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The Influence of Content on the Development of Students' Critical Thinking in the Initial Teaching of Mathematics

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Abstract

This paper points out the importance of developing critical thinking in the initial teaching of mathematics and highlights some of the problems associated with its realization. In this context, the authors draw attention to the definition of critical thinking and the role of content (tasks) in its development. Starting from the specificity of initial teaching of mathematics and students' age, critical thinking is operationalized through specific skills (formulation of the problem, reformulation of the problem, evaluation, sensitivity to the problems), that become apparent in work with mathematical content. On a sample of 246 students (9.5–10.4 years old), an experimental study (experiment with parallel groups) was organized in order to examine whether, with the selection of appropriate content (tasks), critical thinking in the initial teaching of mathematics can be developed. The results show that, with the appropriate choice of content, students' critical thinking, viewed as a whole, can be developed and that, in that process, we can significantly influence the development of each of its skills (formulation of the problem, reformulation of the problem, evaluation, sensitivity to the problems).

Key words: *evaluation; formulation of the problem; mathematics; reformulation of the problem; task.*

Introduction

The time in which we live is marked by an intense and turbulent development in all spheres of life, rapid development of science and technology, increase of knowledge, considerable rise in the amount of information, great influence of media, globalization, mobility of people, rapid flow of ideas, and so on. It is therefore natural that the creators of the educational system are constantly faced with the need to redefine the aims and objectives of education in order to form a personality whose competencies match the requirements and the needs of the time he/she lives in. Today, everybody agrees that the basic feature of the concept of education, of school, teaching and learning must be an increased focus on developing thinking skills, and less on the acquisition of knowledge (Jacobs et al., 2007; Sfard & Kieran, 2001). Hence, a number of recommendations, resolutions and declarations have been put forward by UNESCO and the *European Council for Education*, over the last few years. They draw attention to the need for emphasizing increased activity of students in the learning process, the promotion of gifted students, on personal development, creativity, autonomy, on developing thinking in general, and, in particular, on developing critical thinking skills. In this context, it is emphasized that "critical thinking is not an academic fad; it is an essential skill for living in the information age" (Connor-Greene & Greene, 2002, p. 324) and that "critical thinking provides a vehicle for educating the mind" (Paul & Elder, 2008, p. 88). If critical thinking is not fostered and developed in the classroom, the chances of students' success are reduced (Irfaner, 2006), and therefore, the development of critical thinking skills has become a prerequisite to education (Sezer, 2008).

Teaching of mathematics, today, is also characterized by an increased "focus on developing skills of thought" (Špijunović & Maričić, 2011a, p. 975). This is understandable, taking into account all the more noticeable trends in teaching mathematics where students should not only acquire and master relevant knowledge, but should be enabled, as far as possible, for the application of knowledge, for the critical attitude towards the contents and adequate evaluation in all stages of teaching process. That is, they should be trained for "research, problem solving, creative thinking, information processing, logical reasoning and evaluation of results" (Felda & Cotič, 2012, p. 51).

The demand for the development of critical thinking in mathematics education is not new. In 1938, H. Fawcett promoted the idea that the abilities and critical thinking skills were an integral part of the reasoning in mathematics teaching in which students, on a daily basis, find themselves in a position to conclude, find solutions and make assumptions that must be critically evaluated (Marcut, 2005, p. 61).

However, practice shows that the educational system has not performed well in consistently producing critical thinkers (Barbuto, 2000; Burbach et al., 2004; Lizzio & Wilson, 2007; Paul, 2005; Pithers & Soden, 2000). Emphasis is especially placed on the effects of standardization of teaching and its product of "teaching for the test" as it ignores the process of learning, and emphasizes the content (Landsman & Gorski, 2007; Lundquist, 1999; Sheldon & Biddle, 1998). If we want to develop students' critical thinking in mathematics, then "the goal for mathematics educators who want to instill critical thinking skills in their classrooms is to think of their students not as receivers of information, but as users of information" (Ebiendele, 2012, p. 43).

If teachers are supposed to successfully develop critical thinking skills of their students in the initial teaching of mathematics, first of all, they "must have a clear idea of what critical thinking is" (Lipman, 1988, p. 39).

We find a number of different definitions of critical thinking and of different concepts of critical thinking in literature (Ennis, 1996; Gleser, 1984; Kvaščev, 1969; McPeck, 1981; Meyers, 1988; Siegel, 1988 and others). While some of them are rather general definitions, others are narrow and created for a specific context of use, but neither refer to elementary mathematics education. This diversity suggests a very complex phenomenon, which, according to Facione, "cannot be fully determined by cognitive abilities" (1996, p. 8); and in the opinion of Flores and associates "the concept of critical thinking is too complex to be limited to a narrowly defined construct" (Flores, Gina, Matkin, Burbach, Quinn & Harding, 2012, p. 216).

However, it is necessary, "without going into epistemological, logical and psychological foundation, to, at least in a global sense, define what is meant by critical thinking" (Špijunović & Maričić, 2007, p. 114), that is, to clearly determine which cognitive abilities and skills make its content (Ashton, 1988, p. 2). If we do not clarify the concept of critical thinking, "we will be shooting arrows at a target that we cannot see" (Mulnix, 2012, p. 464).

It means we need to operationalize this concept, in other words, we need to identify skills of critical thinking emphasized in elementary mathematics education classes, taking into account the specificities of elementary mathematics education, specificity of mathematical contents and the age of students. Given the fact we were not able to find a definition of critical thinking that refers to elementary mathematics education, the problem of operationalizing this concept is imposed as the first problem in our aspiration to develop critical thinking in this type of education. For that purpose, and starting from the essence of the term *critical thinking,* definitions of other authors created for a wider context of use, and given the specificities of elementary mathematics education and the age of students, we defined critical thinking as a complex intellectual activity that includes the following abilities:

- formulation of the problem,
- reformulation of the problem,
- evaluation,
- sensitivity to the problems (Maričić, 2011b, p. 133; Špijunović & Maričić, 2011b, p. 67).

Each of the listed components of critical thinking was defined via narrower skills. We will explain and offer an example of a task to illustrate the expression of critical thinking.

Formulation of the problem implies:

a) student's ability to perceive the mathematical problem and to formulate it on the basis of problem situation.
Example: When the car covered 210 kilometers, it ran out of gas. But, it still needs to cover twice as much distance. Write all the things that you can calculate here.

Students should formulate as many problems as they can, based on the problem situation, making sure that all elements from the problem formulation correspond to the initial, given elements, and that they are derived from them, but also that they clearly express the relationship between the given and the required, between the possible and the impossible. Moreover, the problem must be clear, concise and comprehensible, and it must not leave students in any dilemmas or doubts.

b) Detection of mathematical symbolism and transferring that symbolism to spoken language, spotting the connections between mathematical symbols and being critical to presentation of these connections in words.
Example: Circle the letter before the task you think corresponds to the given entry.
900 – (300 + 200) =
a) Marko had 900, he got 300€ from his brother and 200€ from his sister. How much money does Marko have now?
b) Marko had 900€. He gave 300€ to his brother, and he got 200€ from his sister. How much money does Marko have now?
c) Marko had 900€. He gave 300€ to his brother, and 200€ to his sister. How much money does Marko have now?
d) Marko had 900€. He got 300€ from his brother, and he gave 200€ to his sister. How much money does Marko have now?

c) Search for the inherent properties of the formulation of the problem, the identification of shades in the formulation of the problem and the use of precise spoken and precise mathematical language (Maričić 2011a).
Example: Nikola solved 120 problems from a collection of mathematical problems. He has 50 problems left to solve. What can you calculate here (Circle the letters before the correct questions)?
a) How many problems did Nikola solve?
b) How many problems less does Nikola have compared to the number of problems he still needs to solve?
c) What is the total number of problems Nikola has to solve?
d) How much time does Nikola need to solve the problems?
e) How many problems more has Nikola solved, compared to the number of problems he still needs to solve?

The reformulation of the problem includes:

a) linguistic reformulation of a mathematical task.
Example: Milica had so much money that when she gave half of the money to her sister, and the half of the rest to her brother, she was left with 20 €. How much money did Milica have?

Based on the perception of relationships between the given data, the student will formulate the problem differently: *A quarter of the total sum is 20 €.*

b) Drawing conclusions based on the identification of the connections and relationships in the content of a task, explained by clear arguments.

Example: Five competitors finished the race: Milan, Aleksa, Nikola, Saša and Goran, but we do not know in what order. We know Milan finished after Aleksa, but before Nikola and Saša. Nikola is behind Saša, and before Goran, who finished last. What is the order of competitors at the end of the race?

c) Identifying relationships among the terms of a task and turning the cognitive way in the opposite direction.
Example: Petar imagined a number. He subtracted 3 from that number, and added 4 to the obtained difference, divided the result by 2 and got the result 8. What number did Petar imagine?

The evaluation relates to:

a) evaluation of information.
Example: A mountaineer is climbing toward the top of the mountain. He encounters three families. The first family has two kids, the second twice as many, and the third family has as many children as the first two families together. How many persons are climbing toward the top of the mountain?

