{j-1,j}} \cdot \frac{p_{j,j-1}}{p_{j+1,j-1}}.$$

Since  $p_{j-1,j} \leq p_{j-1,j+1}$  and  $p_{j,j-1} \geq p_{j+1,j}$ , holds  $\delta \geq 1$ , and therefore  $p_{j,j+1} = 100 - \frac{100}{1+\delta} \geq 50$ . Further, from  $p_{j-1,j+k} \leq p_{j-1,j+k+1}$  and  $p_{j+k,j-1} \geq p_{j+k+1,j}$ , for every  $k = 1, 2, \dots, n - j - 1$  it follows that

$$\frac{p_{j-1,j+k+1}}{p_{j-1,j}} \cdot \frac{p_{j,j-1}}{p_{j+k+1,j-1}} \geq \frac{p_{j-1,j+k}}{p_{j-1,j}} \cdot \frac{p_{j,j-1}}{p_{j+k,j-1}},$$

which implies  $p_{j,j+k+1} \geq p_{j,j+k}$ . The inequality  $p_{jn} < 100$  is satisfied according to (3). Thus,  $50 \leq p_{j,j+1} \leq p_{j,j+2} \leq \dots \leq p_{jn} < 100$  holds.

Further, from  $p_{kj} = 100 - p_{jk}$ , for  $k = j + 1, \dots, n - j = 1$ , we have  $50 \geq p_{j+1,j} \geq p_{j+2,j} \geq \dots \geq p_{n,j} > 0$ .  $\square$

The weighted coefficient  $W_j$  is given by

$$W_j = \frac{\sum_{k=1, k \neq j}^n p_{jk}}{50n(n - 1)}, \quad (5)$$

for each criterion  $C_j$  ( $j = 1, 2, \dots, n$ ) Also, the following statement is true for weighted coefficients.

**Theorem 3.2.** *The following properties of the weighted coefficients  $W_j$  ( $j = 1, 2, \dots, n$ ) are satisfied:*

(i)  $\sum_{j=1}^n W_j = 1,$

(ii)  $W_1 \geq W_2 \geq \dots \geq W_n.$

*Proof.* (i) Using (5) we have

$$\begin{aligned} \sum_{j=1}^n W_j &= \sum_{j=1}^n \frac{\sum_{k=1, k \neq j}^n p_{jk}}{50n(n-1)} = \frac{1}{50n(n-1)} \cdot \{(p_{12} + p_{13} + \dots + p_{1n}) + (p_{21} + p_{23} + \dots + p_{2n}) + \\ &+ \dots + (p_{n1} + p_{n2} + \dots + p_{n,n-1})\} = \frac{1}{50n(n-1)} \cdot \{(p_{12} + p_{21}) + (p_{13} + p_{31}) + \dots + (p_{n-1,n} + p_{n,n-1})\} = \\ &= \frac{1}{50n(n-1)} \cdot \frac{100n(n-1)}{2} = 1. \end{aligned}$$

(ii) This assertion holds by definition of weighted coefficients (5) and Theorem 3.1.  $\square$

**Corollary 3.3.**  $W_1 = \dots = W_n = \frac{1}{n}$  if and only if  $p_{ij} = 50$ , for all  $i, j = 1, 2, \dots, n, i \neq j$ .

*Proof.* Follows immediately by (5).  $\square$

#### 4. Ranking of Alternatives

For  $i = 1, 2, \dots, m, j = 1, 2, \dots, n$ , let  $q_{ij}$  be the normalized performance value of the alternative  $A_i$  by the criterion  $C_j$  and let  $W_j$  be the weighted coefficient associated to the criterion  $C_j$  according to formula (5). By multiplying  $q_{ij}$  with weight  $W_j$ , we obtain preference value  $e_{ij}$  associated to criterion  $C_j$ , i.e.

$$e_{ij} = W_j * q_{ij}, \quad \text{for all } i = 1, \dots, m, j = 1, \dots, n. \tag{6}$$

In this way, we form Table 5.

