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The Selection of Optimal Parameters of Gerotor Pump by Application of Factorial Experimental Design

The method of factorial experimental design for studying the factor influence on the flow rate and volumetric efficiency of the gerotor pump is applied in this paper. The considered factors are the coefficient of the trochoid radius, the number of revolution of a pump drive shaft and working pressure, where the first factor is a geometrical parameter and the others are working parameters. Factorial design and response surface methodology (RSM) are applied to evaluate and optimize the effects of these parameters on the flow rate and volumetric efficiency. The experimental design 33 is used, where each factor has three levels. Its optimal variant is obtained by the analysis of considered factors: the coefficient of the trochoid radius 1.375, the number of revolution of a pump drive shaft 2000 rpm and the working pressure 1 bar, where the the greatest values of flow rate and volumetric efficiency are obtained. A mathematical prediction model of the flow rate and volumetric efficiency has been developed in terms of above parameters.

Keywords: *factorial experimental design, gerotor pump, response surface methodology, flow rate, volume efficiency.*

1. INTRODUCTION

Gerotor pumps belong to a group of planetary rotary pumps which are characterized by compactness, simple construction and possibility of diverse applications [1]. They represent the mechanism with internal trochoidal gearing which consists of two components, internal and external gears with different load capacities [2]. The internal gear has the role of a rotor and it has one gear tooth less than external gear, which has the role of stator. The gerotor functiones as a pumping mechaninsm, when the fluid from inlet port goes through the gear teeth. The rotor sets in motion the fluid and it creates the inlet and outlet port depending on the position of meshing gear teeth. The operation priciple of gerotor pump has many advantages and therefore it can be fully implemented in the fields where gear pumps with external and internal gearing or piston pumps are used [3-5].

Experimental design is set up with aim to increase the efficiency of gerotor pump where the influence of the factor on the change in the flow rate and volumetric efficiency is analysed. The efficiency of the process and the product quality are incresed by the right selection of experimental design.

Factorial experiment design represents statistical method for processing experimental results when the subject of the study is influenced by several factors on multiple levels [6].

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There are many diverse methods for the selection of the optimal parameters of gerotor pump.

Hsie and *Hwang* have designed the rotor of vacuum pump by introducing cycloidal curves with variable trochoid ratio and they have concluded that the rotor profile is the key factor for improving gerotor pump performances. They have proposed a mathematical model applied to simulate the gerotor pump and cycloidal speed reducer [7-9]. Tong and others are the first to design non circular internal gerotor gear and develop the complete theory, as well as, the algorithm for designing non circular internal gerotor [10]. *Jung Sung-Yuen* and others have used Taguchi method for determing optimal rotor shape and design parameters of the pump. Obtained results allow the designer and the oil pump manufacturer to achieve the greater efficiency [11]. *Bonandrini* and others have presented a complete description of specific geometry that is applied with rotary pumps with internal gearing. The obtained mathematical model allows designer to obtain comletely defined rotor profile in parametric shape [12].

Based on literature review it is concluded that there is no application of factorial experiment for optimization of gerotor pump parameters in available papers. Factorial experiment is applied in this paper, in order to obtain optimal parameter values of gerotor pump and estimate statistical significance of certain factor effects. This method is mainly present in other fields of research and the success of its application is comfirmed [6,13].

Basic research in this paper is analysis and optimization of influential factors of the trochoid radius, the number of revolutions and the pressure on the change in the flow rate and volumetric efficiency.

Application of factorial experiment significantly shortens the time needed to perform the experiment and it takes smaller number of experimental samples than with classical experiment.

Software Minitab 16 is applied for statistical processing of experimental results and optimization of the gerotor pump parameters, the flow rate and volumetric efficiency.

2. FACTORIAL DESIGN OF EXPERIMENTS

Design of Experiments (DOE) is a mathematical methodology which has become one of the most popular statistical technique used in different industries and academic fields. Factorial designs are most frequently employed in engineering and manufacturing experiments. There are three different types of experimental designs: full-factorial, fractional factorial, and Taguchi orthogonal arrays. Factorial design is an efficient tool for estimating the influence of individual variables and studying their interactions using the minimum number of experiments [13,14].

Although the full-factorial design requires a great number of experiments or calculations, it provides very accurate results on the interaction among the factors, so that the conclusions are highly credible and reproducible.