Evaluation is based on careful observation of relationships in the content of the task, separating the given and the required, important and unimportant, identifying surplus data in the task, recognizing the purpose of the information, asking questions that help ascertain the essence of the task.

b) Evaluation of solutions.
Example: An elderly woman planted flowers in the garden, on a triangular surface. What will be the perimeter of the flower garden, if the sides of the triangle are the following whole numbers?
(a) 12cm; b) 9m; c) 2m; d) 6dm; e) 4m)

c) Evaluation of the opinions of the authorities.
Example: A teacher evaluated the following tasks as correct:

a) $4 \cdot 8 - 2 + 0 = 30$ b) $200 + 2 \cdot 50 = 100$ c) $2 \cdot 0 \cdot 3 + 203 = 209$

Do you agree with the teacher's evaluation? Explain your answer.

Sensitivity to the problems implies:

a)assessing the reality of the situation in the problem and the resulting solution, and taking into account the circumstances in which the assignment was given.
Example: A boy scares three sparrows and they fly away. How many boys would it take to scare 9 sparrows?

b) Identification and detection of hidden and implicit information in the formulation, abstaining from fast conclusions, sensitivity to the detection of the way of problem solving.
Example: A customer asked for 20 pens from the salesman, and a box of pens contains 12 items. It takes 1 second to remove a pen from the box. What is the minimum time the salesman needs to remove 20 pens?

c) The ability to identify inconsistencies and contradictions in the problem formulation and problem requirements, identification of the redundant and incomplete resulting data arising from the reality of a given situation and discovering the pitfalls in the formulation of the problem (Maričić, 2009, pp. 485-486; Maričić, 2011b, pp. 134-155).
Example: Mateja has two coins, with the total worth of 7 dinars. One of them is not a 2-dinar coin. What coins does Mateja have?

The above mentioned and some other operationalizations of critical thinking in the initial teaching of mathematics are only the first steps in its development. The next question is *How to develop students' critical thinking in the initial teaching of mathematics?* In this regard, there are a number of discussions, papers and research studies (Abrami et al., 2008; Case, 2005; Connor-Greene & Greene, 2002; Duplass & Ziedler, 2002; Halpern, 1998; Hemming, 2000; Heyman, 2008; Nelson, 1994 and others) but there are no ready recipes and clear answers. Certainly, the approach to the development of critical thinking, "focused on the adoption of rules, procedures and skills of logical thinking is not advisable, because students of this age are not capable of acquiring knowledge by deduction, nor of using strictly logical thinking procedures in reasoning, evaluating and learning" (Maričić, Špijunović, & Malinović Jovanović, 2013, p. 205). Ebiendele (2012, p. 43) points out that "learning environments that actively engage students in the investigation of information and the application of knowledge will promote students' critical thinking skills". A meta-analysis of over 1300 experimental studies conducted in the period from 1960 to 2005, showed that instruction that included critical thinking components, whether delivered implicitly or explicitly, improved learners' critical thinking skills (Abrami et al., 2008).

Certainly the development of critical thinking of students in the initial teaching of mathematics is conditioned by the nature of mathematical content and relies on students' knowledge, because only "by practicing and putting students in a position to think critically on a specific content, can critical thinking be developed" (Maričić, 2009, p. 485). Such view is also supported by McPeck (1981, p. 3), who believes that it is "conceptually impossible to talk about critical thinking at all, because it is, like any other thinking, always thinking about *something*". Therefore, it is necessary for the abilities and skills of critical thinking not to be adopted only within one particular subject, but to "explicitly incorporate them into the process of learning" (Gleser, 1984, p. 93). So, without adequate content "it is unlikely to ensure that any strategy, method, form or educational system, by itself, has a significant influence on the development of critical thinking in the initial teaching of mathematics" (Maričić et al., 2013, p. 206). To that end, in studying the issue of developing critical thinking, we started from the attitude that critical thinking can be successfully developed if it is developed in everyday learning situations and on contents (tasks) whose solution requires skills of critical thinking.

In light of these views, we wanted to examine whether, by carefully planning the selection of appropriate content (tasks), critical thinking in the initial teaching of mathematics can successfully be developed. In addition, the adequacy of the content (tasks) assumes the content the solution of which demands the expressed critical thinking skills listed in the operationalized definition of the term.

Methods

Participants

The study sample consisted of 246 third grade elementary school students (9.5 to 10.4 years old) from three elementary schools in the Republic of Serbia. We randomly selected primary schools in the experimental (N=123) and control group (N=123). The experimental group consisted of students from five classes of one elementary school, and the control group of students from six classes from two schools, because we could not provide an equivalent sample to the experimental group at one elementary school. The unification of the experimental and control group was not done artificially by moving students from one class to another, due to the conditions of work in the school, but we controlled the dependent variable by the statistical approach of analysis of covariance (ANCOVA), because "adjusted" variance corresponds to the variance we would obtain from experimentally homogenized groups. Covariance analysis is based on obtaining a reduced calculation of the experimental error, taking into account the regression of the final measurement (Y) in relation to the initial measurement (X). Students in both groups belonged to the socially homogeneous middle social class.

Data Analysis

The experimental method was used in the study, that is, the experiment with two parallel student groups – experimental and control was applied. We introduced an experimental program into the experimental group, after the initial measurement of the development of critical thinking.

The experimental program was implemented during the 2012/2013 school year within the regular mathematics curriculum in the third grade of elementary school through 27 activities on the following topics:

- Rectangle and square;
- Written addition and subtraction up to 1000;
- Triangle;
- Written multiplication and division up to 1000;
- Fractions;
- Mathematical expressions.

The arrangement of the teaching content in the experimental program was determined according to the current mathematics curriculum for the third grade of elementary schools in Serbia and it fully followed the program. The tasks were carried through individual, group and frontal work with the help of researchers and by giving instructions for work, through frontal and group discussions within the class and clear and meaningful frontal and group feedback for each task in the activity. The experimental program was carried out by class teachers of the experimental group's classes, in accordance with the designed instructions, with complete lesson preparations for carrying out specified teaching units. The activities were designed to include the selected content (tasks) the solution of which emphasized some of the operationalized skills of critical thinking (problem formulation, problem reformulation, evaluation and problem sensitivity). One exercise, which was realized within one school lesson, referred to one skill of critical thinking.

First, a pilot study was conducted on a sample of 55 students, based on which the experimental program was verified and the final forms of the instrument were made, and then, the realization of the experimental program began.

Instruments

The specific research problem required the development of two tests for measuring the development of students' critical thinking in the initial teaching of mathematics:

1) ITCT – test to determine the initial state of critical thinking development of students,
2) FTCT – test to determine the final state of critical thinking development of students.

We constructed the test ourselves, and made two equivalent forms of the test. The structure of the test consisted of 12 tasks. Based on the offered operationalization of critical thinking, we made a selection of tasks, so that the process of solving each task emphasized a certain skill of critical thinking (problem formulation, problem reformulation, evaluation, problem sensitivity), in other words, one of its specialized skills. Each task of the test was scored with 5 points and the maximum number of points that could be achieved on the test was 60.

In order to justify the use of the test, the metric characteristics of the test were determined. The objectivity of the test was provided by placing each student in a roughly equal test situation, by making the independent investigators act by unique instructions and by assessing the tasks in the same way using the key. We determined the logical and content validation of tests by defining the tests' correspondence with the requirements of the curriculum and the contents they refer to. Discrimination (sensitivity) of the test was determined by item analysis, by determining the ease index (p) for each task and the index of difficulty (q) of the given task. The discriminant value coefficient varied between .12 and .25 (Task 1 (.23), Task 2 (.13), Task 3 (.22), Task 4 (.25), Task 5 (.24), Task 6 (.15), Task 7 (.12), Task 8 (.15), Task 9 (.25), Task 10 (.12), Task 11 (.12), Task 12 (.17)). Instrument accuracy was determined by calculating the Cronbach's alpha coefficient (α=.88), which indicates a high instrument reliability. The reliability of the test was also determined by retest procedure, by calculating the correlation coefficient (Pearson's correlation coefficient) between the results achieved by the respondents on the equivalent forms of the critical thinking test. The resulting correlation coefficient was .81.

The first test was carried out before the effects of the experimental factor – the initial, and the second after the end of the experimental program – the final.

The data obtained in the survey were analyzed using the statistical software package IBM Statistics SPSS20, with the use of one factor analysis of variance (ANOVA), the analysis of covariance (ANCOVA) for statistical standardization of groups and longitudinal monitoring of the effects of the experimental program.

Results and Discussion

Before the introduction of the experimental program, initial measurements of the development of students' critical thinking were carried out. On the initial test of critical thinking (ITCT) students in the experimental group scored an average of 18.40 points out of the maximum 60, while students in the control group scored an average of 16.20 points (Table 1). The calculated F test between groups ($F_x(1,244)=3.032$, $p=.083$) indicated that at the initial measurement the groups were rather uniform in the level of critical thinking development and Levene test ($p=.630$) indicates no violation of the assumption of homogeneity of variance (Table 2).

