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Research Article

Fuzzy Approach in Ranking of Banks according to Financial Performances

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Evaluating bank performance on a yearly basis and making comparison among banks in certain time intervals provide an insight into general financial state of banks and their relative position with respect to the environment (creditors, investors, and stakeholders). The aim of this study is to propose a new fuzzy multicriteria model to evaluate banks respecting relative importance of financial performances and their values. The relative importance of each pair of financial performance groups is assessed linguistic expressions which are modeled by triangular fuzzy numbers. Fuzzy Analytic Hierarchical Process (FAHP) is applied to determine relative weights of the financial performances. In order to rank the treated banks, new model based on Fuzzy Technique for Order Performance by Similarity to Ideal Solution (FTOPSIS) is deployed. The proposed model is illustrated by an example giving real life data from 12 banks having 80% share of the Serbian market. In order to verify the proposed FTOPSIS different measures of separation are used. The presented solution enables the ranking of banks, gives an insight of bank's state to stakeholders, and provides base for successful improvement in a field of strategy quality in bank business.

1. Introduction

In an economic theory and practice role of banks can be defined as mediator between the ones with capital surplus and ones in need of capital. A modern competitive environment, globalization, and an unstable financial market have created a wave of important changes in banking sector of each country. The problems that can occur in banking sector [1] directly influence the stakeholders as well as the general economy. Respecting this fact, it is very important to ensure and maintain performances of banks in such manner that banks can contribute in sustainable development of each economy country and overcoming crisis. Assessment of performances may be considered as a significant issue in different organizations on the level of different processes and activities [2]. An assessment of bank's performances, as well as final analysis, needed by both internal and external beneficiaries of information, is very significant for different stakeholders. This is a reason that considered problem has become a topic of research in the last decades.

Improving of bank business strategy can be achieved by managing bank performances, financial and nonfinancial. Analysis of financial performances (FPs) is in its base ratio analysis, which uses financial reports. They give information on capital, capital resources, income, disbursement, gained results, and cash flow. Many researchers suggest that nonfinancial performances can be reflected on financial tables in the long term and lead to a more realistic evaluation of a bank's financial standing [3]. In this paper, FPs and earning capacity of banks are taken into consideration.

The purpose of this study is to propose fuzzy model based on FPs for evaluation bank operating, that is, determining bank performance or financial standing. Theory and practice offer the whole set of techniques and methods of banks evaluation. As is known, it is impossible to manage what cannot be measured. However, measurement of bank performances is the most important activity of bank management team.

As changes in the business environment, especially in financial market, are rapid and continuous, relative importance of FPs cannot be expressed with an exact numerical value. It is easier, for decision makers, to express the subjectivity and imprecision of their assessments by linguistic expressions. The concept of linguistic variables is introduced by [4]; it is very useful in dealing with situations, which are too complex or not well defined to be reasonably described in conventional quantitative expressions [5]. These linguistic terms can be modeled by using the fuzzy sets theory [5, 6] which allows the available information to be represented with the granularity [7]. As it is known, processing of information granules (complex information entities) during the process of data abstraction and derivation of knowledge from input data is defined as granular computing [7].

The fuzzy set theory provides a strict mathematical framework in which vague conceptual phenomena can be precisely and rigorously studied [5]. To outperform competing bank institutions, it requires more emphasis on internal operational performances. This means that it is imperative to develop an effective way to conduct performance evaluations which can measure the overall organizational performance and link it to the corporate goals.

The considered problem can be stated as a multicriteria decision making problem (MCDM) under uncertainties. MCDM refers to finding the best opinion out of all feasible alternatives in the presence of multiple, usually conflicting decision criteria. The necessity of applying MCDM models in the field of financial decision making is supported by the fact that over the past decades the complexity of financial decisions has increased rapidly [8]. The most commonly used MCDM methods for ranking problems in various domains are AHP [9], TOPSIS [10], and the combination of the two MCDM methods. These two methods are used in this paper.

The paper is organized as follows. Section 2 presents the literature review. The basic definitions of the fuzzy set theory and modeling of uncertainties in the relative importance of FP groups are presented in Section 3. In Section 4 the proposed fuzzy TOPSIS approach is introduced. Section 5 represents the application of the recommended algorithm with real-life data. Finally, Section 6 sets the conclusions.

2. Literature Review

There are a large number of papers, found in literature, realized by different methods to measure of bank's performances and ranking of banks. The conventional Analytic Hierarchical Process (AHP) [9] is the most widely used method. Making strategy decisions in a bank with applying both basic and adjusted AHP application models is very significant issue [11]. In that manner, AHP application with specific reference to banking could be used in the finance sector [12]. In banking sector, usage of AHP application is growing, especially in the situation of global financial crisis. Many authors analyzed the bank performances using AHP with the financial and nonfinancial performance criteria [13].

The use of conventional AHP with discrete scale is simple and easy, but it is not sufficient considering uncertainty associated with the mapping of one's perception to a number [14]. Decision makers express their judgments far better by using linguistic expressions than by representing them in terms of precise numbers. These linguistic expressions are modeled by using fuzzy sets theory [5, 6]. In other words, the relative importance of criteria and the preference of alternatives under each criterion are stated by fuzzy pairwise comparison matrices. FAHP enables mapping human perception to a particular number or a ratio and also considering the vagueness in the decision making process.