The advantage of factorial experiement and its main characteristic is that all levels of one factor are combined with all levels of remaining factors. The efficiency of the experiment can significantly increase in conditions of factorial experiment, if favourable type of plan is chosen [6].

Experimental design with three levels is denoted as 3^n factorial experiment. It means that n represents the number of considered factors where each factor is on three levels. The factor levels in this paper are expressed numerically as 0, 1, 2 and values -1, 0, and +1 can be otherwise considered, but it can be confusing in terms of design with two levels where 0 is reserved for the central point. This type of plan is often the basis of complex plans, because of its good characteristics, although it has a less significance than the plan 2^n for the same purpose [13].

The total number of factor level combinations in factorial experiment is indicated by the factor level product. The factorial experiment 3^3 is selected in this paper and it has three factors where each factor has three levels constituting the Matrix 27.

3. EXPERIMENTAL PART

In accordance with defined aims of research it is planned to test the pumps with embedded gear pairs of different gearing geometry. The experimental tests are performed in the laboratory for testing PPT Hydraulics, with simulation of real conditions of pump exploitation. The measurements are performed for two different models of gear pairs based on set experimental design. Individual gear pairs are shown in Figure 1. Testing program is made in accordance with intern standards of PTT Hydraulics [15] and according to set matrix.

The influence of considered factors and its interaction on the change in the flow rate and volumetric efficiency of the gerotor pump are studied, based on the measured values.

The considered factors and its levels are given in Table 1. Three factors are chosen for the experimental design: the coefficient of the trochoid (A), the number of revolutions (B) and the pressure (C) with three different levels respectively.

Table 1. Factors and factor levels

Factor	Factorial symbol	Levels		
		0	1	2
The coefficient of the trochoid (-)	*A*	1.375	1.575	1.675
The number of revolutions (rpm)	*B*	500	1000	2000
The pressure (bar)	*C*	1	5	10

Figure 1 shows the pump models with different coefficient of the trochoid representing the levels of one of the factors which are considered in this paper.

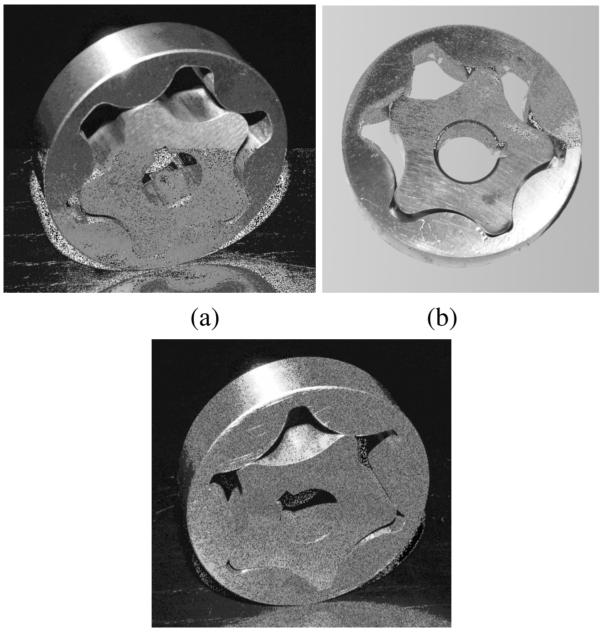


Figure 1. Gear pair models of the gerotor pump (a) the coefficient of the trochoid 1.375, b) the coefficient of the trochoid 1.575, c) the coefficient of the trochoid 1.675

Table 2 shows experimentally measured values of the flow rate while Table 3 shows experimental results of the volumetric efficiency.

Table 2. Experimental results of the flow rate

B (rpm)	*C* (bar)	*A*		
		0	1	2
0	0	6.90	13.45	26.95
0	1	6.50	13.10	26.55
0	2	5.30	12.20	25.70
1	0	6.80	13.55	27.10
1	1	6.30	13.05	26.55
1	2	5.00	12.00	25.35
2	0	6.85	13.40	26.70
2	1	6.45	12.95	26.20
2	2	5.55	12.15	25.30

Obtained experimental results based on set experimental design are processed by statistical software *Minitab* 16 [16].