The experimental program was introduced in the experimental group, while the control group performed the tasks in the usual way, after which the final measuring (FTCT) was carried out. On the final test of critical thinking, students in the experimental group achieved an average of 48.06 points (MAX=60), while students in the control group scored an average of 16.68 points, which approximates to the average number of points they won in the initial measurement (Table 1).

Table 1

The results of the experimental and control group at the initial and final test

Test	Group	N	Mean	Std.Dev	Std.Er.	95% Confidence Interval for Mean		Min	Max
						Lower Bound	Upper Bound		
Initial test	Experimental	123	18.40	10.070	.908	16.60	20.20	0	47
	Control	123	16.20	9.775	.881	14.45	17.94	0	47
	Total	246	17.30	9.965	.635	16.05	18.55	0	47
Final test	Experimental	123	48.06	11.375	1.026	46.03	50.09	5	60
	Control	123	16.68	8.764	.790	15.12	18.25	0	50
	Total	246	32.37	18.702	1.192	30.02	34.72	0	60

The means of the students in the experimental group progressed significantly in the development of critical thinking under the influence of the exercises from the experimental program in relation to the students of the control group (see Figure 1).

The F test between the groups in the final test ($F_y(1,244)=587.140$, $p<.001$) indicates that there are statistically significant differences in the development of critical thinking between the experimental and control group (Table 2).

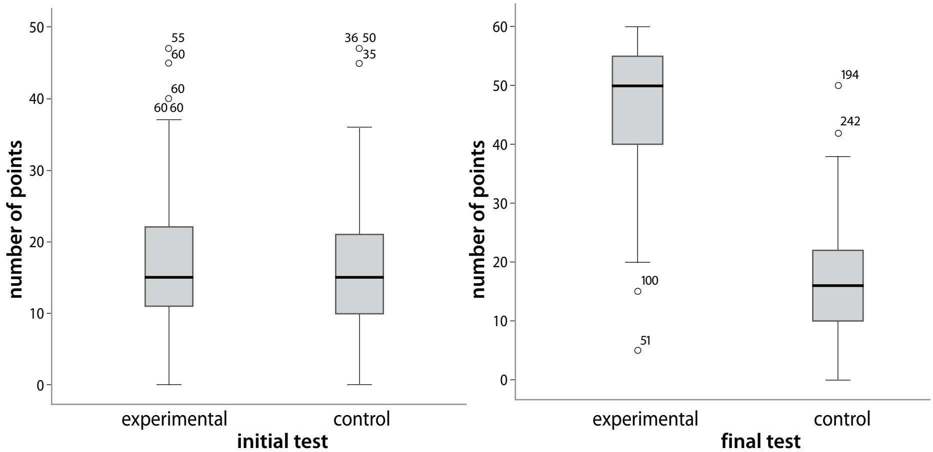

Figure 1 Results of the experimental and control group on testing

Table 2
ANOVA analysis - Test of Homogeneity of Variances

	Levene Statistic	df1	df2	Sig.
Final test	9.876	1	244	.002
Initial test	.233	1	244	.630

		Sum of Squares	df	Mean Square	F	Sig.
Final test	Between Groups	60536.102	1	60536.102	587.140*	.000
	Within Groups	25157.236	244	103.103		
	Total	85693.337	245			
Initial test	Between Groups	298.541	1	298.541	3.032	.083
	Within Groups	24028.797	244	98.479		
	Total	24327.337	245			

*The difference is significant at $p<.001$.

That the difference is real and that it is the result of the action of the experimental program is shown by the covariance (F=857.782, p<.001), which rejects the doubt that the difference is the result of discrepancies in the experimental and the control group (Table 3).

Table 3
ANCOVA analysis

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	70232.144[a]	2	35116.072	551.911	.000	.820
Intercept	27683.215	1	27683.215	435.091	.000	.642
Initial test	9696.043	1	9696.043	152.390	.000	.385
Group	54577.531	1	54577.531	857.782	.000	.779
Error	15461.193	243	63.626			
Total	343455.000	246				
Corrected Total	85693.337	245				

Based on these results we can conclude that the experimental program influenced the development of critical thinking of students in the experimental group and that the appropriate choice of content (tasks) can contribute significantly to the development of critical thinking of students in the initial teaching of mathematics, which confirms the initial idea about the importance of content selection (task selection) for developing critical thinking. Unfortunately, we were unable to compare the obtained results with the results of other research studies, as we failed to find research papers that studied the problem of developing critical thinking for this age group and in these classes.

On further analysis, we wanted to determine whether the choice of content influenced the development of each of the operationalized skills of critical thinking (formulation of the problem, reformulation of the problem, evaluation, sensitivity to the problems).

The analysis of student achievement on critical thinking skills showed approximately uniform results of the experimental and the control group in the initial test and a significant progress of students in the experimental group at the final measurement at all operationalized critical thinking skills in relation to the control group (see Figure 2).

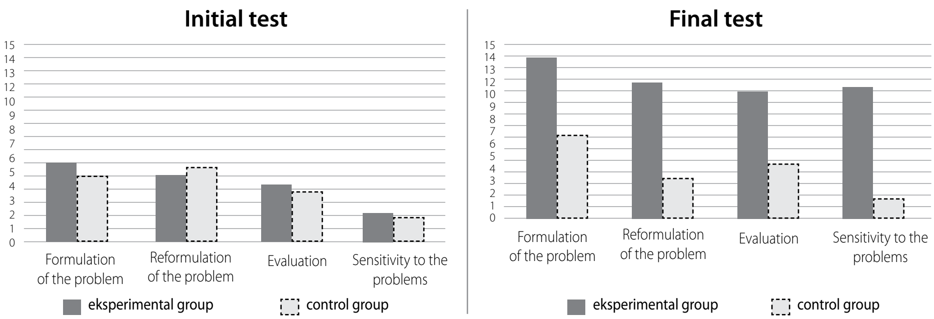

Figure 2 Students' achievement at the initial and final test on critical thinking skills

In the initial test of critical thinking (ITCT) students in the experimental group were most successful in tasks that involved the ability to formulate problems and scored an average of 5.96 points (*SD*=3.372) out of the maximum 15. Students in the control group scored an average of 4.93 points (*SD*=4.136) in the same tasks (Table 4). The variance between the groups in the initial measurement showed that even in the initial measurement there were statistically significant differences between the experimental and control group in the level of development of problem formulating skills (F_x=4.532, p< .05) (Table 5). Such a situation can be coincidental, but it can also be the result of using different textbooks, or the result of the strategy of teachers who create more problem situations in their teaching on the basis of which students should formulate problems. The effects of the experimental program were that students in the experimental group were significantly more successful in the final measurement (*M*=13.83, *SD*=2.268) and gained a significant difference in the development of critical thinking skills compared to students in the control group (*M*=7.08, *SD*=2.763). Based on the *F* ratio between groups at the final measure (F_y=438.405, p<.001), we can conclude that the experimental program significantly affected the development of problem formulating skills. This means that teachers, by choosing appropriate content in the initial teaching of mathematics, can enable their students to perceive mathematical problems, to formulate them on the basis of the problem situation, to critically perceive connections and relationships between mathematical symbols, to design and translate mathematical records to the concrete, real situation familiar to the students, to seek for specific properties in the formulation of the problem, and be able to precisely use mathematics and spoken language to express mathematical truths.

Table 4

Success of students in the experimental and control group on critical thinking skills

Critical thinking abilities	Group	N	Initial test			Final test		
			Mean	Std.Dev	Std. Error	Mean	Std.Dev	Std. Error
Formulation of the problem	Experimental	123	5.96	3.372	.304	13.83	2.268	.204
	Control	123	4.93	4.136	.373	7.08	2.763	.249
Reformulation of the problem	Experimental	123	5.08	4.481	.404	11.67	4.512	.407
	Control	123	5.65	4.340	.391	3.41	4.014	.362
Evaluation	Experimental	123	4.35	4.196	.378	10.93	4.405	.397
	Control	123	3.78	3.526	.318	4.59	4.227	.381
Sensitivity to the problems	Experimental	123	2.20	3.080	.278	11.18	4.879	.440
	Control	123	1.83	3.021	.272	1.59	2.814	.254

Table 5

ANOVA analysis

Critical thinking abilities	Source of Variation	df	Mean Square	F_X	p	Mean Square	F_Y	p
Formulation of the problem	Between groups	1	64.537	4.532**	.034	2800.40	438.405*	.000
	Within groups	244	14.239			6.388		
	Total	245						
Reformulation of the problem	Between groups	1	19.919	1.024	.313	4187.90	229.671*	.000
	Within groups	244	19.456			18.234		
	Total	245						
Evaluation	Between groups	1	19.919	1.326	.251	2473.17	132.710*	.000
	Within groups	244	15.021			18.636		
	Total	245						
Sensitivity to the problems	Between groups	1	8.232	.885	.348	5660.16	356.876*	.000
	Within groups	244	9.306			15.860		
	Total	245						

*The difference is significant at the p<.001
**The difference is significant at the p<.05

In tasks where students were supposed to express criticism of opinion in *reformulating problems*, there were no statistically significant differences (F_x=1.024, p=.313) between students in the experimental (*M*=5.08) and the control group

(*M*=5.65) in the initial measurement. The effects of the experimental program had students in the experimental group achieve significant progress in developing the skill of critical thinking (*M*=11.67), while students in the control group achieved fewer points per pupil compared to the initial measurement (*M*=3.41). The resulting variance between the experimental and the control group (F_y=229.671, *p*<.000) indicates a statistical significance of the observed differences. Therefore, we conclude that under the influence of the experimental program, there has been a development in the skill reformulating problem - as a component of critical thinking. With this we have shown that with adequate choice of content, students can be trained to think more deeply and critically and to perceive more accurately the connections and relationships in the content of the task and express them in the way that helps them understand its essence. They can draw conclusions based on perceived connections and relationships and explain them with clear arguments, training themselves to turn the cognitive way in the opposite direction, to think reversibly and to, more successfully, solve tasks that require problem reformulation.