There are a vast number of papers to be found in literature which use fuzzy AHP (FAHP) method for evaluation and ranking of banks. The evaluation and rank of Turkish banks with respect to many financial and nonfinancial performance criteria are performed by using FAHP [15]. It could be assumed that increased competition among banks and the liberalization of policies helped many institutions to take up the banking business [16]. The rank of banks can be obtained by applying FAHP.

Some researchers solve decision making problems in financial sector, especially in the field of banking, by using TOPSIS method. Prioritizing different kinds of accounts in banks, objectives, alternatives, and criteria can be established from literature review and judgements of bank managers. The TOPSIS/fuzzy TOPSIS [17–19] are widely used for ranking of entities in different research areas [15, 20, 21]. A brief literature review of [22–27] is presented as follows.

In the analyzed papers, the relative importance of attribute is assessed by decision makers that use predefined linguistic expressions. Modeling of these linguistic expressions is based on the fuzzy sets theory. Some authors [17, 18, 25, 28] suggest that the relative importance of attribute can be obtained by direct way. In these papers, determining weight of vectors is stated as fuzzy group decision making problem. In [17, 18, 25] it is assumed that decision makers have equal importance so that the aggregated weight of each attribute can be obtained by using fuzzy averaging method. In [22] a new aggregation procedure is developed. This procedure is applied for calculating of aggregated weights of attributes in [23, 28]. In the rest of the analyzed papers, rating relative importance of attributes should be based on the AHP framework. It appears that the determining of the relative importance of attributes is more reliable when they are obtained by using pairwise comparisons than when they are directly obtained. It is easier to make a comparison between two criteria than to make an overall weight assignment. The weights of considered attributes may be calculated by using the different procedure. In [18], the weights of treated criteria are obtained as the average of the elements of each row from the constructed fuzzy pairwise comparison matrix of the relative importance of attribute. The geometric mean was used in [24] for calculating relative importance of treated attribute which is proposed in [29]. In the rest of mentioned papers, the vectors weights of considered attributes are given by using approach for handling of FAHP which is developed in [30]. The obtained values of vectors weight are not fuzzy numbers [31, 32].

If the values of attributes for considered alternative can be obtained by direct measuring, then these values may be described by precise numbers. This assumption is introduced in [15, 26]. If there is a need, simultaneously, both crisp and uncertain criteria in the considered problem [23] may be analyzed. Under uncertain environment, it is relatively difficult for decision makers to provide exact precise numerical attribute values for treated alternative. Because of that, in the rest of analyzed paper, these values are described by linguistic expressions. These linguistic expressions are modeled by (a) triangular fuzzy numbers (TFNs) in [18, 20, 21, 25, 27] and (b) trapezoidal fuzzy numbers (TRFNs) in [17, 23, 24, 28]. The domains of defined TFNs and TrFNs belong to different interval, such as [0-1] (analogy by [18, 24]), [0–10] (analogy by [17, 20, 28]), and [1–10] (analogy by [25]), [1–9] (analogy by [21, 23, 27]).

As criteria can be benefit and cost type and values of decision matrix can be introduced by different units, it is necessary to perform their normalization. By using the normalization procedure the values of decision matrix are mapped into interval [0-1]. In this way, these values can be compared. Crisp values of alternatives are normalized by using vector normalization procedure in [15, 26] and linear normalization procedure in [23]. Normalization procedures are presented in [33]. In all analyzed papers, the normalized uncertain values are obtained by using linear normalization procedure which is proposed in [34].

Identification of the set Positive Ideal Solutions (PISs) and the Negative Ideal Solutions (NISs) is performed in [15, 26] according to the procedure which is given in conventional TOPSIS [10]. The most likely used technique for defining Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS) is proposed in [35] and it is used in [19, 24]. Authors [17] have developed a new approach for determining FPIS and FNIS. In the rest of analyzed papers [18, 20, 21, 23, 25, 27, 28], FPIS and FNIS are determined in compliance with the method for comparison of fuzzy numbers.

Calculating separation measure of each alternative from PIS and NIS is given by using n-dimensional Euclidean distance in [15, 26]. The separation measures of each alternative from FPIS and FNIS are obtained by using (a) vertex method [22] applied in [19, 20, 25, 28], (b) the proposed expression for the distance between two fuzzy numbers [36] applied in [18, 21, 23, 24], a new approach in [27], and (c) a novel fuzzy distance measure which has been analyzed in [37] and applied in [17]. The total separate measure of each alternative from PIS/FPIS and NIS/FNIS can be calculated as sum of all obtained separate measures from PIS/FPIS and NIS/FNIS. Determining closeness to the ideal solution is based on the total separate measures. The rank of alternative corresponds to closeness coefficient. The selection of existing or development of new approaches, for calculation of separation measures, have become important, because of the importance in choosing the best rankings.

Table 1: The FPs and identified KPIs.