Table 3. Experimental results of the volumetric efficiency

B (rpm)	*C* (bar)	*A*		
		0	1	2
0	0	0.9928	0.9963	1
0	1	0.9352	0.9704	0.9852
0	2	0.7626	0.9037	0.9536
1	0	0.9855	0.9927	1
1	1	0.9130	0.9560	0.9797
1	2	0.7246	0.8791	0.9354
2	0	0.9856	1	1
2	1	0.9281	0.9664	0.9813
2	2	0.7986	0.9067	0.9476

2.1 Results and discussion

The statistical significance of the developed models has been evaluated using an analysis of variance (ANOVA) and the accuracy of the models has been further justified through a regression analysis, and normal plot of residuals.

The key feature of using the RSM method is its ability to identify the combination of variable settings that jointly optimize a single response or a set of responses.

Tables 4 and 5 respectively show obtained results for the change in the flow and the volumetric efficiency based on performed ANOVA analysis.

The factor influence can be determined by Fisher's distribution for 99%, 95% or 90% of probability, based on the number of degrees of freedom of factor and the number of degrees of freedom of the error. The F values, obtained by the analysis, must be higher than the values of Fisher's distribution for adequate number of the degree of freedom [14,16]. Based on Fisher's distribution for 99% of probability, (F=8.40), it can be concluded that the pressure and square contribution of the pressure influence the change in the flow rate, besides the significant influence of the number of revolutions.

Analysis of variance (ANOVA) is applied to estimate the significance of the model. Based on percentage contribution of the factor on the change in the flow rate (Table 4), it can be concluded that the number of revolutions with 99.44 has the greatest influence.

The data analysis for volumetric efficiency is performmed in the same way as the change in the flow rate analysis. The Factor C (the pressure), with percentage contribution of 53.15%, has the greatest factor influence on the volumetric efficiency (Table 5), while the factor B (the number of revolutions) affects it with 19.87%. The interaction $B*C$ with percentage contribution of 14.12% also has a significant influence on the volumetric efficiency . According to Fisher, the pressure has the greatest influence for 99% of probability, followed by the number of revolutions, square contribution of the factor $B*B$ and the ineraction of the factor $B*C$.

Table 4. The results of surface analysis and percentage contribution of the factor for change in the flow

Source	DF	Seq SS	Adj SS	Adj MS	F	P	Pr (%)
Regression	9	1892.71	1892.71	210.30	9557.60	0.000	
Linear	3	1892.40	1794.98	598.33	27192.14	0.000	
A	1	0.08	0.09	0.09	3.91	0.065	0.00
B	1	1882.48	1784.06	1784.06	81080.38	0.000	99.44
C	1	9.84	9.21	9.21	418.73	0.000	0.52
Square	3	0.24	0.24	0.08	3.65	0.034	
A*A	1	0.00	0.00	0.00	0.15	0.701	0.00
B*B	1	0.00	0.00	0.00	0.01	0.929	0.00
C*C	1	0.24	0.24	0.24	10.80	0.004	0.01
Interaction	3	0.08	0.08	0.03	1.19	0.344	
A*B	1	0.07	0.07	0.07	3.38	0.083	0.00
A*C	1	0.00	0.00	0.00	0.02	0.892	0.00
B*C	1	0.00	0.00	0.00	0.16	0.696	0.00
Residual Error	17	0.37	0.37	0.02			0.03
Total	26	1893.09					

Table 5. The results of surface analysis and percentage contribution of the factor for volumetric efficiency

Source	DF	Seq SS	Adj SS	Adj MS	F	P	Pr (%)
Regression	9	0.131479	0.131479	0.014609	30.43	0.000	
Linear	3	0.101989	0.095026	0.031675	65.97	0.000	
A	1	0.000015	0.000009	0.000009	0.02	0.894	0.01
B	1	0.027749	0.033680	0.033680	70.14	0.000	19.87
C	1	0.074225	0.060799	0.060799	126.63	0.000	53.15
Square	3	0.009723	0.009723	0.003241	6.75	0.003	
A*A	1	0.001470	0.001470	0.001470	3.06	0.098	1.05
B*B	1	0.006133	0.006133	0.006133	12.77	0.002	4.39
C*C	1	0.002120	0.002120	0.002120	4.41	0.051	1.52
Interaction	3	0.019766	0.019766	0.006589	13.72	0.000	
A*B	1	0.000025	0.000025	0.000025	0.05	0.823	0.02
A*C	1	0.000028	0.000028	0.000028	0.06	0.811	0.02
B*C	1	0.019713	0.019713	0.019713	41.06	0.000	14.12
Residual Error	17	0.008162	0.008162	0.000480			5.85
Total	26	0.139641					

Response surface plots are used to study the factor interaction between the considered variables.