In the initial check, the experimental (*M*=4.35) and control group (*M*=3.78) reached approximately equal results (F_x=1.326, *p*= .251) in tasks where students were supposed to show their ability to *evaluate*. After the introduction of the experimental program, students in the experimental group achieved a significantly better outcome (*M*=10.93) compared to students in the control group (*M*=4.59) and achieved a statistically significant difference (F_y=132.710, *p*<.001) in the development of this skill of critical thinking. This shows that the choice of appropriate content (tasks) can contribute to students having a more developed critical attitude towards the information, content, task, the obtained solution, and to the opinion of the authority, as well. This proves that the selection of appropriate content (tasks) can help students acquire a more developed critical attitude toward information, task content, obtained solution, but also toward the authorities' opinions.

When it comes to *sensitivity to the problems*, in the initial measurement there were no statistically significant differences in the level of development of critical thinking (F_x= .885, p=.348) between the students in the experimental and control group. Final testing shows that students in the experimental group scored significantly more points (M_y=11.18), compared to the initial measurement (M_x=2.20), while among the students of the control group those differences were minor (M_x=1.83, M_y=1.59). The resulting variance (F_y=356.876, *p*<.001) shows that the activities performed in the experimental program significantly contributed to students' development of sensitivity to the problems. This suggests that the teacher, by choosing appropriate content, can significantly contribute to the development of students' ability to critically assess the reality of the problem situation given in the problem, to be able to assess the reality of the solution taking into account the circumstances in which the problem is given, to notice and reveal the hidden and implicit information in the formulation of the problem, to refrain from the fast conclusions and to observe the inconsistencies, contradictions, and redundant and incomplete data in the formulation of the problem.

Conclusion

Despite the fact that critical thinking is an important task in mathematics education, there is no general consensus on issues related to its development. A number of issues related to its development remain open, and particularly concerning is the fact that teachers are not sufficiently trained to develop critical thinking (Broadbear, 2003; Maričić, 2010; Scriven & Paul, 2007). A part of the problem stems from generalizations and vagueness of the term critical thinking in general, and particularly in a specific area, but also from the lack of a unified point of view with respect to how critical thinking should be developed. In the first part of the paper, we theoretically analyzed the term *critical thinking,* by separating specific skills that comprise its content and that become pronounced when working with mathematical content in elementary mathematics education.

The empirical part of the work aimed to examine whether, by better planning the selection of appropriate content (tasks), critical thinking in the initial teaching of mathematics can successfully be developed. The results of experimental research suggest the following conclusions:

– by proper selection of content in the initial teaching of mathematics, critical thinking can be developed;

– the choice of content in the initial teaching of mathematics significantly influences the development of each operationalized critical thinking skill (formulation of the problem, reformulation of the problem, evaluation, sensitivity to the problems).

This research showed that the selection of content in mathematics education is very important for achieving tasks and objectives and achieving appropriate outcomes of mathematics education, and in this case, it is a successful development of critical thinking skills, which represent a necessary element of thinking in general, and an integral element of mathematical thinking in particular.

The obtained results show that critical thinking can be developed in students of junior grades of elementary school through an adequate selection of content, which further implies the need for educational policy makers to pay more attention to developing critical thinking in elementary school, not only in mathematics education, but in all subject areas. This is even more evident if we consider Plato's attitude that the educational direction a young person takes in his/her youth will greatly determine their future. All this suggests that critical thinking is not a property of adulthood; that it can be developed in students who have just started their education in a way that is acceptable for them. In addition, the results should help not only mathematics teachers, and other teachers, but also all those who are directly or indirectly involved in mathematics education. It offers a more secure support and gives them a clearer guidance in the design and organization of the initial teaching of mathematics, in order to be successful in achieving the requirements for the development of students' critical thinking, and thus contribute to the overall improvement of the initial teaching of mathematics.

If teachers want to develop skills of critical thinking in elementary mathematics education, they should train their students to precisely and critically perceive relationships and connections, hidden and implicit data in the content of the task, to draw conclusions on the basis of perceived connections and relationships, and explain them using clear arguments. They should teach them to refrain from jumping to conclusions, teach them to use reversible thinking, identify mathematical problems, formulate them based on the given problem situation, critically perceive connections and relationships between mathematical symbols, to recognize inconsistencies, irregularities, surplus and incomplete data in problem formulation. Furthermore, they should teach them to devise and translate mathematical inscriptions to a specific and real situation, to search for fundamental characteristics in problem formulation and use both mathematical and verbal language accurately to express mathematical truths. Finally, they should insist that students critically assess the reality of the problem situation given in the problem, the validity of the obtained solution, taking into account the circumstances in which the problem was given.

Of course, teachers should bear in mind that students' critical thinking, in the teaching process, can be "encouraged and developed, that knowledge cannot be built without students' independent intellectual activity, without their direct involvement in the solution of problems and without engaging the intellectual skills and strategies that are inextricably linked to the ability of critical thinking" (Maričić & Špijunović, 2009, p. 69). We hope that we have encouraged other researchers to explore this problem, but also the teachers and textbook authors to pay more attention to the selection of content that contributes to the development of students' critical thinking skills in the initial teaching of mathematics.

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Utjecaj sadržaja na razvijanje kritičkog mišljenja učenika u početnoj nastavi matematike

Sažetak

U radu se ukazuje na važnost razvijanja kritičkog mišljenja učenika u početnoj nastavi matematike i ističu neki od problema koji prate njegovo ostvarivanje. U tom kontekstu autori skreću pozornost na određenje pojma kritičko mišljenje i ulogu sadržaja (zadataka) u njegovu razvijanju. Polazeći od specifičnosti početne nastave matematike i dobi učenika, kritičko mišljenje operacionaliziraju kroz konkretne sposobnosti (formuliranje problema, reformuliranje problema, evaluacija, osjetljivost za probleme) koje dolaze do izražaja u radu s matematičkim sadržajima. Na uzorku od 246 učenika (dobi od 9,5 do 10,4 godina) organizirali su eksperimentalno istraživanje (eksperiment s paralelnim skupinama) s ciljem da ispitaju može li se izborom odgovarajućih sadržaja (zadataka) razvijati kritičko mišljenje učenika u početnoj nastavi matematike. Rezultati istraživanja pokazuju da se adekvatnim izborom sadržaja kritičko mišljenje učenika, promatrano u cjelini, može razvijati i da se pri tome može značajno utjecati na razvijanje svake od njegovih sposobnosti (formuliranje problema, reformuliranje problema, evaluacija, osjetljivost za probleme).

Ključne riječi: *evaluacija; formuliranje problema; matematika; reformulacija problema; zadatak.*

Uvod

Vrijeme u kojem živimo obilježeno je intenzivnim i burnim razvojem u svim sferama života, ubrzanim razvojem znanosti, tehnike i tehnologije, povećavanjem opsega znanja, naglim porastom broja informacija, velikim utjecajem medija, globalizacijom, pokretljivošću ljudi, brzim protokom ideja i tako dalje. Zato je prirodno da se kreatori obrazovnih sustava permanentno suočavaju s potrebom redefiniranja ciljeva i zadataka obrazovanja i odgoja kako bi oblikovali osobnosti čije kompetencije odgovaraju zahtjevima i potrebama takvog vremena. Danas se svi slažu u tome toga da osnovna karakteristika koncepcije obrazovanja, škole, nastave i učenja mora biti sve veća usmjerenost razvijanju sposobnosti mišljenja, a sve manja usmjerenost stjecanju znanja (Jacobs i sur., 2007; Sfard i Kieran, 2001). Otuda i niz preporuka, rezolucija i deklaracija, koje su posljednjih nekoliko godina donijeli UNESCO i *Europsko vijeće za obrazovanje*, skreće pozornost na potrebu da naglasak u području obrazovanja treba biti povećana aktivnost učenika u nastavnom procesu, promocija darovitih učenika, osobni razvoj, kreativnost, samostalnost, razvijanje sposobnosti mišljenja, a posebno razvijanje sposobnosti kritičkog mišljenja. U tom kontekstu ističe se da „kritičko mišljenje nije akademski hir, već bitna vještina za život u informacijskom dobu" (Connor-Greene i Greene, 2002, str. 324) i da „kritičko mišljenje – predstavlja sredstvo za obrazovanje uma" (Paul i Elder, 2008, str. 88). Ako se kritičko mišljenje ne njeguje i ne razvija u učionici, umanjuju se šanse za uspjeh učenika (Irfaner, 2006), pa je stoga razvoj kritičkog mišljenja postao preduvjet za obrazovanje (Sezer, 2008; Tiwari i sur., 2006).