Liquidity of a bank ($j = 1$)	The first liquidity ratio ($k = 1$) The second liquidity ratio ($k = 2$)
Financial structure ($j = 2$)	Leverage ratio 1 ($k = 2$) Leverage ratio 2 ($k = 4$) Leverage ratio 3 ($k = 5$)
Efficiency ($j = 3$)	The first ratio of interest nonbearing costs ($k = 6$) The second ratio of interest nonbearing costs ($k = 7$) The first ratio of interest nonbearing revenue ($k = 8$) The second ratio of interest nonbearing revenue ($k = 9$)
Profitability ($j = 4$)	Assets utilization indicator ($k = 10$) return on operating revenue ($k = 11$) Return on assets ($k = 12$) Share multiplier ($k = 13$) Return on equity ($k = 14$)
Market position ($j = 5$)	Total book value ($k = 15$) Total capital ($k = 16$) Total placement ($k = 17$) Bank obligations ($k = 18$)
Solvency of a bank ($j = 6$)	Capital adequacy ratio ($k = 19$)

3. Modeling of the Relative Importance of FPs

Rating of the relative importance of FPs over time is based on uncertain and imprecise knowledge by financial experts from banking and economy sector. Relative importance of the considered FPs does not depend on banks and it rarely changes. They involve a high degree of subjective judgments, knowledge, and experience of financial experts. It can be assumed that it best suits the nature of human decision to make a comparison between two FPs than to make an overall relative importance assignment. The financial experts express their judgments far better by using linguistic expressions than by representing them in terms of precise numbers. In this paper, the fuzzy ratings of financial experts are described by the linguistic expression modeled by fuzzy sets [5]. A fuzzy set is represented by its membership function which can be obtained by using different approaches. However, subjectivity in determining the membership function has been considered as the weakest point in the fuzzy sets theory.

In the papers which can be found in the literature, triangular and trapezoidal functions are widely used because of the fact that they offer a good compromise between descriptive power and computational simplicity. It is shown that fuzzy sets of higher types and levels have not yet played a significant role in the applications of the fuzzy sets theory [38].

In this paper, AHP framework is used to state relative importance of FPs. It is assumed that financial experts assess, considering variables using predefined linguistic expressions which are modeled by triangular fuzzy numbers (TFNs), $\widetilde{W}_{jj'} = (x; l_{jj'}, m_{jj'}, u_{jj'})$ with the lower and upper bounds $l_{jj'}, u_{jj'}$ and modal value $m_{jj'}$, respectively.

Table 2: KPI values for treated banks.

	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	$k = 8$	$k = 9$
$i = 1$	0.099	0.158	0.162	0.187	0.266	0.808	0.704	0.567	0.127
$i = 2$	0.075	0.120	0.248	0.268	0.398	0.763	0.521	0.429	0.070
$i = 3$	0.063	0.093	0.420	0.490	0.618	0.342	0.051	0.050	0.008
$i = 4$	0.110	0.1375	0.140	0.227	0.175	0.842	0.769	0.660	0.134
$i = 5$	0.135	0.206	0.493	0.609	0.753	0.738	0.696	0.401	0.050
$i = 6$	0.095	0.155	0.226	0.250	0.370	0.712	0.652	0.417	0.076
$i = 7$	0.105	0.152	0.280	0.315	0.402	0.938	0.876	0.809	0.495
$i = 8$	0.082	0.216	0.181	0.197	0.479	0.807	0.734	0.650	0.154
$i = 9$	0.081	0.167	0.110	0.117	0.220	0.626	0.396	0.655	0.158
$i = 10$	0.133	0.214	0.217	0.250	0.350	0.895	0.801	0.768	0.283
$i = 11$	0.147	0.654	0.221	0.124	0.355	0.549	0.456	0.217	0.021
$i = 12$	0.101	0.182	0.256	0.285	0.542	0.864	0.749	0.636	0.153

	$k = 10$	$k = 11$	$k = 12$	$k = 13$	$k = 14$	$k = 15$	$k = 16$	$k = 17$	$k = 18$	$k = 19$
$i = 1$	0.118	0.024	8.796	0.211	0.225	2824	496	2353	1548	19%
$i = 2$	0.244	0.040	5.858	0.234	0.164	1816	497	1498	873	21%
$i = 3$	0.427	0.066	3.607	0.238	0.156	942	384	825	471	38%
$i = 4$	0.087	0.016	10.014	0.163	0.202	1928	287	1490	1232	14%
$i = 5$	0.057	0.007	5.308	0.037	0.124	979	246	690	513	28%
$i = 6$	0.088	0.015	3.515	0.053	0.171	658	167	494	278	35%
$i = 7$	0.064	0.033	3.94	0.144	0.611	1400	403	1157	871	16%
$i = 8$	0.076	0.019	5.516	0.157	0.237	1291	298	1052	577	24%
$i = 9$	0.041	0.011	10.126	0.098	0.235	731	98	588	523	18%
$i = 10$	0.096	0.032	6.670	0.215	0.368	1010	233	773	536	17%
$i = 11$	0.161	0.014	4.549	0.065	0.096	702	167	519	418	20%
$i = 12$	0.123	0.030	5.574	0.157	0.240	811	210	77	339	16%

Table 3

The normalized decision matrix	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$
$i = 1$	0.228	0.175	0.196	0.279	0.433	0.235
$i = 2$	0.173	0.259	0.166	0.277	0.344	0.259
$i = 3$	0.141	0.439	0.388	0.311	0.265	0.469
$i = 4$	0.231	0.159	0.207	0.275	0.289	0.173
$i = 5$	0.305	0.531	0.151	0.233	0.220	0.346
$i = 6$	0.221	0.240	0.167	0.176	0.232	0.432
$i = 7$	0.233	0.287	0.361	0.266	0.279	0.198
$i = 8$	0.238	0.231	0.216	0.228	0.264	0.296
$i = 9$	0.208	0.124	0.249	0.270	0.166	0.224
$i = 10$	0.307	0.232	0.272	0.275	0.221	0.210
$i = 11$	0.583	0.196	0.148	0.203	0.193	0.247
$i = 12$	0.245	0.299	0.210	0.229	0.199	0.198

If the strong relative importance of FP j' over process j holds, then the pairwise comparison scale can be represented by TFN $\widetilde{W}_{jj'} = (\widetilde{W}_{j'j})^{-1} = (1/u_{jj'}, 1/m_{jj'}, 1/l_{jj'})$.