The plots of the change in the flow rate, depending on the number of revolutions and the pressure, are obtained with factor analysis (Figures 2 and 3). It may be noted that the flow rate increases with the increase of the number of revolutions at any pressure value.

The volumetric efficiency increases when the number of revolutions increases, and the pressure decreases, which can be seen in Figures 4 and 5. The minimum volumetric efficiency is obtained at maximum pressure and minimum value of the number of revolutions.

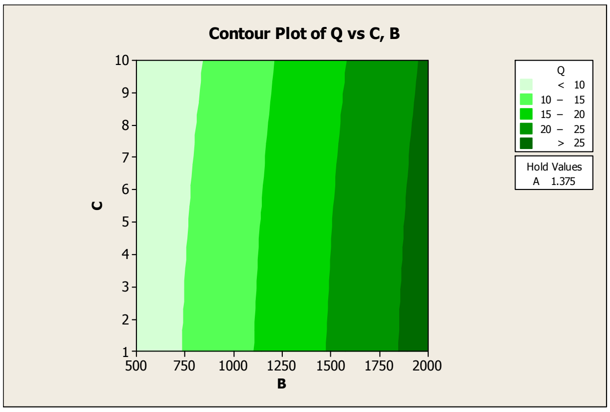


Figure 2. Contour Plot of the change in the flow rate depending on the number of revolutions and the pressure

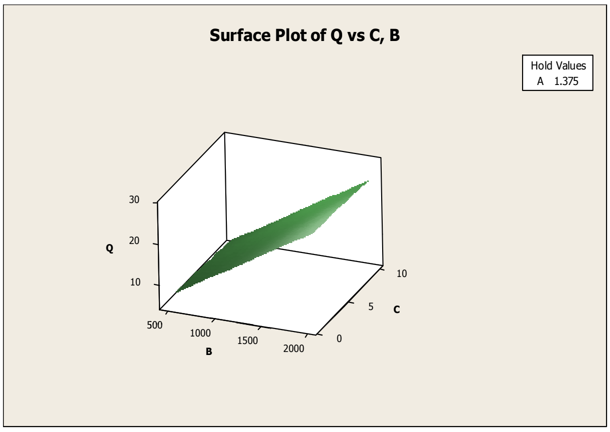


Figure 3. Surface Plot of the change in the flow rate depending on the number of revolutions and the pressure

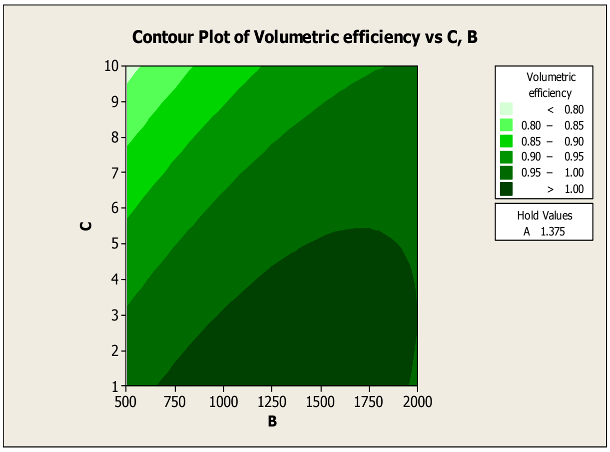


Figure 4. Contour Plot of the volumetric efficiency depending on the number of revolutions and the pressure

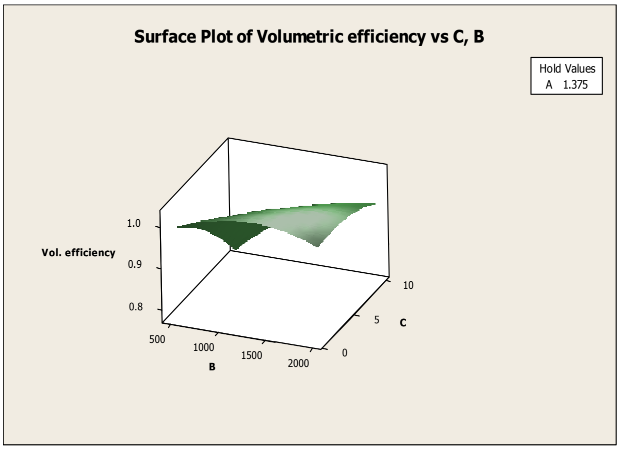


Figure 5. Surface Plot of the volumetric efficiency depending on the number of revolutions and the pressure