I nastavu matematike danas odlikuje sve veća „usmjerenost ka razvijanju sposobnosti mišljenja" (Špijunović i Maričić, 2011, str. 975). To je i razumljivo ako se imaju u vidu sve izraženije tendencije u nastavi matematike prema kojima nije važno samo to da učenici steknu i ovladaju odgovarajućim znanjima, već ih treba u što većoj mjeri osposobiti za primjenu znanja, kritički odnos prema sadržajima i adekvatno vrednovanje i to u svim etapama nastavnog procesa, odnosno da ih treba osposobiti za „istraživanje, rješavanje problema, stvaralačko mišljenje, obradu podataka, logičko zaključivanje i vrednovanje rezultata" (Felda i Ćotić, 2012, str. 51).

Zahtjev za razvijanjem kritičkog mišljenja u matematičkom obrazovanju nije nov. Još 1938. godine H. Fawcett je promovirao ideju o tome da vještine i sposobnosti kritičkog mišljenja predstavljaju sastavni dio rasuđivanja u nastavi matematike u kojoj se učenici svakodnevno nalaze u situaciji da zaključuju, pronalaze rješenja i iznose pretpostavke koje moraju kritički vrednovati (Marcut 2005, str. 61).

Međutim, praksa pokazuje da obrazovni sustav u pravoj mjeri ne doprinosi razvijanju kritičkog mišljenja učenika (Barbuto, 2000; Burbach i sur., 2004; Lizzio i Wilson, 2007; Paul, 2005; Pithers i Soden, 2000). Posebno se ukazuje na efekte koje sa sobom nosi standardizacija nastave i njezin produkt „učenje za test", jer se proces učenja zanemaruje, a naglasak se stavlja na sadržaj (Ladsman i Gorski, 2007; Lundquist, 1999; Sheldon i Biddle, 1998). Ako želimo kod učenika u nastavi matematike razvijati kritičko mišljenje, onda „cilj nastavnika matematike, koji žele razviti vještine kritičkog mišljenja u svojim učionicama je da razmišljaju o svojim učenicima ne kao primateljima informacija, već kao korisnicima informacija" (Ebiendele, 2012, str. 43).

Da bi učitelji u početnoj nastavi matematike uspješno razvijali kritičko mišljenje učenika, prije svega, „moraju imati jasnu predstavu o tome što je to kritičko mišljenje" (Lipman, 1988, str. 39).

U literaturi pronalazimo velik broj različitih određenja pojma *kritičko mišljenje* i različitih koncepcija kritičkog mišljenja. Dok je jedan broj određenja dosta uopćen, druga su uska i nastala za specifičan kontekst upotrebe, ali se nijedno ne odnosi na početnu nastavu matematike. Ta raznolikost govori o tome da je riječ o vrlo složenom fenomenu, koji se, po mišljenje Facionea „ne može do kraja odrediti kognitivnim sposobnostima" (Facione, 1996, str. 8), a po mišljenju L. Kevina i suradnika „koncept kritičko razmišljanje je previše složen da bi se ograničio na usko definiran konstrukt" (Flores, Gina, Matkin, Burbach, Quinn & Harding, 2012, str. 216).

Međutim, neophodno je „ne ulazeći u gnoseološku, logičku i psihološku zasnovanost, makar globalno odrediti što se podrazumijeva pod pojmom kritičko mišljenje" (Špijunović i Maričić, 2007, str. 114), odnosno jasno odrediti koje kognitivne sposobnosti i vještine čine njegov sadržaj (Ashton, 1988, str. 2). Ako ne razjasnimo sam pojam kritičko mišljenje, „mi ćemo bacati strijele na cilj koji ne vidimo" (Mulnix, 2012, str. 464). To znači da je neophodno operacionalizirati taj pojam, odnosno izdvojiti sposobnosti kritičkog mišljenja koje dolaze do izražaja u radu u početnoj nastavi matematike, a uzimajući u obzir specifičnosti početne nastave matematike, specifičnosti matematičkih sadržaja i uzrast učenika. S obzirom na to da nismo pronašli određenje kritičkog mišljenja, koje se odnosi na početnu nastavu matematike, problem operacionalizacije tog pojma nametnuo se kao prvi problem u težnji da razvijamo kritičko mišljenje u ovoj nastavi. U tom smo cilju, polazeći od suštine samog pojma *kritičko mišljenje*, određenja drugih autora nastalih za širi kontekst upotrebe, a imajući u vidu specifičnosti početne nastave matematike i uzrast učenika, kritičko mišljenje definirali kao složenu intelektualnu aktivnost koja obuhvaća sljedeće sposobnosti:

- formuliranje problema
- reformuliranje problema
- evaluacija
- osjetljivost za probleme (Maričić, 2011b, str. 133; Špijunović i Maričić, 2011b, str. 67).

Svaku od navedenih komponenata kritičkog mišljenja odredili smo preko više užih sposobnosti. Mi ćemo ih pojasniti i za svaku navesti primjer zadatka kako bismo ilustrirali ispoljavanje kričikog mišljenja.

Formuliranje problema podrazumijeva:

a) Sposobnost učenika da uoči matematički problem i da ga formulira na temelju problemske situacije
Primjer: Kada je automobil prešao 210 kilometara, nestalo mu je goriva. Preostao mu je još dva puta duži put koji treba prijeći. Što sve možeš izračunati? Napiši.
Učenik na temelju problemske situacije treba formulirati što veći broj problema, vodeći pri tome računa o tome odgovaraju li svi elementi iz formulacije problema polaznim, danim elementima i iz njega slijede, ali i da jasno odražavaju odnos između danog i traženog, mogućeg i nemogućeg. Pri tome se mora voditi računa o tome da problem bude jasan, koncizan i razumljiv i da ne ostavlja nikakve nedoumice i dileme.
b) Otkrivanje značenja matematičke simbolike i prevođenje te simbolike na govorni jezik, uočavanje veza među matematičkim simbolima i kritičnost u iskazivanju tih veza riječima

Primjer: Zaokruži slovo ispred zadatka za koji misliš da odgovara danom zapisu.
900 - (300 + 200) =
a) Marko je imao 900€, dobio je od brata 300€, a od sestre 200€. Koliko eura sada ima Marko?
b) Marko je imao 900€. Dao je bratu 300€ i dobio od sestre 200€. Koliko eura sada ima Marko?
c) Marko je imao 900€. Dao je bratu 300€ i sestri 200€. Koliko eura sada ima Marko?
d) Marko je imao 900€. Dobio je od brata 300€ i dao sestri 200€. Koliko eura sada ima Marko?

c) Traganje za suštinskim svojstvima u formulaciji problema, uočavanje nijansi u formulaciji problema i služenje preciznim govornim i matematičkim jezikom (Maričić, 2011a)
Primjer: Nikola je riješio 120 zadataka iz jedne zbirke. Ostalo mu je da riješi još 50 zadataka. Što možeš izračunaš? (Zaokruži slovo ispred pitanja).
a) Koliko zadataka Nikola nije riješio?
b) Za koliko je zadataka Nikola manje riješio od broja zadataka koji su mu ostali da ih riješi?
c) Koliko ukupno zadataka Nikola treba riješiti?
d) Koliko je vremena Nikoli potrebno da riješi zadatke?
e) Za koliko je zadataka Nikola više riješio od broja zadataka koji su mu ostali da ih riješi?

Reformuliranje problema podrazumijeva:

a) Jezičnu reformulaciju matematičkog zadatka
Primjer: Milica je imala toliko novca da joj je kada je polovinu novca dala sestri i polovinu ostatka bratu njoj ostalo 20€. Koliko je eura imala Milica?

Učenik će na temelju uočavanja odnosa među danim podacima formulirati uvjet na drugačiji način: Četvrtina ukupnog iznosa novca iznosi 20€.

b) Izvođenje zaključaka na temelju uočavanja veza i relacija u sadržaju zadatka obrazloženih jasnim argumentima
Primjer: Utrku je završilo pet natjecatelja: Milan, Aleksa, Nikola, Saša i Goran, ali ne znamo kojim redom. Poznato je da je Milan iza Alekse, a ispred Nikole i Saše. Nikola je iza Saše, a ispred Gorana, koji je na kraju. Kojim su redoslijedom natjecatelji završili utrku?

c) uočavanje odnosa među uvjetima zadatka i vraćanje spoznajnog puta u obrnutom smjeru
Primjer: Petar je zamislio jedan broj. Od njega je oduzeo broj 3, dobivenoj razlici dodao broj 4, sve podijelio sa 2 i dobio 8. Koji je broj Petar zamislio?