If $j = j'$ $(j, j' = 1, \ldots, J)$, then the relative importance of FP j over FP j' is represented by a single point 1 which is TFN $(1, 1, 1)$.

Granularity is defined as the number of TFNs assigned to the relative importance of FPs. In this paper, with respect to the type and size of the considered problem, and opinion of [39], it is assumed that the five linguistic expressions at the most were assigned to the existing linguistic variables.

As it is known, there are different locations in the universe of discourse. In this case, values in the domain of these TFNs belong to a real set within the interval [1–5].

Fuzzy rating of financial experts for each pair of treated FPs is described by linguistic expressions which can be represented as TFNs:

Very low importance: $\widetilde{R}_1 = (x; 1, 1, 2)$.

Low importance: $\widetilde{R}_2 = (x; 1, 2, 3)$.

Moderate importance: $\widetilde{R}_3 = (x; 1.5, 3, 4.5)$.

High importance: $\widetilde{R}_4 = (x; 3, 4, 5)$.

Highest importance: $\widetilde{R}_5 = (x; 4, 5, 5)$.

The vectors weight of FPs is calculated by approach for handling FAHP which is developed in [29]. The used approach has number of advantages over the traditional FAHP [30] which is used in the above analyzed papers. Some advantages are allowing more reasonable description of the decision making process and reflecting the thinking style of human; extent analysis method cannot estimate the true weights from a fuzzy comparison matrix and has led

Table 4: The fuzzy decision matrix.

	$j = 1$	$j = 2$	$j = 3$
$i = 1$	(0.036, 0.068, 0.126)	(0.021, 0.036, 0.069)	(0.024, 0.049, 0.090)
$i = 2$	(0.027, 0.052, 0.096)	(0.031, 0.053, 0.102)	(0.021, 0.041, 0.076)
$i = 3$	(0.022, 0.042, 0.078)	(0.052, 0.090, 0.173)	(0.048, 0.097, 0.178)
$i = 4$	(0.037, 0.069, 0.128)	(0.019, 0.033, 0.063)	(0.026, 0.052, 0.095)
$i = 5$	(0.048, 0.091, 0.169)	(0.063, 0.109, 0.210)	(0.019, 0.038, 0.069)
$i = 6$	(0.035, 0.066, 0.122)	(0.028, 0.049, 0.095)	(0.021, 0.042, 0.077)
$i = 7$	(0.037, 0.070, 0.129)	(0.034, 0.059, 0.113)	(0.045, 0.090, 0.166)
$i = 8$	(0.038, 0.071, 0.132)	(0.027, 0.047, 0.091)	(0.027, 0.054, 0.099)
$i = 9$	(0.033, 0.062, 0.115)	(0.015, 0.025, 0.049)	(0.031, 0.062, 0.114)
$i = 10$	(0.049, 0.092, 0.169)	(0.027, 0.048, 0.092)	(0.034, 0.068, 0.125)
$i = 11$	(0.093, 0.175, 0.322)	(0.023, 0.040, 0.077)	(0.018, 0.037, 0.068)
$i = 12$	(0.039, 0.073, 0.135)	(0.035, 0.061, 0.118)	(0.026, 0.052, 0.097)
$\widetilde{v}_j^+$	(0.093, 0.175, 0.322)	(0.063, 0.109, 0.210)	(0.048, 0.097, 0.178)
$\widetilde{v}_j^-$	(0.022, 0.042, 0.078)	(0.015, 0.025, 0.049)	(0.018, 0.037, 0.068)
	$j = 4$	$j = 5$	$j = 6$
$i = 1$	(0.010, 0.018, 0.038)	(0.014, 0.024, 0.032)	(0.013, 0.030, 0.059)
$i = 2$	(0.010, 0.017, 0.037)	(0.011, 0.019, 0.034)	(0.014, 0.033, 0.065)
$i = 3$	(0.012, 0.019, 0.042)	(0.009, 0.015, 0.026)	(0.027, 0.059, 0.118)
$i = 4$	(0.010, 0.017, 0.037)	(0.010, 0.016, 0.028)	(0.010, 0.022, 0.044)
$i = 5$	(0.009, 0.015, 0.031)	(0.007, 0.012, 0.022)	(0.020, 0.044, 0.087)
$i = 6$	(0.007, 0.011, 0.024)	(0.008, 0.013, 0.023)	(0.025, 0.054, 0.109)
$i = 7$	(0.010, 0.017, 0.036)	(0.009, 0.016, 0.027)	(0.011, 0.025, 0.050)
$i = 8$	(0.008, 0.014, 0.031)	(0.009, 0.015, 0.026)	(0.017, 0.037, 0.075)
$i = 9$	(0.010, 0.017, 0.036)	(0.005, 0.009, 0.016)	(0.013, 0.028, 0.056)
$i = 10$	(0.010, 0.017, 0.037)	(0.007, 0.012, 0.022)	(0.012, 0.026, 0.053)
$i = 11$	(0.008, 0.013, 0.027)	(0.006, 0.011, 0.019)	(0.014, 0.031, 0.062)
$i = 12$	(0.008, 0.014, 0.031)	(0.006, 0.011, 0.019)	(0.011, 0.025, 0.050)
$\widetilde{v}_j^+$	(0.012, 0.019, 0.042)	(0.014, 0.024, 0.032)	(0.027, 0.059, 0.118)
$\widetilde{v}_j^-$	(0.007, 0.011, 0.024)	(0.005, 0.009, 0.016)	(0.010, 0.022, 0.044)

to quite a number of misapplications which may lead to a wrong decision to be made and fuzzy matrices cannot be constructed [39].