Table 5: Preference values associated to criteria

	$C_1$	$C_2$	$\dots$	$C_n$
$A_1$	$e_{11}$	$e_{12}$	$\dots$	$e_{1n}$
$A_2$	$e_{21}$	$e_{22}$	$\dots$	$e_{2n}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$A_m$	$e_{m1}$	$e_{m2}$	$\dots$	$e_{mn}$

Further, we sum up the values  $e_{ij}$  ( $j = 1, 2, \dots, n$ ) to obtain the overall value of the alternative  $A_i$  ( $i = 1, 2, \dots, m$ ), i.e.

$$V(A_i) = \sum_{j=1}^n e_{ij}. \tag{7}$$

The ranking of alternatives  $A_i$  ( $i = 1, 2, \dots, m$ ) is based on the aggregation value function (7) and fulfilment of criteria in order of their importance. In other words, for two alternatives  $A_i$  and  $A_j$  ( $i, j = 1, 2, \dots, m$ ) we say that  $A_j$  is preferred over  $A_i$ , in notation  $A_j \rightarrow A_i$ , if:

$$V(A_i) < V(A_j) \text{ or}$$

$$V(A_i) = V(A_j), e_{i1} < e_{j1} \text{ or}$$

$$V(A_i) = V(A_j), e_{i1} = e_{j1}, e_{i2} < e_{j2} \text{ or}$$

...

$$V(A_i) = V(A_j), e_{i1} = e_{j1}, \dots, e_{i,n-1} = e_{j,n-1}, e_{in} < e_{jn}.$$

Two alternatives  $A_i$  i  $A_j$  ( $i, j = 1, 2, \dots, m$ ) are equivalents if all their values are equal, i.e.  $e_{ik} = e_{jk}$ , for all  $k = 1, 2, \dots, n$ .

It is well known fact that many multi-criteria decision making methods suffer from phenomena called rank reversal. Namely, the order of alternatives can be changed when an alternative is added to the model. The main reason for rank reversal is the use of an inappropriate normalization method. Actually, all most commonly used methods such as vector normalization method, linear max-min normalization, linear sum based normalization, linear max normalization, Gaussian normalization are based on vector normalization, or choice of ideal or nadir solutions created from alternatives, which basically depend on all alternatives included into the model. Normalization process presented in this paper depends only on treated alternative and hypothetical ideal (nadir) solution given by the decision maker. This proves the following.

**Theorem 4.1.** *The rank of alternatives from the set  $\{A_1, A_2, \dots, A_n\}$  remains the same in the case that the starting set of alternatives is expanded by a new alternative  $A$ .*

*Proof.* Let  $\mathcal{A} = \{A_1, A_2, \dots, A_n\}$  and  $\mathcal{B} = \{A_1, A_2, \dots, A_n, A\}$ , and let  $V_{\mathcal{A}}$  and  $V_{\mathcal{B}}$  denote the value functions on  $\mathcal{A}$  and  $\mathcal{B}$  (respectively) calculated by (7), and  $\rightarrow_{\mathcal{A}}$  and  $\rightarrow_{\mathcal{B}}$  denote the preference order relations on  $\mathcal{A}$  and  $\mathcal{B}$  (respectively). Then  $V_{\mathcal{A}}(A_i) = V_{\mathcal{B}}(A_i)$ , for all  $i = 1, 2, \dots, n$ . Therefore, for every  $i, j = 1, 2, \dots, n$  holds  $A_i \rightarrow_{\mathcal{A}} A_j$  if and only if  $A_i \rightarrow_{\mathcal{B}} A_j$ .  $\square$

**Corollary 4.2.** *Adding a new alternative into the multi-criteria model can not lead to a rank reversal.*

**Example 4.3.** In this example, we will rank five alternatives  $A_1, A_2, A_3, A_4$  and  $A_5$  by four criteria  $C_1, C_2, C_3$  and  $C_4$ . This multi-criteria decision making model is given by Table 6. The criterion  $C_1$  is dominant. The criteria  $C_1, C_2$  and  $C_3$  are to be maximized and criterion  $C_4$  is to be minimized.

Table 6: Decision matrix of Example 4.3

	$C_1$	$C_2$	$C_3$	$C_4$
$A_1$	10	7	6	9
$A_2$	8	8	7	8
$A_3$	6	6	10	8
$A_4$	7	6	9	7
$A_5$	4	10	3	5

We will assume that the following values are given by decision maker

$$Q_{j1} = 10, \quad Q_{j2} = 8.5, \quad Q_{j3} = 6.5, \quad Q_{j4} = 4.5, \quad Q_{j5} = 2, \quad \text{for } j = 1, 2, 3,$$

$$Q_{41} = 10, \quad Q_{42} = 8.5, \quad Q_{43} = 6.5, \quad Q_{44} = 3.5, \quad Q_{45} = 1.$$

Thus we have four intervals of importance for all criteria. Also, it is assumed that the decision maker has provided the values of importance in relation to to the first criterion (Table 7).