Evaluacija se odnosi na:

a) Evaluaciju informacija

Primjer: Planinar se penje prema vrhu planine. U susret mu idu tri obitelji. Prva obitelj je imala dvoje djece, druga dva puta više, a treća onoliko koliko prve dvije obitelji zajedno. Koliko osoba ukupno ide prema vrhu planine?

Evaluacija se temelji na pažljivoj uočavanju odnosa koji postoje u sadržini zadatka, odvajanju datog i zadanog, bitnog od nebitnog, otkrivanje suvišnih podataka u zadatku, prepoznavanje smisla informacija, postavljanje razjašnjavajućih pitanja koji pomažu shvaćanje suštine zadatka.

b) Evaluaciju rješenja

Primjer: U dvorištu je baka u površi oblika trokuta zasejala cvijeće. Koliki može biti opseg cvijetnjak ako su mu mjerni brojevi stranica različiti prirodni brojevi? (a) 12cm; b) 9m; c) 2m; d) 6dm; e) 4m)

c) Evaluaciju mišljenja autoriteta

Primjer: Učitelj je na sljedeći način vrednovao zadatke učeniku:

a) $4 \cdot 8 - 2 + 0 = 30$ b) $200 + 2 \cdot 50 = 100$

c) $2 \cdot 0 \cdot 3 + 203 = 209$

Slažeš li se s vrednovanjem učitelja? Obrazloži odgovor.

Osjetljivost za probleme podrazumijeva:

a) Procjenjivanje realnosti situacije u problemu i dobivenog rješenja i uzimanje u obzir okolnosti u kojima je zadatak dan

Primjer: Jedan dječak uplaši 3 vrapca i oni odlete. Koliko dječaka je potrebno da bi se 9 vrabaca uplašilo i odletjelo?

b) Uočavanje i otkrivanje skrivenih i implicitnih podataka u formulaciji, suzdržavanje od prebrzog zaključivanja, osjetljivost za otkrivanje puta rješavanja problema

Primjer: Kupac je tražio od prodavača 20 olovaka, a jedna kutija ima 12 olovaka. Za vađenje jedne olovke prodavatelj utroši jednu sekundu. Koliko je najmanje vremena potrebno da prodavatelj odvoji 20 olovaka?

c) Sposobnost uočavanja nelogičnosti, proturječnosti u formulaciji i zahtjevu problema, uočavanje suvišnih i nepotpunih podataka koji proizlaze iz realnosti dane situacije i otkrivanje mogućih zamki u formulaciji problema (Maričić 2009, str. 485-486; Maričić, 2011b, str. 134-155).

Primjer: Mateja ima dva novčića u ukupnoj vrijednosti od 7 eura. Jedan od njih nema vrijednost 2 eura. Koje kovanice ima Đorđe?

Navedena ili neka druga operacionalizacija kritičkog mišljenja u početnoj nastavi matematike predstavlja tek prvi korak u njegovu razvijanju. Sljedeće pitanje je kako razvijati kritičko mišljenje učenika u početnoj nastavi matematike. U vezi s tim postoji velik broj rasprava, radova i istraživanja (Abrami i sur., 2008; Case, 2005; Connor-Greene i Greene, 2002; Duplass i Ziedler 2002; Halpern, 1998; Hemming, 2000;

Heyman, 2008; Nelson, 1994 i drugi), ali ne postoje gotovi recepti i jasni odgovori. Sigurno je da pristup razvoju kritičkog mišljenja „usmjeren na usvajanje pravila, procedura i vještine logičkog razmišljanja nije preporučljiv, jer učenici ove dobi nisu sposobni za stjecanje znanja dedukcijom, niti sposobnosti da koriste stroge logičke postupke u razmišljanju, vrednovanju i učenju" (Maričić, Špijunović & Malinović Jovanović, 2013, str. 205). Ebiendele (2012) ističe da „okruženje za učenje koje aktivno uključuje učenike u istrazi informacija i primjeni znanja će promovirati vještine kritičkog mišljenja kod učenika" (str. 43). U meta-analizi više od 1.300 eksperimentalnih studija koje su obavljene u razdoblju od 1960. do 2005. pokazano je da nastava koja uključuje komponente kritičkog mišljenja, bilo da se to odvija implicitno ili eksplicitno, poboljšava vještine kritičkog razmišljanja pojedinca (Abrami i sur., 2008).

Sigurno je da je razvijanje kritičkog mišljenja učenika u početnoj nastavi matematike uvjetovano prirodom matematičkih sadržaja i oslonjezino na njihovo poznavanje, jer se samo „vježbanjem i stavljanjem učenika u situaciju da kritički misle na konkretnom sadržaju kritičko mišljenje može razvijati" (Maričić, 2009, str. 485). Takvo stanovište zastupa i J. Mek Pek, koji smatra da je "konceptualno nemoguće govoriti o kritičkom mišljenju uopće, jer je ono, kao i svako mišljenje, uvijek mišljenje o nečemu" (McPeck, 1981, str. 3). Otuda je neophodno da se sposobnosti i vještine kritičkog mišljenja ne usvajaju u okviru posebnog predmeta, već je potrebno da se „eksplicitno ugrade u proces usvajanja znanja" (Gleser, 1984, str. 93). Dakle, bez odgovarajućih sadržaja „teško da će, bilo koja strategija, metoda, oblik ili nastavni sustav, sam po sebi, imati značajniji utjecaj na razvijanje kritičkog mišljenja u početnoj nastavi matematike" (Maričić, i sur., 2013, str. 206). Zbog tih smo razloga, u istraživanju problema razvijanja kritičkog mišljenja pošli od stava da se kritičko mišljenje može uspješno razvijati ako se bude razvijalo u svakodnevnim situacijama učenja i na sadržajima (zadacima) čije rješavanje zahtijeva sposobnosti kritičkog mišljenja.

Polazeći od navedenih stavova, željeli smo ispitati da li se osmišljenijim izborom adekvatnih sadržaja (zadataka) kritičko mišljenje učenika u početnoj nastavi matematike može uspješnije razvijati. Pri tome adekvatnost sadržaja (zadataka) pretpostavlja takve sadržaje u procesu čijeg rješavanja dolaze do izražaja sposobnosti kritičkog mišljenja navedene u operacionaliziranoj definiciji tog pojma.

Metode

Sudionici

Uzorak istraživanja činilo je 256 učenika trećeg razreda osnovne škole (uzrasta 9,5 do 10,4 godine) iz tri osnovne škole u Republici Srbiji. Slučajnim izborom odabrali smo osnovne škole za eksperimentalnu i kontrolnu skupinu. Formirali smo dvije skupine učenika: eksperimentalnu (N=123) i kontrolnu skupinu (N=123). Eksperimentalnu skupinu činili su učenici iz pet odjela iz jedne osnovne škole, a kontrolnu učenici šest odjela iz dvije osnovne škole, jer nismo mogli osigurati ekvivalentan uzorak eksperimentalnoj skupini iz jedne osnovne škole. Ujednačavanje eksperimentalne i kontrolne skupine nije obavljeno umjetno, premještanjem učenika iz jednog odjela u drugi, zbog uvjeta rada u školi, već smo statističkim postupkom analize kovarijance (ANCOVA) statistički kontrolirali zavisnu varijablu, jer "prilagođena" varijanca odgovara varijanci koja bi se dobila na ekperimentalno ujednačenim skupinama. Postupak analize kovarijance temelji se na dobivanju reduciranog proračuna eksperimentalne pogreške uzimajući u obzir regresiju finalne mjere (Y) na inicijalnu mjeru (H). Učenici u obje skupine pripadali su socijalno ujednačenom srednjem socijalnom staležu.

Obrada podataka

U istraživanju smo se koristili eksperimentalnom metodom i to eksperimentom s paralelnim skupinama. Formirali smo dvije skupine učenika: eksperimentalnu i kontrolnu. U eksperimntalnu skupinu smo nakon inicijalnog merenja razvijenosti kritičkog mišljenja uveli eksperimentalni program. Eksperimentalni program realiziran je tijekom školske 2012./2013. godine u okviru redovnog programa nastave matematike u trećem razredu osnovne škole kroz 27 vježbi u okviru sljedećih nastavnih tema:

– Pravokutnik i kvadrat
– Pismeno zbrajanje i oduzimanje do 1000;
– Trokut
– Pismeno množenje i dijeljenje do 1000
– Razlomci
– Matematički izrazi.

Raspored nastavnih sadržaja u eksperimentalnom programu odredili smo na temelju aktualnog programa nastave matematike u trećem razredu osnovne škole u Srbiji i u potpunosti je pratio taj program. Vježbe su realizirane kroz individualni, grupni i frontalni oblik rada uz pomoć eksperimentatora i davanja instrukcija za rad, frontalne i grupne rasprave u okviru odjela i jasne i sadržajne povratne informacije za svaki zadatak unutar vježbe. Eksperimentalni program realizirali su učitelji eksperimentalne skupine prema izrađenoj uputi, s kompletnim pripremama sati za realizaciju navedenih nastavnih jedinica. Vježbe su koncipirane tako da su sadržavale odabrane sadržaje (zadatke) u procesu čijeg je rješavanja dolazila do izražaja neka od opearacionaliziranih sposobnosti kritičkog mišljenja (formuliranje problema, reformuliranje problema, evaluacija, osjetljivost za probleme). Jedna vježba, koja je realizirana u okviru jednog sata odnosila se na jednu sposobnost kritičkog mišljenja. Za svaku od komponenata kritičkog mišljenja osigurali smo jednak broj vježbi i unutar svake vježbe približno jednak broj zadataka po sposobnostima kritičkog mišljenja.