4. Proposed Methodology

The issue of bank ranking is very significant for top management of each bank and for enterprises that have business relations with analyzed banks, too [40].

The banking sector priorities were improving the capital base, time and exchange restructuring of funds with placement structure, and efficient management of both the level of bank exposure to possible risks and the quality of invested funds.

The conditions for harmonizing regulative with Basel II standards were obtained as a consequence of market liberalization, setting up the financial stability of the system, banking industry restructuring and its capital strengthening, advancing supervision function, and introducing international accounting standards. However, it is important to point out that it requires time, not only for regulative harmonization, but also for harmonizing home banks' practice with qualitative and quantitative qualification standards for capital models application and its dynamics.

According to the real practice data, it is known that the most significant business goals of enterprises may be realized only if there is solid cooperation between enterprises and banks. In that manner, the selection of business bank is one of the most significant tasks of top management in any enterprise. This decision can be made by top managers with respect to the rank of considered banks.

In this paper, a new integrated fuzzy multicriteria model for ranking of banks is proposed. Since, the rank of banks is obtained by the exact way, the obtained solution is less burdened by subjective attitudes of decision makers, so it may be assumed that this assumption increases the correctness of solution.

The considered banks are formally presented by set $I = \{1, \dots, i, \dots, I\}$. The index for a bank is denoted as i, and I is the total number of considered banks.

In general, FPs are defined according to the literature and results of good practice. Formally, the FPs are presented by set of indices $J = \{1, \dots, j, \dots, J\}$, where j is index for FP and J is the total number of treated FPs.

Table 5: Comparative results of different distance measurements.

Methods	Banks	c_i	Rank	Methods	Banks	c_i	Rank
[22]	$i = 1$	0.222	9	[43]	$i = 1$	0.224	9
	$i = 2$	0.211	10		$i = 2$	0.205	10
	$i = 3$	0.533	1		$i = 3$	0.539	2
	$i = 4$	0.184	11		$i = 4$	0.186	11
	$i = 5$	0.492	2		$i = 5$	0.491	3
	$i = 6$	0.271	8		$i = 6$	0.269	8
	$i = 7$	0.387	4		$i = 7$	0.431	4
	$i = 8$	0.278	6		$i = 8$	0.282	6
	$i = 9$	0.171	12		$i = 9$	0.169	12
	$i = 10$	0.349	5		$i = 10$	0.344	5
	$i = 11$	0.475	3		$i = 11$	0.544	1
	$i = 12$	0.272	7		$i = 12$	0.271	7
The method based on the hamming distance [44]	$i = 1$	0.225	9	The method based on the Euclidean distance [44]	$i = 1$	0.194	10
	$i = 2$	0.207	10		$i = 2$	0.198	9
	$i = 3$	0.532	1		$i = 3$	0.423	3
	$i = 4$	0.186	11		$i = 4$	0.186	11
	$i = 5$	0.491	2		$i = 5$	0.496	2
	$i = 6$	0.270	7-8		$i = 6$	0.264	7
	$i = 7$	0.387	4		$i = 7$	0.362	4
	$i = 8$	0.277	6		$i = 8$	0.249	8
	$i = 9$	0.171	12		$i = 9$	0.181	12
	$i = 10$	0.331	5		$i = 10$	0.342	5
	$i = 11$	0.476	3		$i = 11$	0.576	1
	$i = 12$	0.270	7-8		$i = 12$	0.286	6

The fuzzy ratings of the relative importance of each pair of FPs are performed by financial experts. It may be assumed that managers of banking sector and main managers of enterprises have the same opinion of FPs' importance. This assumption is based on the fact that bank managers strive to have increasing number of clients so they could put more money on market. On the other hand, enterprise managers strive to decrease the risk of realization of business goals, which may be achieved by selection of the most appropriate bank.

The evaluation of FPs can be performed through predefined key performance indicators (KPIs) which are presented by the set of indices $\kappa_j = \{1, \ldots, k, \ldots, K_j\}$. The total number of KPIs of FP j, $j = 1, \ldots, J$, is K_j, and k is the index of KPI. The KPI values are determined according to financial reports. The KPIs values are given from reasonable financial report. In this paper, treated FIPs can be benefit type and cost type. These values have different units of measures. By using vector normalization procedure [33], the KPI values are mapped into interval [0-1]. The normalized value of each FP j, $j = 1, \ldots, J$, is calculated by using averaging operator.

Calculating of the elements of fuzzy decision matrix is based on using fuzzy algebra rules [5, 6]. These elements are given as product of vector weights of FPs and matrix of these normalized values.

FPIS and FNIS are determined according to the weighted normalized fuzzy decision matrix by using method for comparison of fuzzy numbers in [41, 42].

The distance from FPIS and FNIS is determined by using several different approaches presented in the literature. We considered separation measures which are proposed in [43, 44] and are based on hamming distance and Euclidean distance. These separation measures are applied for TFNs in the proposed model. In this respect, authors can cross-check the results and obtain the most representative ranking.

The closeness coefficient for each bank is calculated by using procedure of conventional TOPSIS [10]. Ranking order of all banks can be determined by using closeness coefficient.