2.2 Regression analysis

It is possible to predict by a mathematical model how the output variables (responses) change and how they are related to other influential factors. Therefore, the dependence of change in the flow rate, that is to say, the volumetric efficiency, on the influential factors is presented by the mathematical model. Table 6 shows the results of the regression analysis for the change in the flow rate. While Table 7 shows the results of the regression analysis for the volumetric efficiency. The most influential factors and its interactions are used to display the mathematical model, that is to say, the factors and factors interactions of small values of percentage contribution are not taken into consideration in regression equations (see Tables 4 and 5).

Mathematical models enable to identify the independent variables and variable interactions which mostly influence the responses. The significance of the independent variables on the responses depends on the regression coefficient.

The adequate response equation, i.e. change in the flow rate (Q), is obtained by the regression analysis of statistical data and it is showed in the equation (1). The equation form of the regression analysis for volumetric efficiency (η) is given in equation (2).

$$Q = 0.185802 + 0.0133897\,B - 0.0526852\,C - 0.00996914\,C*C \quad (1)$$

$$\eta = 0.947807 + 0.00015609\,B - 0.0279708\,C - 6.51185e\text{-}008\,B*B + 1.17685e\text{-}005\,B*C \quad (2)$$

The higher value of the regression coefficient, the greater effect of the independent variable on the response is obtained. With model for the change in the flow rate equations (1), the number of revolutions has the highest value of the regression coefficient, followed by the pressure. The pressure has a significant value of the regression coefficient with volumetric efficiency model (equation 2).

Figure 6 (a) and (b) show the normal probability plots of the residual for the gerotor pump with respect to the change in the flow rate and volumetric efficiency respectively.

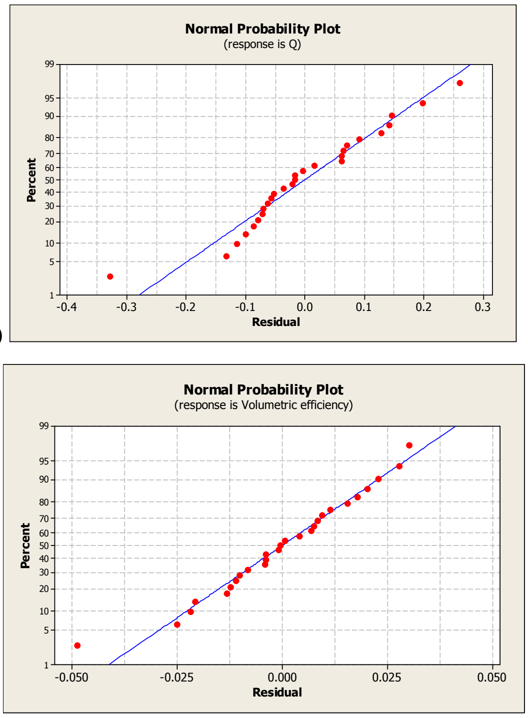
Table 6. Coefficients of the regression analysis

Term	Coef	SE Coef	T	P
Constant	0.185802	0.0928696	2.001	0.057
B	0.013390	0.0000469	285.372	0.000
C	-0.052685	0.0356161	-1.479	0.153
C*C	-0.009969	0.0031098	-3.206	0.004

Table 7. Coefficients of the regression analysis

Term	Coef	SE Coef	T	P
Constant	0.947807	0.0292896	32.3599	0.000
B	0.000156	0.0000511	3.0525	0.006
C	-0.027971	0.0025702	-10.8825	0.000
B*B	-0.000000	0.0000000	-3.3787	0.003
B*C	0.000012	0.0000019	6.0571	0.000

The residuals in each plot generate near the straight line, implying that the errors are distributed normally. The observations of this plots responses indicate that every single model suggested are adequate and satisfied.