Najprije je realizirano pilot istraživanje na uzorku od 55 učenika, na osnovi koga je verificiran eksperimentalni program i izrađene su konačne forme instrumenata, a zatim se pristupilo realizaciji eksperimentalnog programa.

Instrumenti

Specifičnost problema istraživanja zahtijevala je izradu dva testa za mjerenje razvijenosti kritičkog učenika u početnoj nastavi matematike:

1) TIKM – test za utvrđivanje inicijalnog stupnja razvijenosti kritičkog mišljenja učenika,

2) TFKM – test za utvrđivanje finalnog stupnja razvijenosti kritičkog mišljenja učenika.

Testove smo sami konstruirali. Napravili smo dvije ekvivalentne forme testa. Strukturu testa činilo je 12 zadataka. Na temelju ponuđene operacionalizacije kritičkog mišljenja napravili smo izbor zadataka tako da je u procesu rješavanja svakog zadatka dolazila do izražaja određena sposobnost kritičkog mišljenja (formuliranje problema, reformuliranje problema, evaluacija i osjetljivost za probleme), odnosno njezina uža vještina. Svaki zadatak iz testa bodovan je maksimalno s 5 bodova. Maksimalan broj bodova, koji je mogao biti ostvaren na testu, iznosio je 60 bodova.

Da bi upotreba testa bila opravdana, utvrđene su metrijske karakteristike testa. Objektivnost testa osigurana je tako što je svaki učenik stavljen u približno jednaku ispitnu situaciju, time što su neovisni ispitivači postupali po jedinstvenim uputama i što je ocjenjivanje zadataka obavljeno na isti način na temelju ključa. Utvrdili smo logičku i sadržajnu provjeru valjanosti testova utvrđivanjem slaganja testova sa zahtjevima kurikula i sadržajima na koje se odnose. Diskriminativnost (osjetljivost) testa utvrđena je putem item analize određujući za svaki zadatak indeks lakoće (r) i indeks težine danog zadatka (q). Koeficijent diskriminativne vrijednosti varirao je između ,12 i ,25 (1. zadatak (,23), 2. zadatak (,13), 3. zadatak (,22), 4. zadatak (,25), 5. zadatak (,24), 6. zadatak (,15), 7. zadatak (,12), 8. zadatak (,15), 9. zadatak (,25), 10. zadatak (,12), 11. zadatak (,12), 12. zadatak (,17)). Pouzadnost instrumenta utvrđena je izračunavanjem Kronbachov alfa koeficijenta (α=,88) koji ukazuje na visoku pouzdanost instrumenta. Relijabilnost (pouzdanost) testa utvrdili smo i retest postupkom izračunavanjem koeficijenta korelacije (Pearsonov koeficijent korelacije) između rezultata koje su ispitanici postigli na ekvivalentnim formama testa kritičkog mišljenja. Dobiveni koeficijent korelacije iznosio je ,81.

Prvo testiranje izvršeno je prije početka djelovanja eksperimentalnog faktora – inicijalno, a drugo nakon završetka eksperimentalnog programa – finalno.

Podaci dobiveni istraživanjem obrađeni su upotrebom statističkog programskog paketa IBM Statistics SPSS20, pri čemu se koristila jednofaktorska analiza varijance (ANOVA) i analiza kovarijance (ANCOVA) za statističko ujednačavanje skupina i longitudinalno praćenje učinaka eksperimentalnog programa.

Rezultati istraživanja i rasprava

Prije uvođenja eksperimentalnog programa obavljena su inicijalna mjerenja razvijenosti kritičkog mišljenja učenika. Na inicijalnom testu kritičkog mišljenja (TIKM) učenici eksperimentalne grupe postigli su prosječno po 18.40 bodova od maksimalnih 60, a učenici kontrolne grupe osvojili su prosječno po 16.20 bodova (Tablica 1). Izračunati *F* omjer (F_x(1,244)=3.032, *p*=,083) ukazuje na to da su skupine na inicijalnom mjerenju prilično ujednačene u razini razvijenosti kritičkog mišljenja, a Levenov test (*p*=,630) pokazuje da nije prekšena pretpostavka o homogenosti varijanci (Tablica 2).

U eksperimentalnu skupinu uveden je eksperimentalni program, a kontrolna je skupina radila na ustaljen način, nakon čega su obavljena finalna mjerenja (TFKM). Na finalnom testu kritičkog mišljenja učenici eksperimentalne skupine postigli su prosječno po 48.06 bodova (Max=60), a učenici kontrolne skupine prosječno po 16.68 bodova, što je približno prosječnom broju bodova koji su osvojili na inicijalnom mjerenju (Tablica 1).

Tablica 1

Znači, učenici eksperimentalne skupine su pod utjecajem vježbi iz eksperimentalnog programa znatno napredovali u razvijenosti kritičkog mišljenja, u odnosu na učenike kontrolne grupe (Grafikon 1).

Grafikon 1

F-omjer na finalnom testu (F_y(1,244)=587.140, *p*<,001) ukazuje na to da postoje statistički značajne razlike u razvijenosti kritičkog mišljenja između eksperimentalne i kontrolne skupine (Tablica 2).

Tablica 2

Da je razlika stvarna i rezultat djelovanja eksperimentalnog programa pokazuje kovarijanca (*F*=857.782, *p*<,001) koja odbacuje sumnju u to da su razlike rezultat neujednačenosti eksperimentalne i kontrolne skupine (Tablica 3).

Tablica 3

Na temelju dobivenih rezultata možemo zaključiti da je eksperimentalni program utjecao na razvijanje kritičkog mišljenja učenika u eksperimentalnoj skupini, odnosno da se adekvatnim izborom sadržaja (zadataka) značajno može doprinijeti razvijanju sposobnosti kritičkog mišljenja učenika u početnoj nastavi matematike, čime smo potvrdili polaznu ideju o važnosti izbora sadržaja (zadataka) na uspješno razvijanje kritičkog mišljenja. Nažolost, nismo mogli usporediti dobivene rezultate s rezultatima drugih istraživača, jer nismo pronašli istraživačke radove koji su problem razvijanja kritičkog mišljenja istraživali na navedenom uzrastu i u navedenoj nastavi.

Daljnjom analizom željeli smo utvrditi utječe li izbor sadržaja na razvijanje svake od operacionaliziranih sposobnosti kritičkog mišljenja (formuliranje problema, reformuliranje problema, evaluacija, osjetljivost za probleme).

Analiza uspjeha učenika po sposobnostima kritičkog mišljenja pokazuje približno ujednačene rezultate ekperimentalne i kontrolne skupine na inicijalnom testu i znatan napredak učenika eksperimentalne skupine na finalnom mjerenju po svim operacionaliziranim sposobnostima kritičkog mišljenja u odnosu na kontrolnu skupinu (vidi Grafikon 2).

Grafikon 2

Na inicijalnom testu kritičkog mišljenja (TIKM) učenici eksperimentalne skupine bili su najuspješniji na zadacima koji podrazumijevaju sposobnost *formuliranja problema* i prosječno su postigli 5,96 bodova (*SD*=3,372) od maksimalnih 15 bodova, a učenici kontrolne skupine na istim zadacima prosječno su postigli po 4.93 boda (*SD*=4,136) (Tablica 4). Varijanca između grupa na inicijalnom mjerenju pokazuje da i na inicijalnom mjerenju postoje statistički značajne razlike između eksperimentalne i kontrolne grupe u razini razvijenosti sposobnosti formuliranja problema (F_x=4,532, *p*<,05) (Tablica 5). Takva situacija može biti slučajna, ali može biti i rezultat korištenja različitim udžbenicima u radu ili strategijama rada učitelja koji u nastavi postavljaju više problemskih situacija na temelju kojih učenici trebaju formulirati probleme. Nakon djelovanja eksperimentalnog programa učenici eksperimentalne skupine na finalnom mjerenju bili su znatno uspješniji (*M*=13,83 , *SD*=2,268) i ostvarili značajnu razliku u razvijenosti sposobnosti kritičkog mišljenja u odnosu na učenike kontrolne skupine (*M*=7,08, *SD*=2,763). Na temelju varijance između grupa na finalnom mjerenju (F_y=438,405, *p*< ,001) možemo zaključiti da je eksperimentalni program značajno utjecao na razvijanje sposobnosti formuliranja problema. To znači da učitelji izborom odgovarajućih sadržaja u početnoj nastavi matematike mogu osposobiti učenike za to da uočavaju matematičke probleme, da ih formuliraju na temelju problemske situacije, kritički uočavaju veze i odnose među matematičkim simbolima, smišljaju i prevode matematičke zapise na konkretnu realnu situaciju koja je učeniku bliska, da tragaju za suštinskim svojstvima u formulaciji problema i precizno se služe matematičkim i govornim jezikom u izražavanju matematičkih istina.