4.1. The Proposed Algorithm. The algorithm of the proposed fuzzy TOPSIS method is carried out in the following steps.

Step 1. Calculate the vectors weight of FPs by applying modified procedure which is developed in [15]:

Order:

$$\alpha_j = \left[\prod_{j=1}^{J} l_{jj'}\right]^{1/J}, \quad \beta_j = \left[\prod_{j=1}^{J} m_{jj'}\right]^{1/J}, \quad \chi_j = \left[\prod_{j=1}^{J} u_{jj'}\right]^{1/J}, \quad j = 1, \ldots, J, \tag{1}$$

$$\alpha = \sum_{j=1}^{J} \alpha_j, \quad \beta = \sum_{j=1}^{J} \beta_j, \quad \chi = \sum_{j=1}^{J} \chi_j, \quad j = 1, \ldots, J.$$

Then the vectors weight of FPs, $\widetilde{W} = [\widetilde{w}_j]_{1\times J}$, can be obtained:

$$\widetilde{w}_j = \left(\alpha_j \cdot \chi^{-1}, \beta_j \cdot \beta^{-1}, \chi_j \cdot \alpha^{-1}\right) = \left(y; l_j, m_j, u_j\right), \quad j = 1, \ldots, J. \tag{2}$$

Step 2. Transform all crisp values of KPIs, z_{ik} into R_{ik}, whose domains are defined on a common scale [0, 1] by applying the vector normalization method [33]:

(a) For benefit type KPI k, $k = 1, \ldots, K$:

$$R_{ik} = \frac{z_{ik}}{\sqrt{\sum_{i=1}^{I} z_{ik}^2}}. \tag{3}$$

Table 6: Comparative analyses of the proposed FTOPSIS.

Methods	Application area	Attribute	Alternative	Separation measures	Solution methods
[28]	Supplier selection in supply chains	5	5, uncertain, modeled by TFNs, the linear normalization procedure [34]	Vertex method	The proposed aggregation method, FTOPSIS
[18]	Decision making model with fuzzy data	5	3, uncertain, modeled by TFNs, the linear normalization procedure [34]	Procedure is proposed in [36]	The fuzzy averaging method, FTOPSIS
[26]	Evaluation of hazardous waste	8	5, crisp, the vector normalization procedure [33]	n-dimensional Euclidean	FAHP, TOPSIS
[15]	Evaluation of performances in banking sector	13	5, crisp and uncertain, vector normalization procedure [33]	n-dimensional Euclidean	FAHP (handling by [30]), TOPSIS
[17]	Assessment of traffic police centers performance	4	30, uncertain, modeled by TrFNs, the linear normalization procedure [34]	Vertex method	The fuzzy averaging method, FTOPSIS
[19]	Illustrative example	2	10, uncertain, modeled by TFNs	Vertex method	FAHP (handling by [30]), FTOPSIS
[25]	Personnel selection	11	4, uncertain, modeled by TFNs, the linear normalization procedure [34]	Vertex method	The fuzzy averaging method, FTOPSIS
[20]	Energy planning	4	7, uncertain, modeled by TFNs, the linear normalization procedure [34]	Vertex method	FAHP (handling by [30]), FTOPSIS
[27]	Organizational strategy development in distribution channel management	4	5, uncertain, modeled by TFNs, a proposed normalization procedure [27]	A proposed procedure [27]	FAHP (handling by [30]), FTOPSIS
[21]	Ranking of quality goals	8	10, uncertain, modeled by TFNs, the linear normalization procedure [34]	Vertex method	FAHP (handling by [30]), FTOPSIS
[23]	Supplier selection of medical devices	5	7, crisp, linear normalization procedure; uncertain, modeled by TrFNs, the linear normalization procedure [25]	Procedure is proposed in [36]	Proposed method proposed in [28], FTOPSIS
[24]	Ranking of resilience factors	6	13, uncertain, modeled by TFNs, the linear normalization procedure [25]	Vertex method	FAHP (handling by [29]), FTOPSIS
The proposed model	Ranking of banks	5	12, crisp, the vector normalization procedure [24]	Vertex method Procedure is proposed in [36]. The extension of method based on the Hamming distance [34] The extension of method based on the Euclidean distance [34]	FAHP (handling by [29]), FTOPSIS, the sensitively analyses

(b) For cost type KPI k, $k = 1, \ldots, K$:

$$R_{ik} = \frac{1/z_{ik}}{\sqrt{\sum_{i=1}^{I} \left(1/z_{ik}\right)^2}}. \tag{4}$$

Step 3. Calculate the normalized values of FPs, r_{ij}, $i = 1, \ldots, I$, $j = 1, \ldots, J$:

$$r_{ij} = \sum_{k=1}^{K_j} R_{ik} \quad i = 1, \ldots, I; \; k = 1, \ldots, K; \; j = 1, \ldots, J. \tag{5}$$

Step 4. Construct the weighted normalized fuzzy decision matrix:

$$[\tilde{v}_{ij}]_{I\times J}, \tag{6}$$

where

$$\tilde{v}_{ij} = \tilde{w}_j \cdot r_{ij},$$
$$\tilde{v}_{ij} = (y; l_{ij}, m_{ij}, u_{ij}), \quad j = 1, \ldots, J;\ i = 1, \ldots, I. \tag{7}$$