(a)

(b)

Figure 6. Normal probability plot of the residual for the gerotor pump for each response of (a) change in the flow rate (b) volumetric efficiency

The optimal factor variant for using RSM may be obtained by this statistical analysis in *Minitab*. The use of RSM provides optimization of the response by manipulating independent variable, which is a key advantage. Figure 7 depicts the optimization value of all the responses.

The optimization plot (Figure 7) is very useful in terms of design, as it allows adjusting the values of the input variables and simultaneously seeing the effects of these changes in the responses, without running more simulations. It is necessary to select the optimal values of the coefficient of the trochoid radius, the number of revolutions and the pressure, in order to obtain the maximum values of change in the flow rate and volumetric efficiency. In order to obtain the maximum values for change in the flow rate and volumetric efficiency, the same factor levels are suitable, i.e. the coefficient of the trochoid radius 1.375, the number of revolutions 2000 rpm and the pressure of 1 bar. The experiment is performed to increase the value of change in the flow rate and volumetric efficiency of the gerotor pump.

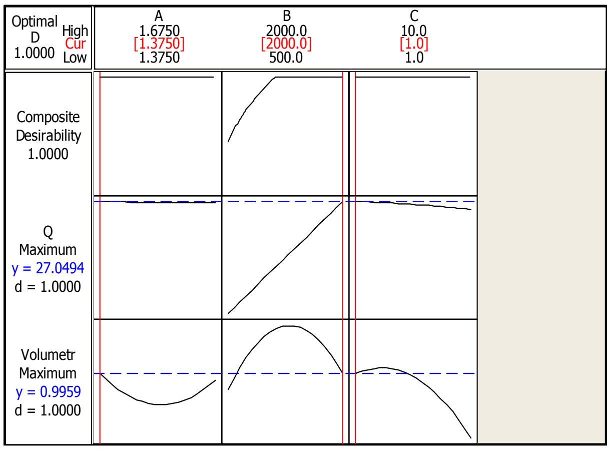


Figure 7. Optimization results

4. CONCLUSION

This study has shown that factorial experimental design approach is an excellent tool and can successfully be used to develop empirical equation for the prediction and to discover the optimal parameter values of the gerotor pump.

Based on the performed analysis, it can be concluded that the number of revolutions mostly influence the chnage in the flow rate. And based on the displayed contour plot and surface plot, it can be noted that the change in the flow rate increases as the number of revolutions increases at all pressure values of the gerotor pump.

By analyzing data for volumetric efficiency, it is concluded that pressure of the gerotor pump has the greatest influence. The volumetric efficiency increases when the number of revolutions increases while the pressure decreases.

The optimal factor variant of the gerotor pump is the same in both cases of analysis. The obtained optimal values of the factor are: the coefficient of the trochoid radius 1.375, the number of revolutions 2000 rpm and the pressure 1 bar. The highest value of the change in the flow rate and the highest value of the volumetric efficiency are obtained, at these factor levels.

The regression equation is also given in the paper by which it is possible to follow the output value in the function of influential factors.

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ИЗБОР ОПТИМАЛНИХ ПАРАМЕТАРА ГЕРОТОР ПУМПЕ ПРИМЕНОМ ФАКТОРИЈАЛНОГ ЕКСПЕРИМЕНТА

Л. Ивановић, С. Величковић, Б. Стојановић, М. Кандева, К. Јакимовска

У овом раду је примењена метода факторијалног експеримента за проучавање утицаја фактора на проток и запремински степен искоришћења геротор пумпе. Разматрани фактори су коефицијент полупречника трохоиде, број обртаја погонског вратила пумпе и радни притисак, при чему је први фактор геометријски параметар, а остали су радни параметри. Факторијални дизајн и метод површинског одзива су примењени за процену и оптимизацију ефеката параметара на проток и запремински степен искоришћења. Коришћен је дизајн експеримента 3^3 код кога сваки фактор има три нивоа. Анализом разматраних фактора добијена је њихова оптимална варијанта, и то: коефицијент полупречника трохоиде 1.375, број обртаја погонског вратила пумпе 2000 o/min и радни притисак 1 bar, при којој се добија највећа вредност протока и највећа вредност запреминског степена искоришћења. Развијен је математички модел за предвиђање протока и запреминског степена искоришћења у погледу већ наведених параметара.