Tablica 4 i 5

Na zadacima u kojima su učenici trebali pokazati kritičnost u mišljenju pri *reformuliranju problema* između učenika eksperimentalne (M=5,08) i kontrolne skupine (*M*=5,65) na inicijalnom mjerenju ne postoje statistički značajne razlike (F_x=1,024, p= ,313). Nakon djelovanja eksperimentalnog programa učenici eksperimentalne grupe ostvarili su značajan napredak u razvijanju sposobnosti kritičkog mišljenja (*M*=11,67), a učenici su kontrolne grupe ostvarili manji broj bodova po učeniku u odnosu na inicijalno mjerenje (*M*=3,41). F-omjer između eksperimentalne i kontrolne skupine (F_y=229,671, *p*<,000) ukazuje na statističku značajnost uočenih razlika. Prema tome, zaključujemo da je pod utjecajem eksperimentalnog programa došlo do razvijanja sposobnosti reformuliranja problema kao komponente kritičkog mišljenja. Time smo pokazali da se odgovarajućim izborom sadržaja učenici mogu osposobiti za to da dublje, kritičnije i preciznije uočavaju veze i odnose u sadržaju zadatka i izražavaju ih na način koji im pomaže u shvaćanju njegove suštine, izvode zaključke na temelju uočenih veza i odnosa i obrazložu ih jasnim argumentima, osposobljavaju se da spoznajni put vrate u obratnom smjeru, reverzibilno misle i uspješnije rješavaju zadatke koji zahtijevaju reformuliranje problema.

I na zadacima u kojima su učenici trebali pokazati sposobnost *evaluacije* na inicijalnom mjerenju eksperimentalna (*M*=4,35) i kontrolna skupina (*M*=3,78) postigle su približno ujednačene rezultate (F_x=1,326, *p*= ,251). Nakon uvođenja eksperimentalnog programa učenici eksperimentalne grupe postigli su znatno bolji rezultat (*M*=10,93) u odnosu na učenike kontrolne grupe (*M*=4,59) i ostvarili statistički značajnu razliku (F_y=132,710, *p*< ,001) u razvijenosti sposobnosti kritičkog mišljenja. Time smo pokazali da se izborom adekvatnih sadržaja (zadataka) može doprinijeti tome da učenici imaju razvijeniji kritički stav prema informacija, sadržaju zadatka, prema dobivenom rješenju, ali i prema mišljenju autoriteta.

Kada je u pitanju *osjetljivost za probleme* na inicijalnom mjerenju između učenika eksperimentalne i kontrolne skupine, ne postoje statistički značajne razlike u razini razvijenosti te sposobnosti kritičkog mišljenja (F_x= ,885, p= ,348). Finalno testiranje pokazuje da su učenici eksperimentalne skupine osvojili znatno više bodova (M_y=11,18), u odnosu na inicijalno mjerenje (M_x=2,20), a kod učenika kontrolne skupine te su razlike neznatne (M_x=1,83, M_y=1,59). Dobivena varijanca (F_y=356,876, p<,001) pokazuje da su vježbe izvedene u okviru eksperimentalnog programa značajno utjecale na razvijanje osjetljivosti za probleme kod učenika. To upućuje na zaključak da učitelj izborom odgovarajućih sadržaja može značajno doprinijeti razvijanju sposobnosti kod učenika za to da kritički procjenjuje realnost problemske situacije dane u problemu, procjenjuje realnost dobivenog rješenja, uzimajući u obzir okolnosti u kojima je problem dan, da uočava i otkriva skrivene i implicitne podatke u formulaciji problema, suzdržava se od zaključivanja na brzinu, uočava nelogičnosti, proturječnosti, suvišne i nepotpune podatke u formulaciji problema.

Zaključak

Unatoč činjezinici da razvijanje kritičkog mišljenja predstavlja važan zadatak matematičkog obrazovanja ne postoji opći konsenzus o pitanjima vezanim uz njegovo razvijanje. Brojna pitanja vezana uz njegovo razvijanje i dalje su otvorena, a posebno zabrinjava činjenica da nastavnici nisu u dovoljnoj mjeri osposobljeni za razvijanje kritičkog mišljenja (Broadbear, 2003; Maričić, 2010; Scriven i Paul, 2007). Dio problema proizlazi i iz postojanja velike općenitosti i neodređenosti pojma kritičko mišljenje u općem smislu, a posebno u nekom konkretnom području, ali i nepostojanja jedinstvenog stava o načinu na koji ga treba razvijati. U prvom dijelu rada teorijski je rasvijetljen pojam *kritičko mišljenje* izdvajanjem konkretnih sposobnosti koje čine njegov sadržaj i koje dolaze do izražaja u radu s matematičkim sadržajima u početnoj nastavi matematike.

Empirijski dio rada imao je za cilj ispitati može li se izborom adekvatnih sadržaja (zadataka) utjecati na razvijanje kritičkog mišljenja u početnoj nastavi matematike. Rezultati eksperimentalnog istraživanja upućuju na sljedeće zaključke:

– adekvatnim izborom sadržaja u početnoj nastavi matematike kritičko mišljenje učenika se može razvijati

– izbor sadržaja u početnoj nastavi matematike značajno utječe na razvijanje svake od operacionaliziranih sposobnosti kritičkog mišljenja (formuliranje problema, reformuliranje problema, evaluacija, osjetljivost za probleme).

Istraživanje je pokazalo da je izbor sadržaja u nastavi matematike vrlo važan za ostvarivanje ciljeva i zadataka, kao i za postizanje odgovarajućih ishoda nastave matematike, a u ovom slučaju uspješno razvijanje sposobnosti kritičkog mišljenja, koje predstavljaju neophodan element mišljenja uopće, a osobito neophodan element matematičkog mišljenja. Dobiveni rezultati pokazali su da se adekvatnim izborom sadržaja kritičko mišljenje može razvijati na uzrastu učenika mlađih razreda osnovne škole, što implicira potrebu da kreatori obrazovnih politika trebaju više pažnje pokloniti razvijanju kritičkog mišljenja u osnovnoj školi i to ne samo u nastavi matematike, već u svim predmetnim područjima. To još više dolazi do izražaja ako se ima u vidu Platonov stav da smjer obrazovanja kojim čovjek krene u mladosti određuje njegovu budućnost. Sve to ukazuje na to da kritičko mišljenje ne predstavlja karakteristiku odrasle dobi, da se može razvijati kod učenika s početkom školovanja na način koji je prihvatljiv učenicima. Osim toga, dobiveni rezultati trebali bi, prije svega učiteljima matematike, ali i svima onima koji su neposredno ili posredno uključeni u proces matematičkog obrazovanja pružiti sigurniji oslonac i dati jasniju orijentaciju pri koncipiranju i organizaciji početne nastave matematike, kako bi bili uspješniji u ostvarivanju zahtjeva za razvijanje kritičkog mišljenja učenika, a time pridonijeti i sveukupnom unapređivanju početne nastave matematike.

Ako učitelj želi u početnoj nastavi matematike razvijati sposobnosti kritičkog mišljenja, treba učenike osposobljavati za to da precizno i kritički uočavaju veze i odnose, skrivene i implicitne podatke u sadržaju zadatka, izvode zaključke na temelju uočenih veza i odnosa i obrazlažu ih jasnim argumentima. Zatim da ih nauči kako da se suzdržavaju od prebrzog zaključivanja, reverzibilno misle, uočavaju matematičke probleme, formuliraju ih na temelju dane problemske situacije, kritički uočavaju veze i odnose među matematičkim simbolima, uočavaju nelogičnosti, proturječnosti, suvišne i nepotpune podatke u formulaciji problema, osmišljavaju i prevode matematičke zapise na konkretnu realnu situaciju, tragaju za suštinskim svojstvima u formulaciji problema i precizno se služe matematičkim i govornim jezikom u izražavanju matematičkih istina. Na kraju treba inzistirati na tome da učenici kritički procjenjuju realnost problemske situacije dane u problemu, realnost dobivenog rješenja, uzimajući u obzir okolnosti u kojima je problem dan.

Naravno, učitelji trebaju imati u vidu da se kritičko mišljenje učenika u procesu nastave može „poticati i razvijati, da se znanje ne može izgraditi bez samostalne intelektualne aktivnosti učenika, bez njihove neposredne uključenosti u rješavanje različitih problema i angažiranja intelektualnih vještina i strategija koje su neodvojivo povezane sa sposobnostima kritičkog mišljenja" (Maričić i Špijunović 2009, str. 69).

Nadamo se da smo potaknuli i druge istraživače da navedeni problem istražuju, ali i učitelje i autore udžbenika da više pažnje posvete izboru sadražaja koji doprinose razvijanju kritičkog mišljenja učenika u početnoj nastavi matematike.