Step 5. Identify FPIS, $\tilde{v}_j^+$, and FNIS, $\tilde{v}_j^-$, with respect to fuzzy decision matrix as

$$\tilde{v}_j^+ = (\tilde{v}_1^+, \ldots, \tilde{v}_j^+, \ldots, \tilde{v}_J^+),$$
$$\tilde{v}_j^- = (\tilde{v}_1^-, \ldots, \tilde{v}_j^-, \ldots, \tilde{v}_J^-), \tag{8}$$

where

$$\tilde{v}_j^+ = \max_{i=1,\ldots,I} \tilde{v}_{ij},$$
$$\tilde{v}_j^+ = (y; l_j^+, m_j^+ \cdot u_j^+),$$
$$\tilde{v}_j^- = \min_{i=1,\ldots,I} \tilde{v}_{ij}, \quad i = 1, \ldots, I;\ j = 1, \ldots, J,$$
$$\tilde{v}_j^- = (y; l_j^-, m_j^- \cdot u_j^-). \tag{9}$$

The values of $\tilde{v}_j^+$, $\tilde{v}_j^-$, $j = 1, \ldots, J$, are given by using method developed in [41, 42].

Step 6. Calculate separation measures: the distance of each treated bank from $\tilde{v}_j^+$ and $\tilde{v}_j^-$ and $d_i^+$and d_i^-, respectively.

Separation Measure Based on Vertex Method [22]. Consider the following:

$$d_i^+ = \sum_{j=1}^{J} \sqrt{\frac{1}{3} \cdot \left[(l_{ij} - l_j^+)^2 + (m_{ij} - m_j^+)^2 + (u_{ij} - u_j^+)^2 \right]},$$
$$d_i^- = \sum_{j=1}^{J} \sqrt{\frac{1}{3} \cdot \left[(l_{ij} - l_j^-)^2 + (m_{ij} - m_j^-)^2 + (u_{ij} - u_j^-)^2 \right]}. \tag{10}$$

Separation Measure Based on [43]. Consider the following:

$$d_i^+ = \sum_{j=1}^{J} \sqrt{\frac{1}{6} \cdot \left\{ (l_{ij} - l_j^+)^2 + 2 \cdot (m_{ij} - m_j^+)^2 + (u_{ij} - u_j^+)^2 + (m_j^+ - m_{ij}) \cdot \left[(l_j^+ - l_{ij}) + (u_j^+ - u_{ij}) \right] \right\}},$$
$$d_i^- = \sum_{j=1}^{J} \sqrt{\frac{1}{6} \cdot \left\{ (l_{ij} - l_j^+)^2 + 2 \cdot (m_{ij} - m_j^+)^2 + (u_{ij} - u_j^+)^2 + (m_{ij} - m_j^-) \cdot \left[(l_{ij} - l_j^-) + (u_{ij} - u_j^-) \right] \right\}}. \tag{11}$$

Separation Measure Based on the Hamming Distance [44]. Consider the following:

$$d_i^+ = \frac{1}{2} \cdot \sum_{j=1}^{J} \left[\max\left(|l_{ij} - l_j^+|, |m_{ij} - m_j^+| \right) + \max\left(|m_{ij} - m_j^+|, |u_{ij} - u_j^+| \right) \right],$$
$$d_i^- = \frac{1}{2} \cdot \sum_{j=1}^{J} \left[\max\left(|l_{ij} - l_j^-|, |m_{ij} - m_j^-| \right) + \max\left(|m_{ij} - m_j^-|, |u_{ij} - u_j^-| \right) \right]. \tag{12}$$

Separation Measure Based on the Euclidean Distance [44]. Consider the following:

$$d_i^+ = \sqrt{\frac{1}{2} \cdot \sum_{j=1}^{J} \left[\max\left(|l_{ij} - l_j^+|, |m_{ij} - m_j^+| \right)^2 + \max\left(|m_{ij} - m_j^+|, |u_{ij} - u_j^+| \right)^2 \right]},$$
$$d_i^- = \sqrt{\frac{1}{2} \cdot \sum_{j=1}^{J} \left[\max\left(|l_{ij} - l_j^-|, |m_{ij} - m_j^-| \right)^2 + \max\left(|m_{ij} - m_j^-|, |u_{ij} - u_j^-| \right)^2 \right]}. \tag{13}$$

Step 7. Calculate a closeness coefficient for each bank, c_i, $i = 1, \ldots, I$:

$$c_i = \frac{d_i^-}{d_i^- + d_i^+}. \quad (14)$$

Step 8. Using closeness coefficient, banks can be ranked in decreasing order.

5. Numerical Example of Bank Evaluation

Regardless of the world financial crisis and complex problems in home economy, banking system in Serbia may be considered as stable. A prompt recognition of business and other challenges of the environment and defining and maintaining certain strategies as a successful answer to those challenges improved the banking sector performances up to the level of sustainable development in the last decade.

Using analytical approach to consider market position and business results in the period 1/10/2010–31/10/2015, 12 banks were found to show leading position as far as sector structure is concerned. Only those banks having 80% share of the Serbian market are taken into consideration in this paper. Their business results show a significant influence on the overall financial results at the sector level.

Common KPIs (presented in Table 1) from financial reports from National bank of Serbia have been analyzed in the period of 1/10/2010–31/10/2015.