Table 7: The most important criterion w.r.t. other criteria in Example 4.3

$p_{12} = 50$	$p_{13} = 60$	$p_{14} = 70$
$p_{21} = 50$	$p_{31} = 40$	$p_{41} = 30$

In this example, we will use linear functions for normalization process. In relation to a maximizing criterion  $C_j$  ( $j = 1, 2, 3$ ), the functions  $f_j$  are defined in the following way:

$$f_j(a_{ij}) = \begin{cases} 1, & a_{ij} \geq Q_{j1}; \\ \frac{a_{ij} - Q_{j5}}{Q_{j1} - Q_{j5}}, & a_{ij} \in I_{jk}, \quad k = 1, 2, 3, 4; \\ 0, & a_{ij} \leq Q_{j5}. \end{cases} \quad (8)$$

and in relation to the minimizing  $C_4$  criterion, the function  $f_4^*$  can be defined by:

$$f_4^*(a_{ij}) = \begin{cases} 0, & a_{ij} \geq Q_{j1}; \\ \frac{Q_{j1} - a_{ij}}{Q_{j1} - Q_{j5}}, & a_{ij} \in I_{jk}, \quad k = 1, 2, 3, 4; \\ 1, & a_{ij} \leq Q_{j5}. \end{cases} \quad (9)$$

In this way, we obtain normalized decision matrix given by Table 8.

Table 8: Normalized decision matrix of Example 4.3

	$C_1$	$C_2$	$C_3$	$C_4$
$A_1$	1	0.625	0.5	0.111
$A_2$	0.75	0.75	0.625	0.222
$A_3$	0.5	0.5	1	0.222
$A_4$	0.625	0.5	0.875	0.333
$A_5$	0.25	1	0.125	0.556

Further, starting with Table 7 and using formula (3), we obtain Table 9. Now, by (5) we have

Table 9: Pairwise comparison of criteria in Example 4.3

$p_{12} = 50$	$p_{13} = 60$	$p_{14} = 70$
$p_{21} = 50$	$p_{31} = 40$	$p_{41} = 30$
	$p_{23} = 60$	$p_{24} = 70$
	$p_{32} = 40$	$p_{42} = 30$
		$p_{34} = 61$
		$p_{43} = 39$

$$W_1 = 0.300, \quad W_2 = 0.300, \quad W_3 = 0.235 \quad \text{and} \quad W_4 = 0.165.$$

Finally, by (6) we obtain Table 10 with overall values of the alternatives.

Table 10: Preference values associated to criteria and overall scores of alternatives in Example 4.3

	$C_1$	$C_2$	$C_3$	$C_4$	$\Sigma$
$A_1$	0.300	0.188	0.118	0.018	$V(A_1) = 0.624$
$A_2$	0.225	0.225	0.147	0.037	$V(A_2) = 0.634$
$A_3$	0.150	0.150	0.235	0.037	$V(A_3) = 0.572$
$A_4$	0.188	0.150	0.206	0.055	$V(A_4) = 0.599$
$A_5$	0.075	0.300	0.029	0.092	$V(A_5) = 0.496$

Since all values  $V(A_i)$  ( $i = 1, 2, \dots, 5$ ) are different we have the total order of alternatives

$$A_2 \rightarrow A_1 \rightarrow A_4 \rightarrow A_3 \rightarrow A_5,$$

and therefore,  $A_2$  is the best alternative solution.

## 5. Conclusion

The suggested method allows a high level of influence of personal preferences of the decision maker and helps him to find a solution that best suits his goal and his understanding of the problem. This results in higher quality of decisions reached. Also, the advantage of this method is that it is easier to understand and it can effectively handle both qualitative and quantitative data. It can be expected that this method will be applicable in many areas (science, technology, business decision making, military doctrine, etc.) because introducing new alternatives does not require additional calculations and comparisons to previously introduced alternatives and does not change the established order. It can be noticed that complexity of methods such as AHP, PPOMETHEE, and others, depends on number of mutually comparisons of alternatives, while this is not a case with the complexity of the proposed new method, since it linearly depends on the number of alternatives included into the model.

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