In compliance with the proposed algorithm, the following steps are presented. The fuzzy pairwise matrix of the relative importance of the considered FPs is stated:

$$\begin{bmatrix} (x;1,1,1) & \tilde{R}_2 & \tilde{R}_1 & \tilde{R}_4 & \tilde{R}_5 & \tilde{R}_3 \\ \dfrac{1}{\tilde{R}_2} & (x;1,1,1) & \dfrac{1}{\tilde{R}_2} & \tilde{R}_3 & \tilde{R}_4 & \tilde{R}_1 \\ \dfrac{1}{\tilde{R}_1} & \tilde{R}_2 & (x;1,1,1) & \tilde{R}_4 & \tilde{R}_5 & \tilde{R}_2 \\ \dfrac{1}{\tilde{R}_4} & \dfrac{1}{\tilde{R}_3} & \dfrac{1}{\tilde{R}_4} & (x;1,1,1) & \tilde{R}_1 & \dfrac{1}{\tilde{R}_2} \\ \dfrac{1}{\tilde{R}_5} & \dfrac{1}{\tilde{R}_4} & \dfrac{1}{\tilde{R}_5} & \dfrac{1}{\tilde{R}_1} & (x;1,1,1) & \dfrac{1}{\tilde{R}_2} \\ \dfrac{1}{\tilde{R}_3} & \dfrac{1}{\tilde{R}_1} & \dfrac{1}{\tilde{R}_2} & \tilde{R}_2 & \tilde{R}_2 & (x;1,1,1) \end{bmatrix}. \quad (15)$$

By using procedure for handling of FAHP defined in [43], the vectors weights of FP groups are

$$\begin{aligned} \widetilde{w}_1 &= (0.321, 0.300, 0.553), \\ \widetilde{w}_2 &= (0.118, 0.205, 0.395), \\ \widetilde{w}_3 &= (0.124, 0.250, 0.460), \\ \widetilde{w}_4 &= (0.037, 0.063, 0.135), \\ \widetilde{w}_5 &= (0.033, 0.056, 0.098), \\ \widetilde{w}_6 &= (0.057, 0.126, 0.252). \end{aligned} \quad (16)$$

The values of identified KPIs are presented in Table 2 as mean values of KPIs in the period of 1/10/2010–31/10/2015 from evidence of National bank of Serbia.

Applying procedures defined in Algorithm (Step 2 and Step 3), normalized decision matrix is obtained, as presented in Table 3.

Applying procedures defined in Algorithm (Step 4 and Step 5), fuzzy weighted normalized decision matrix is constructed, as presented in Table 4.

The following results are obtained, applying procedures shown in Step 6 to Step 8 of the developed model, and given in Table 5.

The results of the sensitivity analyses are showed in Table 5. The resolving closeness coefficient values are used in order to verify the proposed model. Considering the calculated closeness coefficient values, it is observed that various resolving closeness coefficient values almost do not affect the ranking order of the treated banks. Also, ranking order of banks slightly changes with separation measure. Finally, when separation measures based on vertex method are used [22], as well as separation measures based on the Hamming distance [44], bank ($i = 3$) is ranked at the first place. When other two separation measures are employed, bank ($i = 11$) is ranked at the first place. With respect to only FPs, there is only one most appropriate bank. According to obtained result, it can be concluded that it is necessary to analyze nonfinancial performances in order to determine the rank of banks.

With respect to bank rank, main manager and financial manager of each firm may choose bank ($i = 3$) or ($i = 11$) which presents partner of enterprise.

The next step of any investigations could be the comparison of the proposed model to other similar FTOPSIS which can be found in the literature. The presented FTOPSIS methods are very different compared with each other: research domains, the number of attributes and alternatives, determining of weights of attribute, alternative values, determining separation measures, and so forth. These main differences for comparative analyses are presented in Table 6.

Basic difference between methods which can be found in literature and the proposed model may be stated as follows: (1) FPIS and FNIS are determined by analogy conventional TOPSIS and (2) proposed method is verified by using different separation measures.

6. Conclusions

Changes that occur in the business environment, in financial markets, especially globalization, increase demand on continuous improvement of bank effectiveness. It can be achieved by applying performance measurement and managing them. Employment of analytic methods may be a good choice when solution of some issue needs to be found in the exact way because it is less burdened by subjective judgments of decision makers. The goal of presented paper is to introduce a fuzzy model for ranking of banks. This issue is very important and may be illustrated by the following facts: (1) appropriate improvement strategy which has key impact on achievement and maintaining of organizational sustainability and market position over time and (2) stakeholders, where the business processes are advanced.

The AHP framework is used for the rating of relative importance of FP. It can be assumed that decision makers have specific preferences and demand prerequisites in relation to profile of ideal solution. It is close to human way of thinking that decision makers used predefined linguistic expressions which are modeled in this paper by TFNs. Handling of fuzziness and uncertainties is based on using of geometric mean that enables making full advantage of the fuzzy sets theory. In the last part of this paper, the proposed FTOPSIS method is used for ranking of banks in terms of their FPs.

The method could be very useful for management of banks to improve their management processes and organizational structure, market position, and so forth; the main constraint of the proposed model is that it only deals with financial performances, although the illustrative example has shown that nonfinancial performances should be taken into account when banks need to be ranked. The flexibility of the model is reflected by the fact that all changes, such as number and type of performances and their importance, can be easily incorporated into it. The proposed fuzzy model is also suitable for software development. Also, it can be mentioned that proposed model can be easily extended to analyze other management decision problems in the different areas.

Further research will be focused on (1) extending of proposed model with nonfinancial performances, (2) developing software based on proposed model, and (3) developing and applying fuzzy version of other MCDM methods as ELECTREE and PROMETHEE and comparison of results obtained in this paper with the ones from other proposed fuzzy MCDM methods.

Competing Interests

The authors declare that they have no competing interests.

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